Econometric Homework 3

Assumption for Regression Model

A0: X is non-random

A1: $Y_i = \beta_1 + \beta_2 X_i + u_i$

A2: Some Xi's are different.

A3: $E(u_i) = 0$

A4: $Var(u_i) = \sigma_u^2$

A5: $Cov(u_i, u_j) = 0$ for $i \neq j$

A6: u_i is normally distributed.

1. Let *X* be ages of Thai workers and *Y* be incomes of Thai workers. Suppose we have only two samples of (*X*, *Y*). Under assumptions A0-A6 of regression model **together** with the following additional assumption

$$\text{A7: } \beta_1 = 0 .$$

Let $b_2 = \dfrac{Y_1 - Y_2}{X_1 - X_2}$ and $c_2 = \dfrac{Y_1 + Y_2}{X_1 + X_2}$ are two estimators of β_2. Find $E(b_2), E(c_2), Var(b_2)$ and $Var(c_2)$. Can you decide whether b_2 or c_2 is a better estimator?

2.Suppose we have only 2 samples of (*X*, *Y*). Under assumptions A0-A6 of regression model together with the following assumption

$$\text{A7: } \beta_1 = 0 .$$

Show that $b = c_1 Y_1 + c_2 Y_2$ is the minimum-variance linear unbiased estimator for β_2 if $c_1 = \dfrac{X_1}{X_1^2 + X_2^2}$ and $c_2 = \dfrac{X_2}{X_1^2 + X_2^2}$.

by the following steps:

1) Show that unbiasedness implies $c_1 X_1 + c_2 X_2 = 1$.

2) Show that $Var(b) = \sigma_u^2 (c_1^2 + c_2^2)$

3) Using *calculus* to choose c_1 and c_2 that minimizes $V(b)$ under the condition that $c_1 X_1 + c_2 X_2 = 1$. Formally your minimization problem is:

$$\min_{c_1, c_2} \sigma_u^2 (c_1^2 + c_2^2)$$

$$\text{subject to } c_1 X_1 + c_2 X_2 = 1 .$$

3) Suppose Y = $\beta_1 + \beta_2 X_i + u_i$. β_1 and β_2 are constant. u_i is independently random and has the following probability distribution $prob(u_i = 1) = prob(u_i = -1) = \frac{1}{2}$. An economist has 2 samples of (X,Y). He estimates β_2 using $b_2 = \frac{Y_2 - Y_1}{X_1 - X_2}$. Find the probability that b_2 will be exactly equal to β_2.

4. Use the data set provided on the website to answer the following question. The data set provides the information about the average of gold price (in US$) in each year, consumer price index (CPI) and New York Stock Exchange (NYSE) Index. Use the data set to answer the following questions:

1. Apply the OLS method to estimate the models:
   - Goldprice = α_1 + α_2NYSE (1)
   - Goldprice = β_1 + β_2CPI (2)
2. Interpret the estimation results of both (1) and (2).
3. Choose only one model, between (1) and (2), which one is the best? Why so?
4. One claims that the average of gold price has 1-1 positive relationship with the CPI; precisely, the one unit increasing of CPI implies one US$ increasing of the average of gold price. Is it true? Why?