Homework 1: Due Wed 20 in TA class

1. A random variable X is defined to be the larger of the two values when two dice are thrown, or the value if the values are the same. Find the probability distribution for X. Find the expected value of X and variance of X

2. Here is a bivariate probability distribution of X, Y.

	X=1	X=2
Y=1	0.2	0.4
Y=2	0.3	0.1

The numbers in the table show the probability in each case. Are X and Y independent? Find the following: $\text{cov}(X,Y)$ $VAR(X+Y)$ and ρ_{XY}

3. Suppose a variable Y is an exact linear function of X: $Y = \lambda + \mu X$ where λ and μ are constants, and suppose that Z is a third variable. Show that $\rho_{XZ} = \rho_{YZ}$

4. An estimator is consistent if it is unbiased and has zero variance the number of observation is equal to infinity. A random variable X has unknown population mean μ_X and population variance σ_X^2. A sample of n observations $\{X_1, ..., X_n\}$ is generated. Show that Z = $X_1/2 + X_2/4 + X_3/8 + \dots$ is a biased but consistent estimator for μ_X.

5. A random variable X has a continuous uniform distribution over the interval from 0 to θ, where θ is an unknown parameter. The following three estimators are used to estimate θ, given a sample of n observations on X:
(a) twice the sample mean
(b) the largest value of X in the sample
Explain verbally whether or not each estimator is (1) unbiased (2) consistent.

6. Suppose there is an unfair coin with two faces: 0 and 1. The probability that face one shows up is ¼.
-A researcher who has just got the coin and don't know anything about it wants to estimate probability that face 1 shows up. Let Y_1, Y_2 are the number he gets from tossing the coin for the first time, second time and third time, respectively.
He uses the following 2 estimators:

$$m_1 = \frac{Y_1}{2} + \frac{Y_2}{2}, \ m_2 = \sqrt{Y_1 Y_2}$$

-On expectation (average), what is the value the researcher would get from the two estimators? Are the two estimators biased?
-Find the variance of the two estimators. From your point of view, which estimator is better?