Chapter 7 The Cost of Production

Read Pindyck and Rubinfeld (2013), Chapter 7

CHAPTER 7 OUTLINE

7.1 Measuring Cost: Which Costs Matter?

- **Economic Cost versus Accounting Cost**

- **accounting cost** Actual expenses plus depreciation charges for capital equipment.
- **economic cost** Cost to a firm of utilizing economic resources in production.

Opportunity Cost

- **opportunity cost** Cost associated with opportunities forgone when a firm's resources are not put to their best alternative use.

The concept of opportunity cost is particularly useful in situations where alternatives that are forgone do not reflect monetary outlays.

Economic cost = Opportunity cost

EXAMPLE 7.1 CHOOSING THE LOCATION FOR A NEW LAW SCHOOL BUILDING

The Northwestern University Law School has long been located in Chicago, along the shores of Lake Michigan. However, the main campus of the university is located in the suburb of Evanston. In the mid-1970s, the law school began planning the construction of a new building.

The downtown location had many prominent supporters. They argued in part that it was cost-effective to locate the new building in the city because the university already owned the land. A large parcel of land would have to be purchased in Evanston if the building were to be built there.

Does this argument make economic sense? No. It makes the common mistake of failing to appreciate opportunity cost. From an economic point of view, it is very expensive to locate downtown because the opportunity cost of the valuable lakeshore location is high: That property could have been sold for enough money to buy the Evanston land with substantial funds left over.

In the end, Northwestern decided to keep the law school in Chicago. This was a costly decision. It may have been appropriate if the Chicago location was particularly valuable to the law school, but it was inappropriate if it was made on the presumption that the downtown land had no cost.

Sunk Costs

- **sunk cost** Expenditure that has been made and cannot be recovered.

Because a sunk cost cannot be recovered, it should not influence the firm's decisions.

For example, consider the purchase of specialized equipment for a plant. Suppose the equipment can be used to do only what it was originally designed for and cannot be converted for alternative use. The expenditure on this equipment is a sunk cost. *Because it has no alternative use, its opportunity cost is zero.* Thus it should not be included as part of the firm's economic costs.

A *prospective* sunk cost is an *investment.* Here the firm must decide whether that investment in specialized equipment is *economical.*

Fixed Costs and Variable Costs

- **total cost (TC or C)** Total economic cost of production, consisting of fixed and variable costs.

- **fixed cost (FC)** Cost that does not vary with the level of output and that can be eliminated only by shutting down.

- **variable cost (VC)** Cost that varies as output varies.

The only way that a firm can eliminate its fixed costs is by shutting down.

Fixed Costs and Variable Costs

Shutting Down

Shutting down doesn't necessarily mean going out of business.

By reducing the output of a factory to zero, the company could eliminate the costs of raw materials and much of the labor. The only way to eliminate fixed costs would be to close the doors, turn off the electricity, and perhaps even sell off or scrap the machinery.

Fixed or Variable?

How do we know which costs are fixed and which are variable?

Over a very short time horizon—say, a few months—most costs are fixed.

Over such a short period, a firm is usually obligated to pay for contracted shipments of materials.

Over a very long time horizon—say, ten years—nearly all costs are variable.

Workers and managers can be laid off (or employment can be reduced by attrition), and much of the machinery can be sold off or not replaced as it becomes obsolete and is scrapped.

- **Fixed versus Sunk Costs**

Shutting down doesn't necessarily mean going out of business. Fixed costs can be avoided if the firm shuts down a plant or goes out of business.

Sunk costs, on the other hand, are costs that have been incurred and *cannot be recovered*.

When a firm's equipment is too specialized to be of use in any other industry, most if not all of this expenditure is sunk, i.e., cannot be recovered.

Why distinguish between fixed and sunk costs? Because fixed costs affect the firm's decisions looking forward, whereas sunk costs do not. Fixed costs that are high relative to revenue and cannot be reduced might lead a firm to shut down—eliminating those fixed costs and earning zero profit might be better than incurring ongoing losses. Incurring a high sunk cost might later turn out to be a bad decision (for example, the unsuccessful development of a new product), but the expenditure is gone and cannot be recovered by shutting down. Of course a *prospective* sunk cost is different and, as we mentioned earlier, would certainly affect the firm's decisions looking forward.

AMORTIZING SUNK COSTS

- **amortization** Policy of treating a one-time expenditure as an annual cost spread out over some number of years.

Amortizing large capital expenditures and treating them as ongoing fixed costs can simplify the economic analysis of a firm's operation. As we will see, for example, treating capital expenditures this way can make it easier to understand the tradeoff that a firm faces in its use of labor versus capital.

For simplicity, we will usually treat sunk costs in this way as we examine the firm's production decisions. When distinguishing sunk from fixed costs does become essential to the economic analysis, we will let you know.

EXAMPLE 7.2 SUNK, FIXED, AND VARIABL E COSTS: COMPUTERS, SOFTWARE, AND PIZZAS

It is important to understand the characteristics of production costs and to be able to identify which costs are fixed, which are variable, and which are sunk.

Good examples include the personal computer industry (where most costs are variable), the computer software industry (where most costs are sunk), and the pizzeria business (where most costs are fixed).

Because computers are very similar, competition is intense, and profitability depends on the ability to keep costs down. Most important are the cost of components and labor.

A software firm will spend a large amount of money to develop a new application. The company can recoup its investment by selling as many copies of the program as possible.

For the pizzeria, sunk costs are fairly low because equipment can be resold if the pizzeria goes out of business. Variable costs are low—mainly the ingredients for pizza and perhaps wages for a workers to produce and deliver pizzas.

Marginal and Average Cost

Marginal Cost (MC)

- **marginal cost (MC)** Increase in cost resulting from the production of one extra unit of output.

Because fixed cost does not change as the firm's level of output changes, marginal cost is equal to the increase in variable cost or the increase in total cost that results from an extra unit of output.

We can therefore write marginal cost as

$$\mathrm{MC} = \Delta\mathrm{VC}/\Delta q = \Delta\mathrm{TC}/\Delta q$$

Marginal and Average Cost

Average Total Cost (ATC)

- **average total cost (ATC)** Firm's total cost divided by its level of output.
- **average fixed cost (AFC)** Fixed cost divided by the level of output.
- **average variable cost (AVC)** Variable cost divided by the level of output.

Marginal and Average Cost

Marginal Cost (MC)

TABLE 7.1 A Firm's Costs

Rate of Output (Units per Year)	Fixed Cost (Dollars per Year)	Variable Cost (Dollars per Year)	Total Cost (Dollars per Year)	Marginal Cost (Dollars per Unit)	Average Fixed Cost (Dollars per Unit)	Average Variable Cost (Dollars per Unit)	Average Total Cost (Dollars per Unit)
	(FC) (1)	(VC) (2)	(TC) (3)	(MC) (4)	(AFC) (5)	(AVC) (6)	(ATC) (7)
0	50	0					
1	50	50					
2	50	78					
3	50	98					
4	50	112					
5	50	130	180	18	10	26	36
6	50	150	200	20	8.3	25	33.3
7	50	175	225	25	7.1	25	32.1
8	50	204	254	29	6.3	25.5	31.8
9	50	242	292	38	5.6	26.9	32.4
10	50	300	350	58	5	30	35
11	50	385	435	85	4.5	35	39.5

Marginal and Average Cost

Marginal Cost (MC)

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Rate of Output (Units per Year)	Fixed Cost (Dollars per Year) (FC) (1)	Variable Cost (Dollars per Year) (VC) (2)	Total Cost (Dollars per Year) (TC) (3)	Marginal Cost (Dollars per Unit) (MC) (4)	Average Fixed Cost (Dollars per Unit) (AFC) (5)	Average Variable Cost (Dollars per Unit) (AVC) (6)	Average Total Cost (Dollars per Unit) (ATC) (7)
0	50	0	50	--	--	--	--
1	50	50	100	50	50	50	100
2	50	78	128	28	25	39	64
3	50	98	148	20	16.7	32.7	49.3
4	50	112	162	14	12.5	28	40.5
5	50	130	180	18	10	26	36
6	50	150	200	20	8.3	25	33.3
7	50	175	225	25	7.1	25	32.1
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E1. Joe quits his computer programming job, where he was earning a salary of $50,000 per year, to start his own computer software business in a building that he owns and was previously renting out for $24,000 per year. In his first year of business he has the following expenses, salary paid to himself, $40,000; rent, $0; other expenses, $25,000. Find the accounting cost and the economic cost associated with Joe's computer software business.

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ANS. The accounting cost includes only the explicit expenses, which are Joe's salary and his other expenses: $40,000 + 25,000 = $65,000. Economic cost includes these explicit expenses plus opportunity costs. Therefore, economic cost includes the $24,000 Joe gave up by not renting the building and an extra $10,000 because he paid himself a salary $10,000 below market ($50,000 – 40,000). Economic cost is then $40,000 + 25,000 + 24,000 + 10,000 = $99,000.

2. The owner of a small retail store does her own accounting work. How would you measure the opportunity cost of her work?

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ANS. The economic, or opportunity, cost of doing accounting work is measured by computing the monetary amount that the owner's time would be worth in its *next best use*. For example, if she could do accounting work for some other company instead of her own, her opportunity cost is the amount she could have earned in that alternative employment. Or if she is a great stand-up comic, her opportunity cost is what she could have earned in that occupation instead of doing her own accounting work.

5. A recent issue of Business Week reported the following: During the recent auto sales slump, GM, Ford, and Chrysler decided it was cheaper to sell cars to rental companies at a loss than to lay off workers. That's because closing and reopening plants is expensive, partly because the auto makers' current union contracts obligate them to pay many workers even if they're not working. When the article discusses selling cars "at a loss," is it referring to accounting profit or economic profit? How will the two differ in this case? Explain briefly.

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ANS. When the article refers to the car companies selling at a loss, it is referring to accounting profit. The article is stating that the price obtained for the sale of the cars to the rental companies was less than their accounting cost. Economic profit would be measured by the difference between the price and the opportunity cost of producing the cars. One major difference between accounting and economic cost in this case is the cost of labor. If the car companies must pay many workers even if they are not working, the wages paid to these workers are sunk. If the automakers have no alternative use for these workers (like doing repairs on the factory or preparing the companies' tax returns), the opportunity cost of using them to produce the rental cars is zero. Since the wages would be included in accounting costs, the accounting costs would be higher than the economic costs and would make the accounting profit lower than the economic profit..

Q3. Please explain whether the following statements are true or false.

a) If the owner of a business pays himself no salary, then the accounting cost is zero, but the economic cost is positive.

b) A firm that has positive accounting profit does not necessarily have positive economic profit.

c) If a firm hires a currently unemployed worker, the opportunity cost of utilizing the worker's services is zero.

Q3. Please explain whether the following statements are true or false.

a) If the owner of a business pays himself no salary, then the accounting cost is zero, but the economic cost is positive.

ANS. True. Since there is no monetary transaction, there is no accounting, or explicit, cost. However, since the owner of the business could be employed elsewhere, there is an economic cost. The economic cost is positive, reflecting the opportunity cost of the owner's time. The economic cost is the value of the owner's time in his next best alternative, or the amount that the owner would earn if he took the next best job.

b) A firm that has positive accounting profit does not necessarily have positive economic profit.

ANS. True. Accounting profit considers only the explicit, monetary costs. Since there may be some opportunity costs that were not fully realized as explicit monetary costs, it is possible that when the opportunity costs are added in, economic profit will become negative. This indicates that the firm's resources are not being put to their best use.

c) If a firm hires a currently unemployed worker, the opportunity cost of utilizing the worker's services is zero.

ANS. False. From the firm's point of view, the wage paid to the worker is an explicit cost whether she was previously unemployed or not. The firm's opportunity cost is equal to the wage, because if it did not hire this worker, it would have had to hire someone else at the same wage. The opportunity cost from the worker's point of view is the value of her time, which is unlikely to be zero. By taking this job, she cannot work at another job or take care of a child or elderly person at home.
If her best alternative is working at another job, she gives up the wage she would have earned.
If her best alternative is unpaid, such as taking care of a loved one, she will now have to pay someone else to do that job, and the amount she has to pay is her opportunity cost.

4. Suppose a firm must pay an annual tax, which is a fixed sum, independent of whether it produces any output.

a. How does this tax affect the firm's fixed, marginal, and average costs?

b. Now suppose the firm is charged a tax that is proportional to the number of items it produces. Again, how does this tax affect the firm's fixed, marginal, and average costs?

4. Suppose a firm must pay an annual tax, which is a fixed sum, independent of whether it produces any output.

a. How does this tax affect the firm's fixed, marginal, and average costs?

ANS. This tax is a fixed cost because it does not vary with the quantity of output produced. If T is the amount of the tax and F is the firm's original fixed cost, the new total fixed cost increases to $TFC = T + F$. The tax does not affect marginal or variable cost because it does not vary with output. The tax increases both average fixed cost and average total cost by T/q.

b. Now suppose the firm is charged a tax that is proportional to the number of items it produces. Again, how does this tax affect the firm's fixed, marginal, and average costs?

ANS. Let t equal the per unit tax. When a tax is imposed on each unit produced, variable cost increases by tq and fixed cost does not change. Average variable cost increases by t, and because fixed costs are constant, average total cost also increases by t. Further, because total cost increases by t for each additional unit produced, marginal cost increases by t.

The Determinants of Short-Run Cost

The change in variable cost is the per-unit cost of the extra labor w times the amount of extra labor needed to produce the extra output ΔL. Because $\Delta VC = w\Delta L$, it follows that

$$MC = \Delta VC/\Delta q = w\Delta L/\Delta q$$

The extra labor needed to obtain an extra unit of output is $\Delta L/\Delta q = 1/MP_L$. As a result,

$$MC = w/MP_L \qquad (7.1)$$

Diminishing Marginal Returns and Marginal Cost

Diminishing marginal returns means that the marginal product of labor declines as the quantity of labor employed increases.

As a result, when there are diminishing marginal returns, marginal cost will increase as output increases.

The Shapes of the Cost Curves

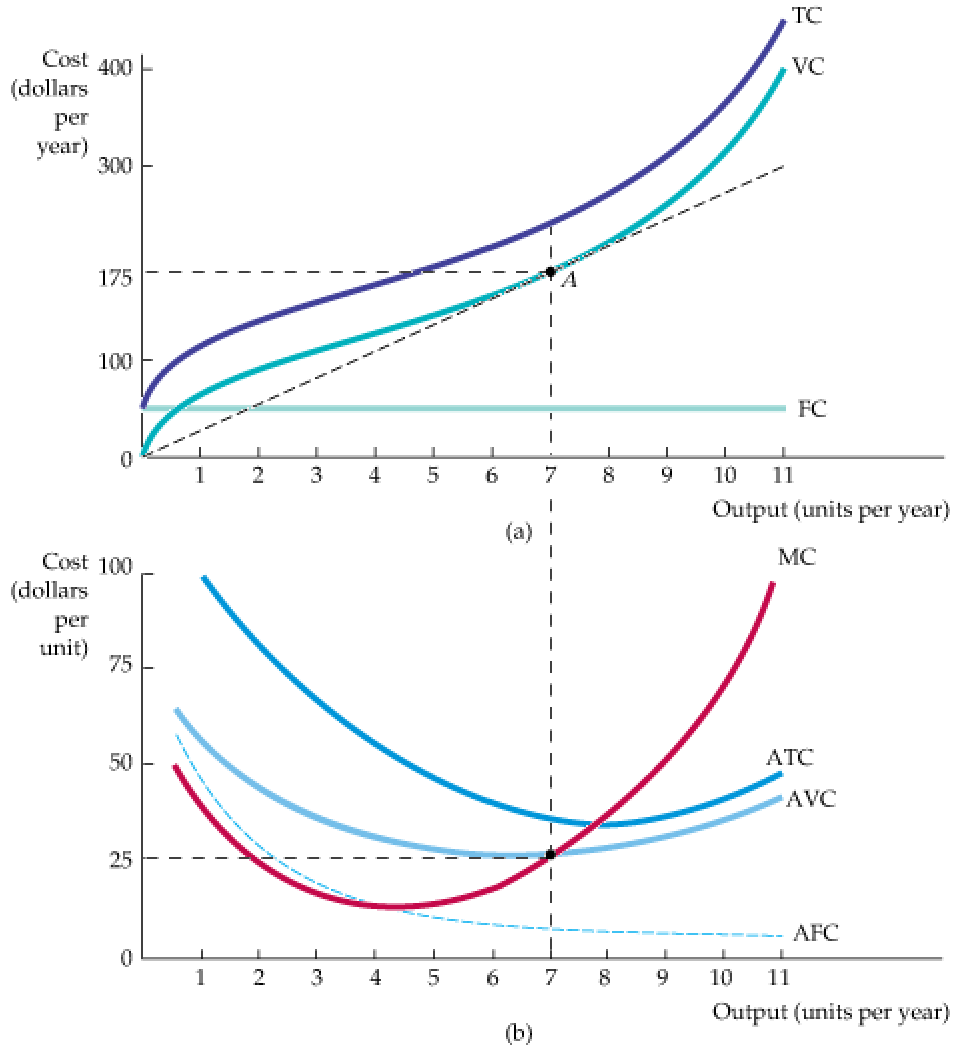
Figure 7.1

Cost Curves for a Firm

In **(a)** total cost TC is the vertical sum of fixed cost FC and variable cost VC.

In **(b)** average total cost ATC is the sum of average variable cost AVC and average fixed cost AFC.

Marginal cost MC crosses the average variable cost and average total cost curves at their minimum points.



THE AVERAGE-MARGINAL RELATIONSHIP

Marginal and average costs are another example of the average-marginal relationship described in Chapter 6 (with respect to marginal and average product).

Because average total cost is the sum of average variable cost and average fixed cost and the AFC curve declines everywhere, the vertical distance between the ATC and AVC curves decreases as output increases.

TOTAL COST AS A FLOW

Total cost is a flow: the firm produces a certain number of units *per year*. Thus its total cost is a flow—for example, some number of dollars per year. For simplicity, we will often drop the time reference, and refer to total cost in dollars and output in units.

Knowledge of short-run costs is particularly important for firms that operate in an environment in which demand conditions fluctuate considerably. If the firm is currently producing at a level of output at which marginal cost is sharply increasing, and if demand may increase in the future, management might want to expand production capacity to avoid higher costs.

E9. The short-run cost function of a company is given by the equation TC = 200 + 55*q*, where TC is the total cost and *q* is the total quantity of output, both measured in thousands.

a) What is the company's fixed cost?

b) If the company produced 100,000 units of goods, what would be its average variable cost?

c) What would be its marginal cost of production?

d) What would be its average fixed cost?

e) Suppose the company borrows money and expands its factory. Its fixed cost rises by $50,000, but its variable cost falls to $45,000 per 1000 units. The cost of interest (*i*) also enters into the equation. Each 1-point increase in the interest rate raises costs by $3,000. Write the new cost equation.

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a) What is the company's fixed cost?

ANS. When $q = 0$, $TC = 200$, so fixed cost is equal to 200 (or \$200,000).

b) If the company produced 100,000 units of goods, what would be its average variable cost?

ANS. With 100,000 units, $q = 100$. Variable cost is $55q = (55)(100) = 5500$ (or \$5,500,000). Average variable cost is $\frac{TVC}{q} = \frac{\$5500}{100} = \$55$, or \$55,000.

c) What would be its marginal cost of production?

ANS. With constant average variable cost, marginal cost is equal to average variable cost, \$55 (or \$55,000).

d) What would be its average fixed cost?

ANS. At $q = 100$, average fixed cost is $\frac{TFC}{q} = \frac{\$200}{100} = \$2$ or (\$2,000).

e) Suppose the company borrows money and expands its factory. Its fixed cost rises by \$50,000, but its variable cost falls to \$45,000 per 1000 units. The cost of interest (*i*) also enters into the equation. Each 1-point increase in the interest rate raises costs by \$3,000. Write the new cost equation.

ANS. Fixed cost changes from 200 to 250, measured in thousands. Variable cost decreases from 55 to 45, also measured in thousands. Fixed cost also includes interest charges: $3i$. The cost equation is

$$TC = 250 + 45q + 3i.$$

The User Cost of Capital

- **user cost of capital** Annual cost of owning and using a capital asset, equal to economic depreciation plus forgone interest.

The user cost of capital is given by the *sum of the economic depreciation and the interest (i.e., the financial return) that could have been earned had the money been invested elsewhere.* Formally,

$$\text{User Cost of Capital} = \text{Economic Depreciation} + (\text{Interest Rate})(\text{Value of Capital})$$

We can also express the user cost of capital as a *rate* per dollar of capital:

$$r = \text{Depreciation rate} + \text{Interest rate}$$

Example: Purchase of a Boeing 777 for $150 million

Life of the airplane = 30 years
Annual Amortized cost is $5 million per year
Interest rate = 10%

user cost of capital = economic depreciation + foregone interest

In the first year:
user cost of capital = economic depreciation + (interest rate)(value of capital)
= *$5 million + (0.10)($150 million)*
= *$20 million*

In the tenth year:
user cost of capital = economic depreciation + (interest rate)(value of capital)
= *$5 million + (0.10)($100 million)*
= *$15 million*

We can also express the user cost of capital as a *rate* per dollar of capital:

r = depreciation rate + interest rate

r = 1/30+0.10
= 13.33%

The Cost-Minimizing Input Choice

We now turn to a fundamental problem that all firms face: *how to select inputs to produce a given output at minimum cost.*

For simplicity, we will work with two variable inputs: labor (measured in hours of work per year) and capital (measured in hours of use of machinery per year).

The Price of Capital

The price of capital is its *user cost*, given by r = Depreciation rate + Interest rate.

The Rental Rate of Capital

- **rental rate** Cost per year of renting one unit of capital.

If the capital market is competitive, *the rental rate should be equal to the user cost, r.* Why? Firms that own capital expect to earn a competitive return when they rent it. *This competitive return is the user cost of capital.*

Capital that is purchased can be treated as though it were rented at a rental rate equal to the user cost of capital.

• The Isocost Line

- **isocost line** Graph showing all possible combinations of labor and capital that can be purchased for a given total cost.

To see what an isocost line looks like, recall that the total cost *C* of producing any particular output is given by the sum of the firm's labor cost *wL* and its capital cost *rK:*

$$C = wL + rK \tag{7.2}$$

If we rewrite the total cost equation as an equation for a straight line, we get

$$K = C/r - (w/r)L$$

It follows that the isocost line has a slope of $\Delta K/\Delta L = -(w/r)$, which is the ratio of the wage rate to the rental cost of capital.

The Isocost Line

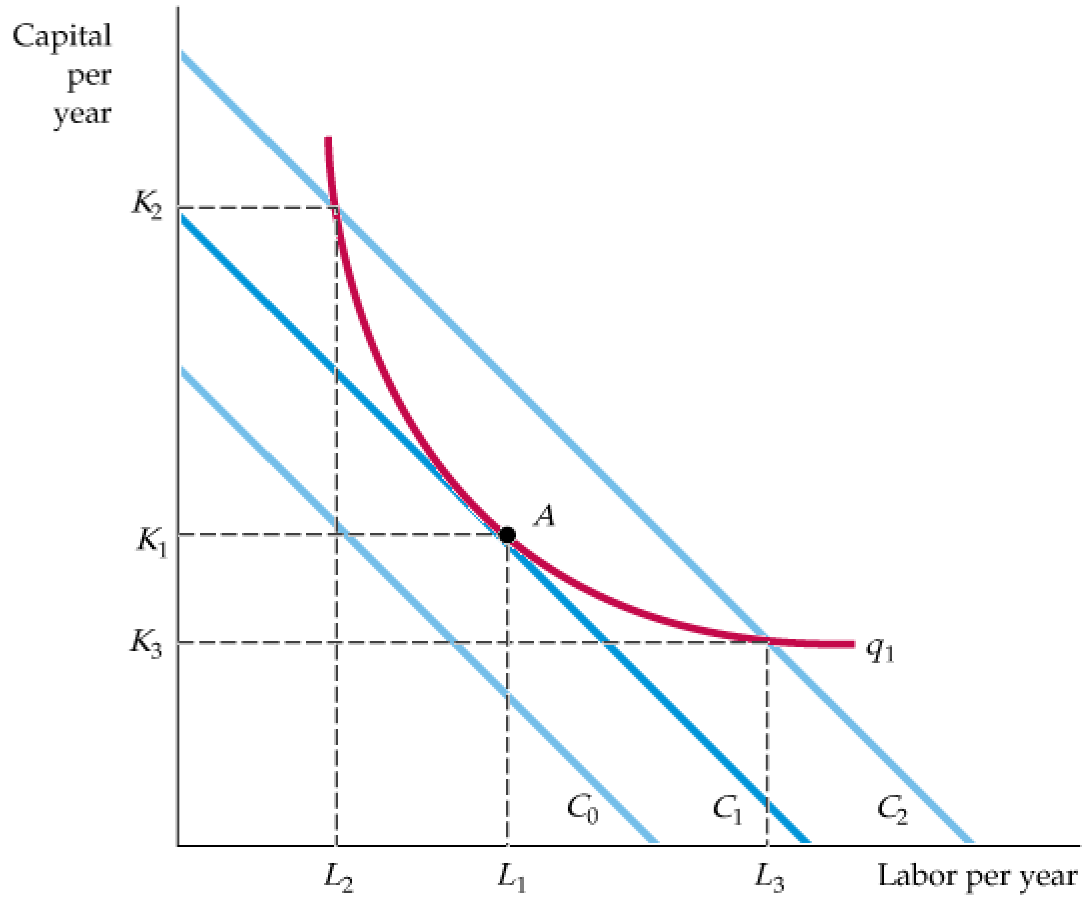
Figure 7.3

Producing a Given Output at Minimum Cost

Isocost curves describe the combination of inputs to production that cost the same amount to the firm.

Isocost curve C_1 is tangent to isoquant q_1 at A and shows that output q_1 can be produced at minimum cost with labor input L_1 and capital input K_1.

Other input combinations–L_2, K_2 and L_3, K_3–yield the same output but at higher cost.



Choosing Inputs

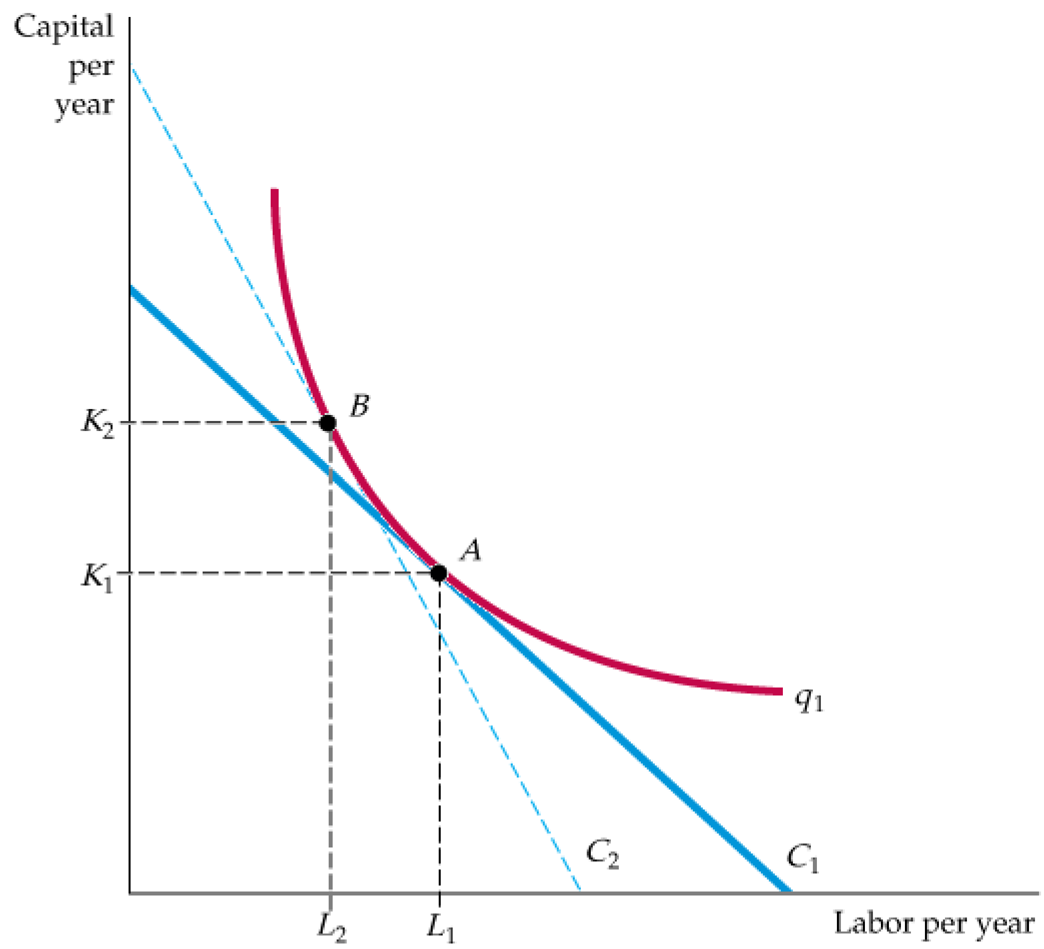
Figure 7.4

Input Substitution When an Input Price Changes

Facing an isocost curve C_1, the firm produces output q_1 at point A using L_1 units of labor and K_1 units of capital.

When the price of labor increases, the isocost curves become steeper.

Output q_1 is now produced at point B on isocost curve C_2 by using L_2 units of labor and K_2 units of capital.



Recall that in our analysis of production technology, we showed that the marginal rate of technical substitution of labor for capital (MRTS) is the negative of the slope of the isoquant and is equal to the ratio of the marginal products of labor and capital:

$$\mathrm{MRTS} = -\Delta K/\Delta L = \mathrm{MP}_L/\mathrm{MP}_K \qquad \textbf{(7.3)}$$

It follows that when a firm minimizes the cost of producing a particular output, the following condition holds:

$$\mathrm{MP}_L/\mathrm{MP}_K = w/r$$

We can rewrite this condition slightly as follows:

$$\boxed{\mathrm{MP}_L/w = \mathrm{MP}_K/r \qquad \textbf{(7.4)}}$$

EXAMPLE 7.4 THE EFFECT OF EFFLUENT FEES ON INPUT CHOICES

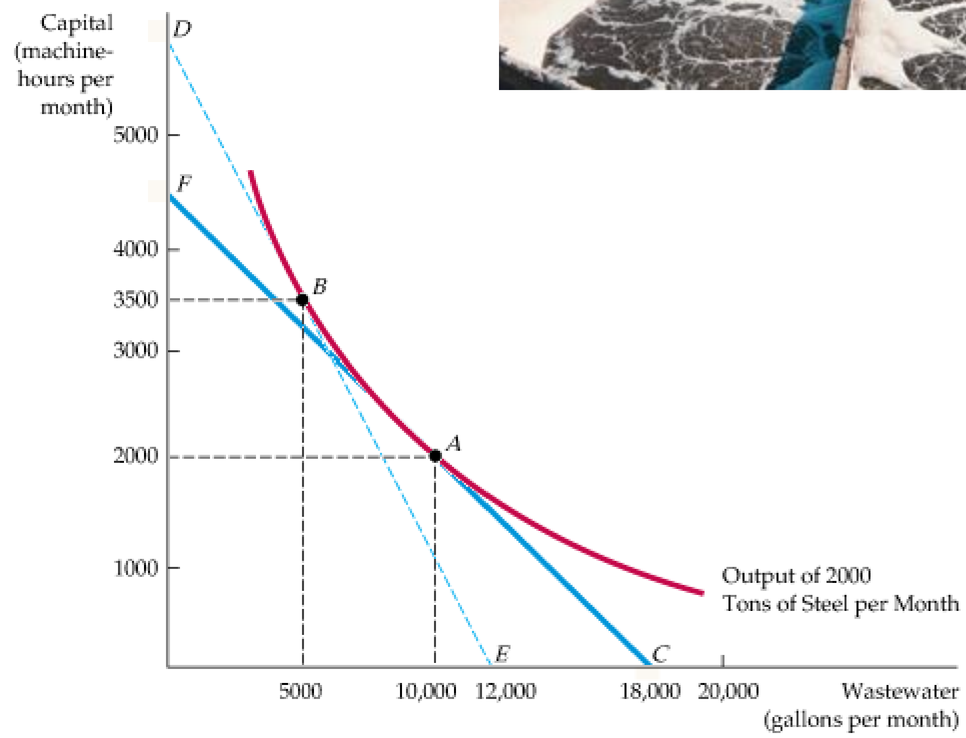
An effluent fee is a per-unit fee that the steel firm must pay for the effluent that goes into the river.

FIGURE 7.5

THE COST-MINIMIZING RESPONSE TO AN EFFLUENT FEE

When the firm is not charged for dumping its wastewater in a river, it chooses to produce a given output using 10,000 gallons of wastewater and 2000 machine-hours of capital at *A*.

However, <u>an effluent fee raises the cost of wastewater, shifts the isocost curve from *FC* to *DE*</u>, and causes the firm to produce at *B*—a process that results in much less effluent.



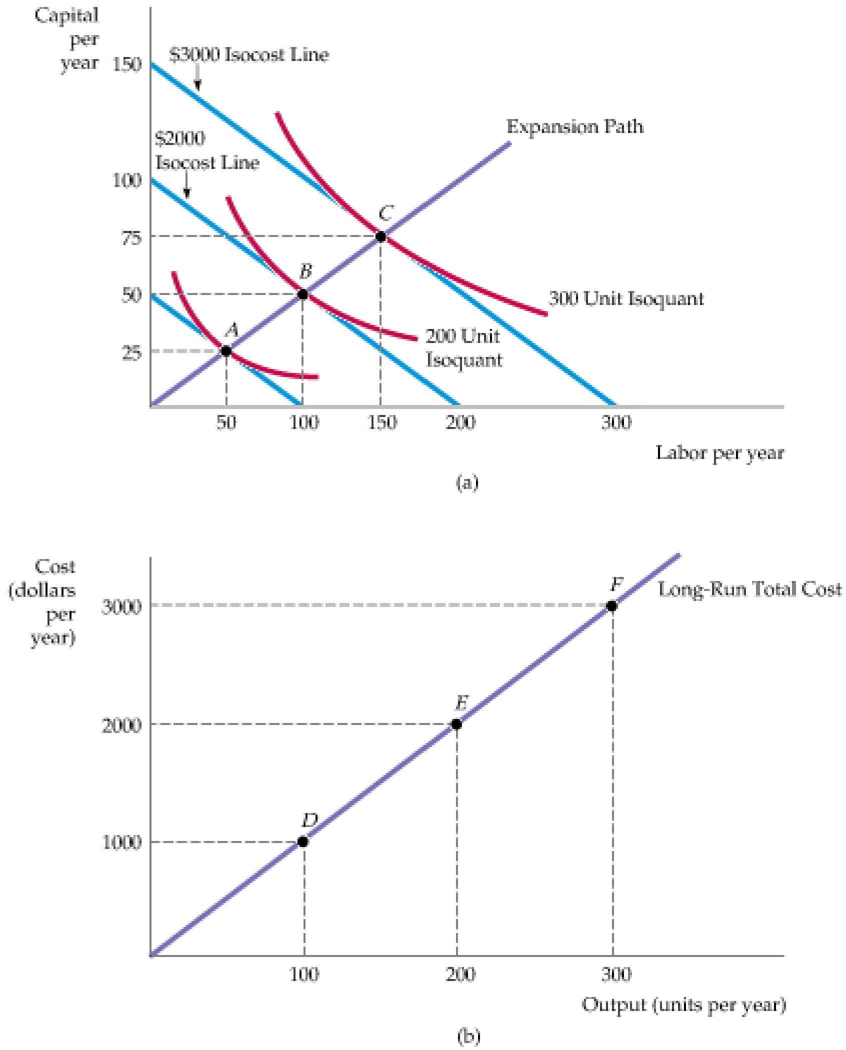
Cost Minimization with Varying Output Levels

Figure 7.6

A Firm's Expansion Path and Long-Run Total Cost Curve

In **(a)**, the expansion path (from the origin through points *A*, *B*, and *C*) illustrates the lowest-cost combinations of labor and capital that can be used to produce each level of output in the long run—i.e., when both inputs to production can be varied.

In **(b)**, the corresponding long-run total cost curve (from the origin through points *D*, *E*, and *F*) measures the least cost of producing each level of output.



Cost Minimization with Varying Output Levels

- **expansion path** Curve passing through points of tangency between a firm's isocost lines and its isoquants.

The firm can hire labor L at w = \$10/hour and rent a unit of capital K for r = \$20/hour. Given these input costs, we have drawn three of the firm's isocost lines. Each isocost line is given by the following equation:

$$C = (\$10/\text{hour})(L) + (\$20/\text{hour})(K)$$

The expansion path is a straight line with a slope equal to

$$\Delta K/\Delta L = (50 - 25)/(100 - 50) = \frac{1}{2}$$

The Expansion Path and Long-Run Costs

To move from the expansion path to the cost curve, we follow three steps:

1. Choose an output level represented by an isoquant. Then find the point of tangency of that isoquant with an isocost line.
2. From the chosen isocost line, determine the minimum cost of producing the output level that has been selected.
3. Graph the output-cost combination.

The Inflexibility of Short-Run Production

Figure 7.7

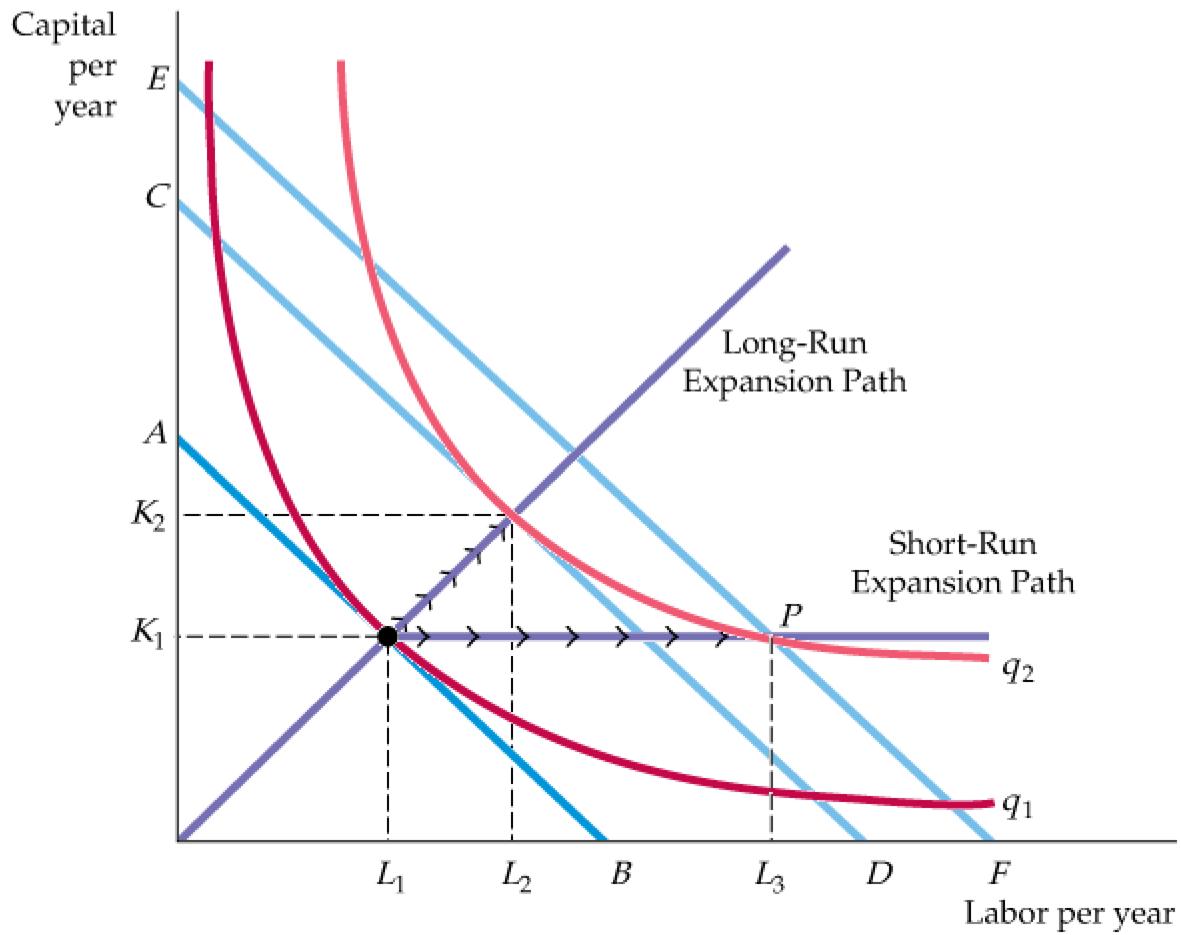
The Inflexibility of Short-Run Production

When a firm operates in the short run, its cost of production may not be minimized because of inflexibility in the use of capital inputs.

Output is initially at level q_1.

In the short run, output q_2 can be produced only by increasing labor from L_1 to L_3 because capital is fixed at K_1.

In the long run, the same output can be produced more cheaply by increasing labor from L_1 to L_2 and capital from K_1 to K_2.



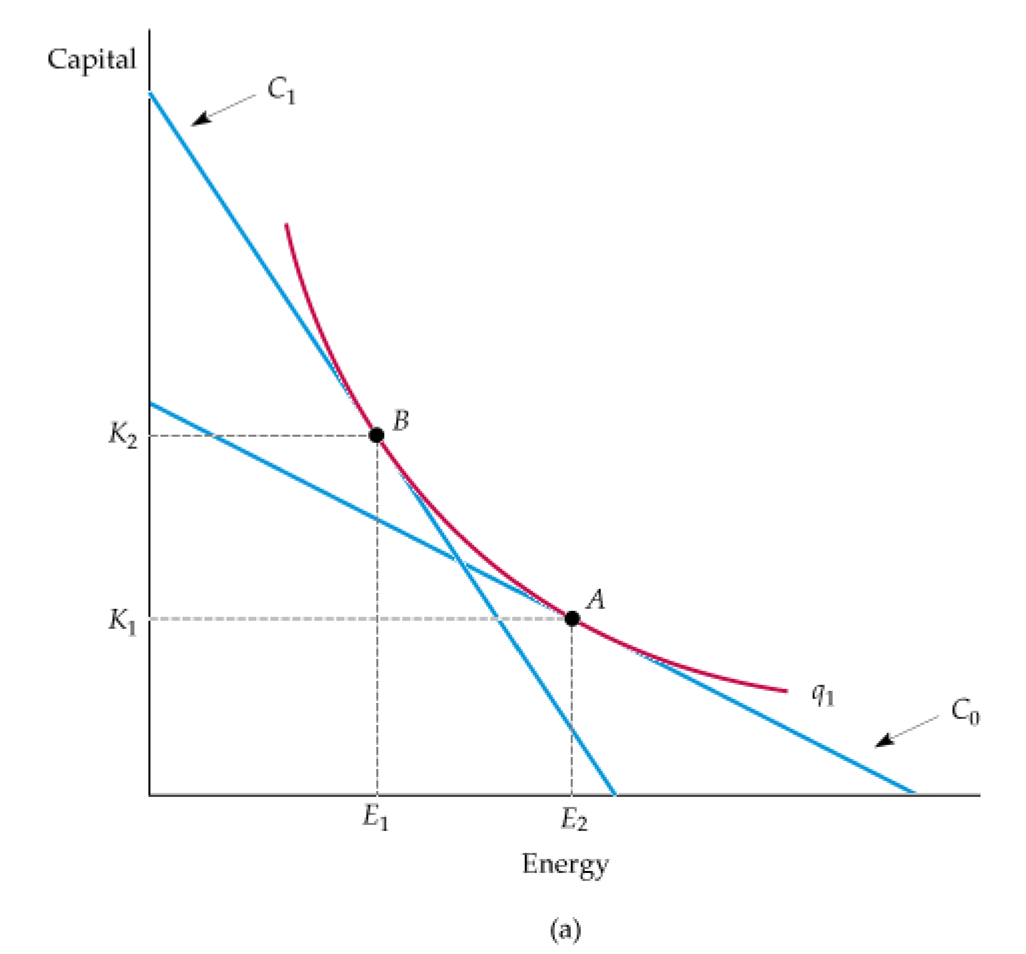
EXAMPLE 7.5 **REDUCING THE USE OF ENERGY**

FIGURE 7.7a

ENERGY EFFICIENCY THROUGH CAPITAL SUBSTITUTION FOR LABOR

Greater energy efficiency can be achieved if capital is substituted for energy.

This is shown as a movement along isoquant q_1 from point A to point B, with capital increasing from K_1 to K_2 and energy decreasing from E_2 to E_1 in response to a shift in the isocost curve from C_0 to C_1.

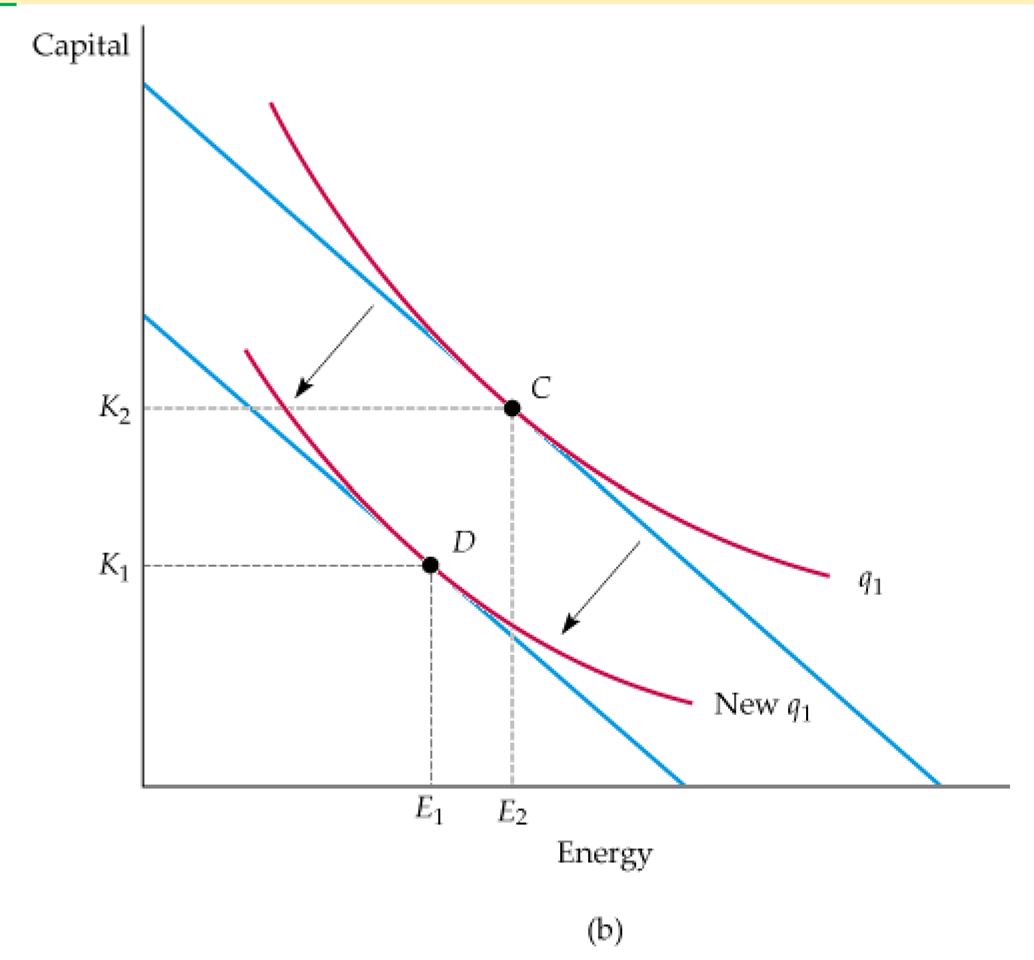


EXAMPLE 7.5 REDUCING THE USE OF ENERGY

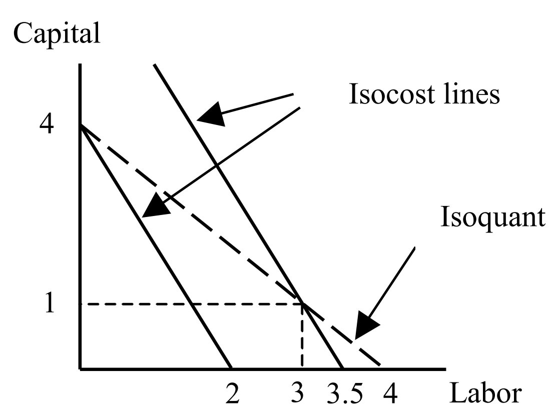
FIGURE 7.7b

ENERGY EFFICIENCY THROUGH TECHNOLOGICAL CHANGE

Technological change implies that the same output can be produced with smaller amounts of inputs. Here the isoquant labeled q_1 shows combinations of energy and capital that will yield output q_1; the tangency with the isocost line at point C occurs with energy and capital combinations E_2 and K_2. Because of technological change the isoquant shifts inward, so the same output q_1 can now be produced with less energy and capital, in this case at point D, with energy and capital combination E_1 and K_1.



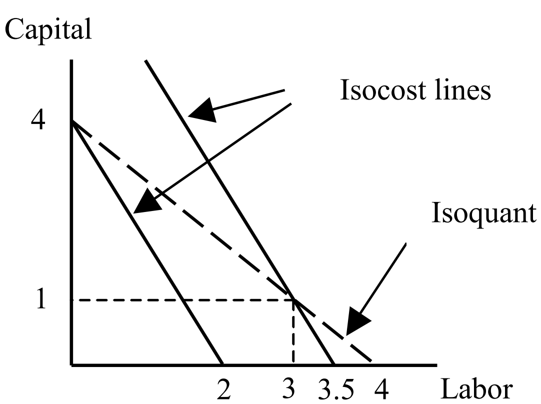
10. A chair manufacturer hires its assembly-line labor for $30 an hour and calculates that the rental cost of its machinery is $15 per hour. Suppose that a chair can be produced using 4 hours of labor or machinery in any combination. If the firm is currently using 3 hours of labor for each hour of machine time, is it minimizing its costs of production? If so, why? If not, how can it improve the situation? Graphically illustrate the isoquant and the two isocost lines for the current combination of labor and capital and for the optimal combination of labor and capital.



10. A chair manufacturer hires its assembly-line labor for \$30 an hour and calculates that the rental cost of its machinery is \$15 per hour. Suppose that a chair can be produced using 4 hours of labor or machinery in any combination. If the firm is currently using 3 hours of labor for each hour of machine time, is it minimizing its costs of production? If so, why? If not, how can it improve the situation? Graphically illustrate the isoquant and the two isocost lines for the current combination of labor and capital and for the optimal combination of labor and capital.

ANS. If the firm can produce one chair with either four hours of labor or four hours of machinery (i.e., capital), or any combination, then the isoquant is a straight line with a slope of -1 and intercepts at $K = 4$ and $L = 4$, as depicted by the dashed line.

The isocost lines, $TC = 30L + 15K$, have slopes of -30/15 = -2 when plotted with capital on the vertical axis and intercepts at $K = TC/15$ and $L = TC/30$. The cost minimizing point is the corner solution where $L = 0$ and $K = 4$, so the firm is not currently minimizing its costs. At the optimal point, total cost is \$60. Two isocost lines are illustrated on the graph. The first one is further from the origin and represents the current higher cost (\$105) of using 3 labor and 1 capital. The firm will find it optimal to move to the second isocost line which is closer to the origin, and which represents a lower cost (\$60). In general, the firm wants to be on the lowest isocost line possible, which is the lowest isocost line that still intersects the given isoquant.



E11. Suppose that a firm's production function is $q = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$. The cost of a unit of labor is \$20 and the cost of a unit of capital is \$80.

a) The firm is currently producing 100 units of output and has determined that the cost-minimizing quantities of labor and capital are 20 and 5, respectively. Graphically illustrate this using isoquants and isocost lines.

b) The firm now wants to increase output to 140 units. If capital is fixed in the short run, how much labor will the firm require? Illustrate this point graphically and find the firm's new total cost.

c) Graphically identify the cost-minimizing level of capital and labor in the long run if the firm wants to produce 140 units.

d) If the marginal rate of technical substitution is K/L, find the optimal level of capital and labor required to produce the 140 units of output.

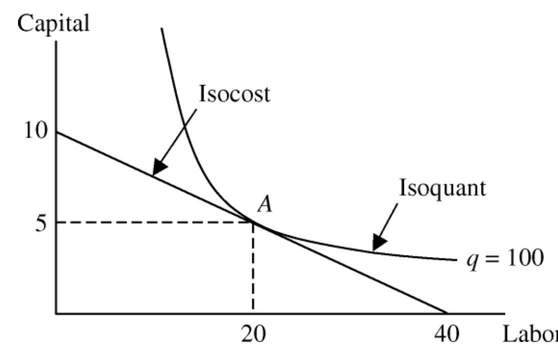
E11. Suppose that a firm's production function is $q = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$. The cost of a unit of labor is \$20 and the cost of a unit of capital is \$80.

a) The firm is currently producing 100 units of output and has determined that the <u>cost-minimizing</u> quantities of labor and capital are 20 and 5, respectively. Graphically illustrate this using isoquants and isocost lines.

ANS. To graph the isoquant, set $q = 100$ in the production function and solve it for K. Solving for K:

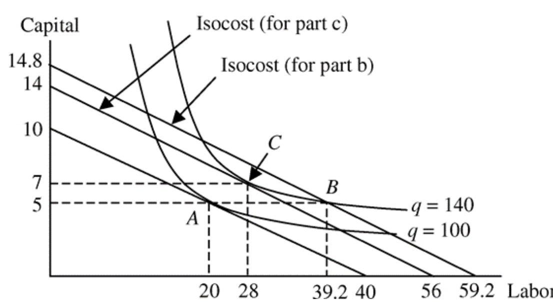
$K^{1/2} = \dfrac{q}{10L^{1/2}}$ Substitute 100 for q and square both sides. The isoquant is $K = 100/L$.

Choose various combinations of L and K and plot them. The isoquant is convex. The optimal quantitiesof labor and capital are given by the point where the isocost line is tangent to the isoquant. The isocost line has a slope of -1/4, given labor is on the horizontal axis. The total cost is TC = (\$20)(20) + (\$80)(5) = \$800, so the isocost line has the equation $20L + 80K = 800$, or $K = 10 - 0.25L$, with intercepts $K = 10$ and $L = 40$. The optimal point is labeled A on the graph.



b) The firm now wants to increase output to 140 units. If capital is fixed in the short run, how much labor will the firm <u>require</u>? Illustrate this point graphically and find the firm's new total cost.

ANS. The new level of labor is 39.2. To find this, use the production function $q = 10L^{\frac{1}{2}}K^{\frac{1}{2}}$ and substitute 140 for output and 5 for capital; then solve for L. The new cost is TC = (\$20)(39.2) + (\$80)(5) = \$1184. The new isoquant for an output of 140 is above and to the right of the original isoquant. Since capital is fixed in the short run, the firm will move out horizontally to the new isoquant and new level of labor. This is point B on the graph below. This is not the long-run cost- minimizing point, but it is the best the firm can do in the short run with K fixed at 5. You can tell that this is not the long-run optimum because the isocost is not tangent to the isoquant at point B. Also there are points on the new ($q = 140$) isoquant that are below the new isocost (for part b) line. These points all involve hiring more capital and less labor.



c) Graphically identify the cost-minimizing level of capital and labor in the long run if the firm wants to produce 140 units.

ANS. This is point *C* on the graph above. When the firm is at point *B* it is not minimizing cost. The firm will find it optimal to hire more capital and less labor and move to the new lower isocost (for part c) line that is tangent to the q = 140 isoquant. Note that all three isocost lines are parallel and have the same slope.

d) If the marginal rate of technical substitution is K/L, find the optimal level of capital and labor required to produce the 140 units of output.

ANS. Set the marginal rate of technical substitution equal to the ratio of the input costs so that $\frac{K}{L} = \frac{20}{80} \Rightarrow K = \frac{L}{4}$. Now substitute this into the production function for *K*, set *q* equal to 140, and solve for *L*: $140 = 10L^{\frac{1}{2}}\left(\frac{L}{4}\right)^{\frac{1}{2}} \Rightarrow L = 28,\ K = 7$. This is point *C* on the graph. The new cost is

$TC = (\$20)(28) + (\$80)(7) = \$1120$, which is less than in the short run (part b), because the firm can adjust all its inputs in the long run.

Long-Run Average Cost

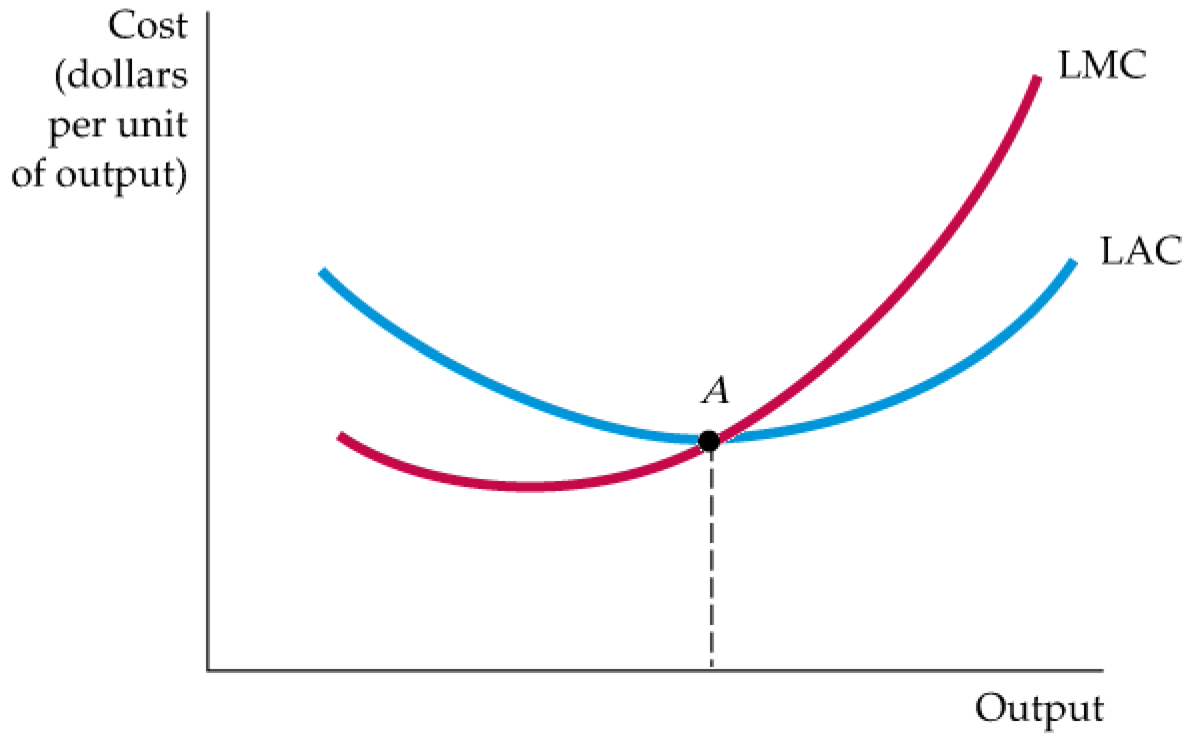
Figure 7.8

Long-Run Average and Marginal Cost

When a firm is producing at an output at which the long-run average cost LAC is falling, the long-run marginal cost LMC is less than LAC.

Conversely, when LAC is increasing, LMC is greater than LAC.

The two curves intersect at *A*, where the LAC curve achieves its minimum.



Long-Run Average Cost

- **long-run average cost curve (LAC)** Curve relating average cost of production to output when all inputs, including capital, are variable.

- **short-run average cost curve (SAC)** Curve relating average cost of production to output when level of capital is fixed.

- **long-run marginal cost curve (LMC)** Curve showing the change in long-run total cost as output is increased incrementally by 1 unit.

Economies and Diseconomies of Scale

- **economies of scale** Situation in which output can be doubled for less than a doubling of cost.

- **diseconomies of scale** Situation in which a doubling of output requires more than a doubling of cost.

Increasing Returns to Scale:	Output more than doubles when the quantities of all inputs are doubled..
Economies of Scale:	A doubling of output requires less than a doubling of cost.

- **Economies of Scale**

As output increases, the firm's average cost of producing that output is likely to decline, at least to a point.

This can happen for the following reasons:

1. If the firm operates on a larger scale, <u>workers can specialize</u> in the activities at which they are most productive.
2. <u>Scale can provide flexibility</u>. By varying the combination of inputs utilized to produce the firm's output, managers can <u>organize the production process more effectively</u>.
3. The firm may be able to acquire some production inputs at lower cost because it is <u>buying them in large quantities</u> and can therefore negotiate better prices. The mix of inputs might change with the scale of the firm's operation if managers take advantage of lower-cost inputs.

• Diseconomies of Scale

At some point, however, it is likely that the average cost of production will begin to increase with output.

There are three reasons for this shift:

1. At least in the short run, <u>factory space and machinery</u> may make it more difficult for workers to do their jobs effectively.

2. Managing a larger firm may become <u>more complex and inefficient</u> as the number of tasks increases.

3. The advantages of buying in bulk may have disappeared once certain quantities are reached. At some point, <u>available supplies of key inputs</u> may be limited, pushing their costs up.

Economies and Diseconomies of Scale

Economies of scale are often measured in terms of a cost-output elasticity, E_C. E_C is the percentage change in the cost of production resulting from a 1-percent increase in output:

$$E_C = (\Delta C / C) / (\Delta q / q) \qquad (7.5)$$

To see how EC relates to our traditional measures of cost, rewrite the equation as follows:

$$E_C = (\Delta C / \Delta q) / (C / q) = \mathrm{MC} / \mathrm{AC} \qquad (7.6)$$

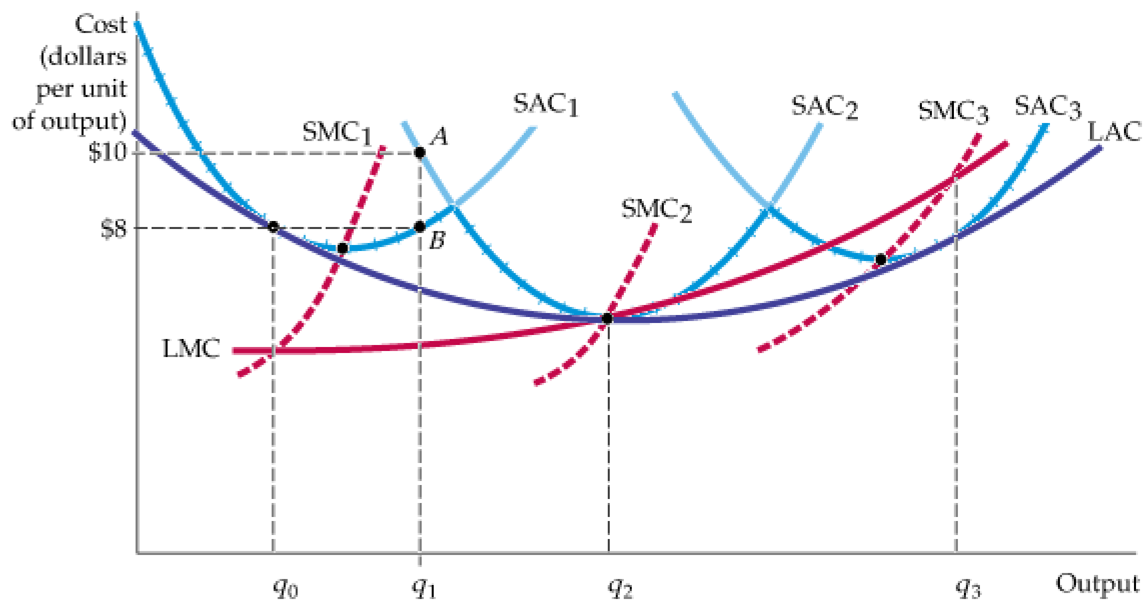
EC < 1 economies to scale
EC > 1 diseconomies to scale

The Relationship Between Short-Run and Long-Run Cost

Figure 7.9

Long-Run Cost with Economies and Diseconomies of Scale

The long-run average cost curve LAC is the envelope of the short-run average cost curves SAC_1, SAC_2, and SAC_3. With economies and diseconomies of scale, the minimum points of the short-run average cost curves do not lie on the long-run average cost curve.



Product Transformation Curves

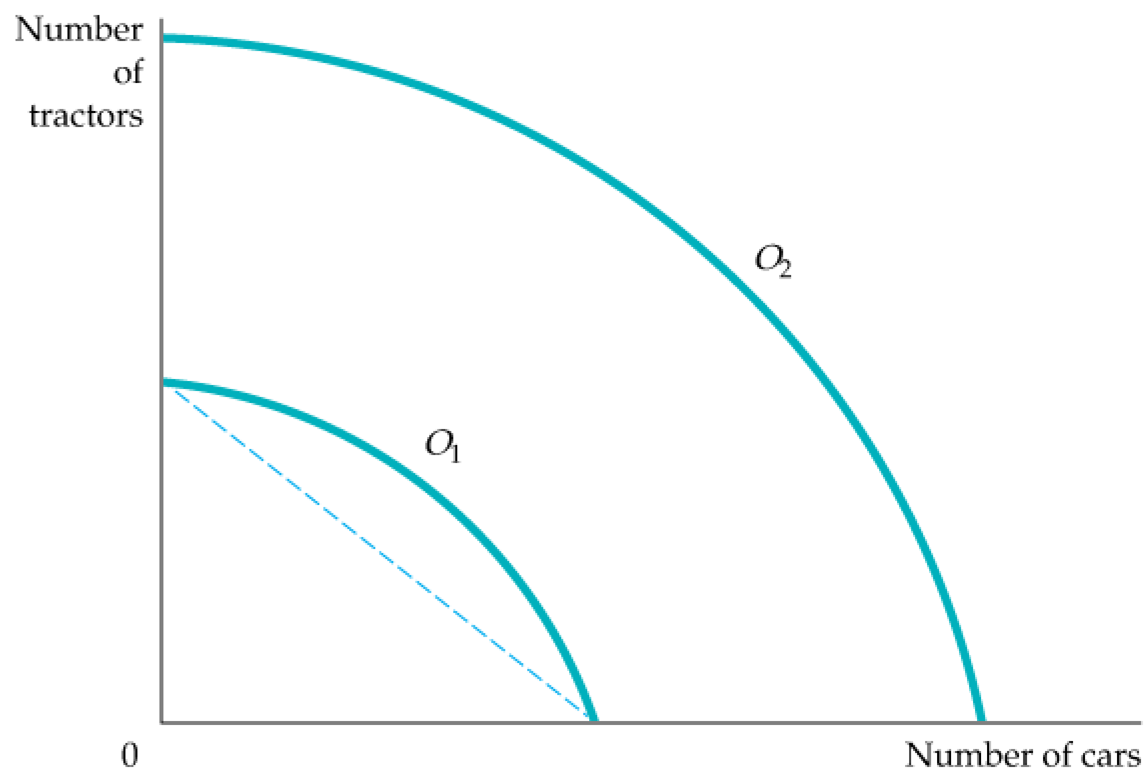
- **product transformation curve** Curve showing the various combinations of two different outputs (products) that can be produced with a given set of inputs.

Figure 7.10

Product Transformation Curve

The product transformation curve describes the different combinations of two outputs that can be produced with a fixed amount of production inputs.

The product transformation curves O_1 and O_2 are bowed out (or concave) because there are economies of scope in production.



7.5

PRODUCTION WITH TWO OUTPUTS—ECONOMIES OF SCOPE

- **Economies and Diseconomies of Scope**

● **economies of scope** Situation in which joint output of a single firm is greater than output that could be achieved by two different firms when each produces a single product.

● **diseconomies of scope** Situation in which joint output of a single firm is less than could be achieved by separate firms when each produces a single product.

The Degree of Economies of Scope

To measure the *degree* to which there are economies of scope, we should ask what percentage of the cost of production is saved when two (or more) products are produced jointly rather than individually.

$$SC = \frac{C(q_1) + C(q_2) - C(q_1, q_2)}{C(q_1, q_2)} \quad (7.7)$$

- **degree of economies of scope (SC)**
  Percentage of cost savings resulting when two or more products are produced jointly rather than Individually.

- **SC > 0 economies of scope**
  SC < 0 diseconomies of scope

EXAMPLE 7.6 ECONOMIES OF SCOPE IN THE TRUCKING INDUSTRY

In the trucking business, several related products can be offered, depending on the size of the load and the length of the haul. This range of possibilities raises questions about both economies of scale and economies of scope.

The scale question asks whether large-scale, direct hauls are more profitable than individual hauls by small truckers. The scope question asks whether a large trucking firm enjoys cost advantages in operating direct quick hauls and indirect, slower hauls.

Because large firms carry sufficiently large truckloads, there is usually no advantage to stopping at an intermediate terminal to fill a partial load.

Because other disadvantages are associated with the management of very large firms, the economies of scope get smaller as the firm gets bigger.

The study suggests, therefore, that to compete in the trucking industry, a firm must be large enough to be able to combine loads at intermediate stopping points.

E14. A computer company produces hardware and software using the same plant and labor. The total cost of producing computer processing units H and software programs S is given by

$$TC = aH + bS - cHS$$

where *a*, *b*, and *c* are positive. Is this total cost function consistent with the presence of economies or diseconomies of scale? With economies or diseconomies of scope?

E14. A computer company produces hardware and software using the same plant and labor. The total cost of producing computer processing units H and software programs S is given by

$$TC = aH + bS - cHS$$

where *a*, *b*, and *c* are positive. Is this total cost function consistent with the presence of economies or diseconomies of scale? With economies or diseconomies of scope?

ANS. If each product were produced by itself there would be neither economies nor diseconomies of scale. To see this, define the total cost of producing H alone (TC_H) to be the total cost when $S = 0$. Thus $TC_H = aH$. Similarly, $TC_S = bS$. In both cases, doubling the number of units produced doubles the total cost, so there are no economies or diseconomies of scale.

Economies of scope exist if $S_C > 0$, where, from equation (7.7) in the text:

$$SC = \frac{C(q_1) + C(q_2) - C(q_1, q_2)}{C(q_1, q_2)}$$

In our case, $C(q_1)$ is TC_H, $C(q_2)$ is TC_S, and $C(q_1,q_2)$ is TC. Therefore,

$$SC = \frac{aH + bS - (aH + bS - cHS)}{aH + bS - cHS} = \frac{cHS}{aH + bS - cHS}$$

Because cHS (the numerator) and TC (the denominator) are both positive, it follows that $SC > 0$, and there are economies of scope.

RECAP: CHAPTER 7

- Measuring Cost: Which Costs Matter?
- Cost in the Short Run
- Cost in the Long Run
- Long-Run versus Short-Run Cost Curves
- Production with Two Outputs—Economies of Scope