



# 8. Qualitative Information Binary or Dummy Variables

Read Wooldridge (2013), Chapter 8

# Outline

- I. Describing Qualitative Information
- II. A Single Dummy Independent Variable
- III. Using Dummy Variable for Multiple Categories
- IV. Interactions Involving Dummy Variables
- V. Chow Test

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# I. Describing Qualitative Information

- Quantitative vs. Qualitative Variables
  - Quantitative** - *wage, educ, sales, exper, tenure, profit*
  - Qualitative** - gender, race, or policy evaluation
- Examples: male/female; married/single; part time/full time; capital intensive/labor intensive
- Qualitative factors often come in the form of binary information – zero or one (0 or 1)
  - female* = 1    Otherwise = 0
  - married* = 1    Otherwise = 0

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**Table 7.1**  
A Partial Listing of the Data in WAGE1.RAW

<i>person</i>	<i>wage</i>	<i>educ</i>	<i>exper</i>	<i>female</i>	<i>married</i>
1	3.10	11	2	1	0
2	3.24	12	22	1	1
3	3.00	11	2	0	0
4	6.00	8	44	0	1
5	5.30	12	7	0	1
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-
525	11.56	16	5	0	1
526	3.50	14	5	1	0

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## II. A Single Dummy Independent Variable



- Consider a model

$$wage = \beta_0 + \delta_0 female + u$$

binary variable

$female = 1$ , if female

$female = 0$ , otherwise (if male)

- Comparing mean wages of male and female

$$E(wage | female=1) = \beta_0 + \delta_0 \quad (female=1)$$

$$E(wage | female=0) = \beta_0 \quad (female=0)$$

$$\delta_0 = E(wage | female) - E(wage | male)$$

$\delta_0$  : the difference in hourly wage between female and **male** (the base group)

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Example: Comparison-of-means-test between two groups:

$$wage = \beta_0 + \delta_0 female + u$$



$$w\hat{a}ge = 7.099 - 2.51female$$

s.e. (.210) (.3034)

t-stat [33.81] [-8.28]

n=526  $R^2=.115667$   $R^2\text{-bar} = .113979$

- 1) What is  $\hat{\delta}_0$ ?
- 2) What is the average hourly wage for male and female?

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Example: add *educ* to the model:

$$wage = \beta_0 + \delta_0 female + \beta_1 educ + u$$



$$w\hat{a}ge = .6228 - 2.27female + .50652educ$$

s.e. (.672) (.2790) (.05391)

[t-stat] [0.527] [-3.15] [10.05]

n=526  $R^2=.2588$   $R^2\text{-bar}=.2559$

Interpretation:

- 1) What is  $\hat{\beta}_0$ ?
- 2) What is  $\hat{\delta}_0$ ?
- 3) What is  $\hat{\beta}_1$ ?
- 4) Graph with different intercepts.

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Different Intercept but same slope



$$E(wage | female = 1, educ) = \beta_0 + \delta_0 + \beta_1 educ$$

$$= -1.6472 + .5065educ$$

$$E(wage | female=0, educ) = \beta_0 + \beta_1 educ$$

$$= .6228 + .5065educ$$

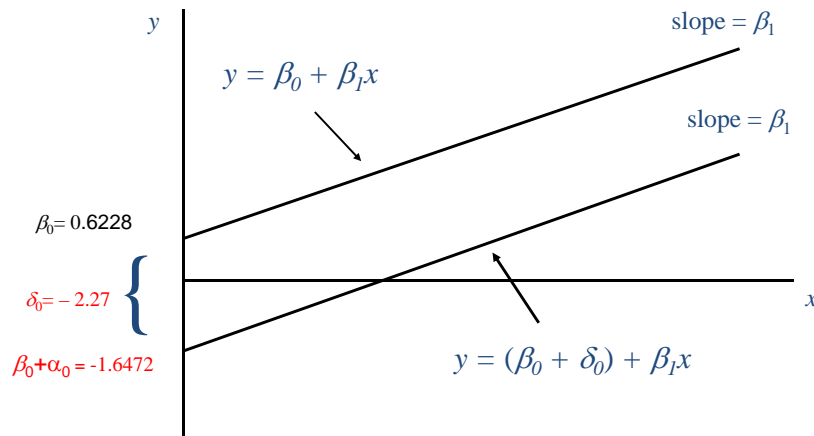
Can you test whether the mean wage for female with high school diploma is equal to zero?

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## Example of $\delta_0 < 0$



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Can we include both *male* and *female* in the model?

No! problem of perfect collinearity (dummy variable trap)

$$\widehat{wage} = \beta_0(1) + \gamma_0 \text{female} + \beta_1 \text{educ} + u$$

$$\therefore \text{male} + \text{female} = 1$$

- Different Base group  

$$\widehat{wage} = \alpha_0 + \gamma_0 \text{male} + \beta_1 \text{educ} + u$$

$$\text{male} = 1 \quad \text{if male}$$

$$= 0 \quad \text{otherwise (if female)}$$

$$\gamma_0 = E(\widehat{wage} | \text{male}=1, \text{educ}) - E(\widehat{wage} | \text{male}=0, \text{educ})$$

- What is  $\gamma_0$ ?

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Dependent variable :  $\log(y)$

$$\log(\widehat{wage}) = \hat{\beta}_0 + \hat{\delta}_0 \text{female} + \hat{\beta}_1 \text{educ} + \hat{\beta}_2 \text{exper} + \hat{\beta}_3 \text{tenure}$$

$$= .501 - .301 \text{female} + .087 \text{educ} + .0046 \text{exper} + .0174 \text{tenure}$$

t-stat [4.92] [-8.09] [12.6] [2.85] [5.84]



- Interpret the coefficient of female ( $\hat{\delta}_0$ ).  
 Women earn about 30.1% less than men, for the same levels of *educ*, *exper*, and *tenure*.

- Approximate vs. Exact proportionate change

$$\log(\text{wage}_F) - \log(\text{wage}_M) = -0.301$$

$$[\text{wage}_F / \text{wage}_M - 1] \approx [\exp(-0.301) - 1] = -0.2599$$

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Regress  $\log(\text{wage})$  on *female*, *educ*, *exper*, and *tenure*



Dependent Variable: LOG(WAGE)				
Sample: 1 526				
Included observations: 526				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.501348	0.101902	4.919885	0
FEMALE	-0.30115	0.037246	-8.085414	0
EDUC	0.087462	0.006939	12.60463	0
EXPER	0.004629	0.001627	2.845087	0.0046
TENURE	0.017367	0.002976	5.835235	0
R-squared	0.39227	Mean dependent var	1.623268	
Adjusted R-squared	0.387604	S.D. dependent var	0.531538	
S.E. of regression	0.415959	Akaike info criterion	1.093001	
Sum squared resid	90.14445	Schwarz criterion	1.133545	
Log likelihood	-282.459	F-statistic	84.07213	
Durbin-Watson stat	1.775217	Prob(F-statistic)	0	

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# Other use: Project Evaluation



- Example: The goal is to study the effect of capital intensity on state enterprise employment.

Control group : labor intensive project

Experiment group: capital intensive project

$$empnum = \beta_0 + \delta_0 K510 + \beta_1 invsize + \beta_2 years + u$$

$K510 = 1$  capital intensive project  
 $K510 = 0$  otherwise (labor intensive)  
 $empnum$  workers employed (persons)  
 $invsize$  amount of investment (million baht)  
 $years$  duration of the project (years)

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$$\widehat{empnum} = 760.2 - 203K510 + 0.148invsize - 123.7years$$

s.e. (182.6) (183.4) (.0168) (33.09)  
 t-stat [4.16] [-1.11] [8.80] [-3.74]  
 n=106 R<sup>2</sup>=.433282 R<sup>2</sup>-bar=.416614



- Interpretation:

- 1) Interpret the coefficient on  $K510$ ?
- 2) Is  $K510$  statistically significant?
- 3) Interpret the coefficients on  $invsize$  and  $years$ ?

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## Regress $empnum$ on $K510$ , $invsize$ , $years$



Dependent Variable: EMPNUM  
 Method: Least Squares  
 Sample: 1 154  
 Included observations: 106  
 Excluded observations: 48

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	760.1944	182.6296	4.162492	0.0001
K510	-203.3838	183.3694	-1.109148	0.27
INVSIZ	0.147838	0.016804	8.798054	0
YEARS	-123.7395	33.08859	-3.739644	0.0003
R-squared	0.433282	Mean dependent var		622.8517
Adjusted R-squared	0.416614	S.D. dependent var		997.0641
S.E. of regression	761.5549	Akaike info criterion		16.14561
Sum squared resid	59156512	Schwarz criterion		16.24611
Log likelihood	-851.7172	F-statistic		25.99455
Durbin-Watson stat	1.240076	Prob(F-statistic)		0

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## III. Using Dummy Variables for Multiple Categories



- Consider the model:

$$\log(wage) = \beta_0 + \delta_0 female + \delta_1 married + u$$

Two dummy variables:  $female$  and  $married$

- $\delta_1$  is the proportional differential in wages between those who are married and those who are not (the base group).

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## Multiple Categories

- Example: Hourly wage equation with two categories

$$\log(\text{wage}) = 1.653 - .359\text{female} + .233\text{married}$$

s.e.	(.0417)	(.0426)	(.0436)
t-stat	[39.66]	[-8.44]	[5.36]

$$n = 526; R^2 = .18438; R^2\text{-bar} = .18126$$

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## Regress log(wage) on female and married

Dependent Variable: LOG(WAGE)				
Method: Least Squares				
Sample: 1 526				
Included observations: 526				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.653447	0.041688	39.66281	0
FEMALE	-0.35934	0.04257	-8.441102	0
MARRIED	0.233371	0.043568	5.356471	0
R-squared	0.18438	Mean dependent var	1.623268	
Adjusted R-squared	0.181261	S.D. dependent var	0.531538	
S.E. of regression	0.480958	Akaike info criterion	1.379614	
Sum squared resid	120.9807	Schwarz criterion	1.403941	
Log likelihood	-359.839	F-statistic	59.1149	
Durbin-Watson stat	1.792627	Prob(F-statistic)	0	

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## Example: Effect of Capital Intensity

- Suppose we want to divide investment projects into three categories
  - 1) 0-30 percent of total project fund invested in capital
  - 2) 30-60 percent invested in capital
  - 3) 60-100 percent invested in capital
- Dummy variables: 0-30% capital investment as the base group
 

K36	=1	30-60 percent capital investment
	=0	Otherwise
K610	=1	60-100 percent capital investment
	=0	Otherwise

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$$\text{empnum} = \beta_0 + \delta_0 K36 + \delta_1 K610 + \beta_1 \text{invsize} + \beta_2 \text{years} + u$$

$$\widehat{\text{empnum}} = 967.9 - 359.3K36 - 397.3K610 + .148\text{invsize} - 126.0\text{years}$$

t-stat	[3.17]	[-1.10]	[-1.31]	[8.79]	[-3.78]
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$$n=106, R^2=0.436065, R^2\text{-bar}=.413731$$

### Interpretation:

- 1) What is the economic significance of  $\hat{\delta}_0$ ? Interpret .
- 2) What is the economic significance of  $\hat{\delta}_1$ ? Interpret .
- 3) Should we include K36 and K610 in the model?

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## View/Coefficient Tests/Redundant Variables..



Redundant Variables: K36 K610			
F-statistic	0.861298	Probability	0.425694
Log likelihood ratio	1.792628	Probability	0.408071

## View/Coefficient Tests/Wald - Coefficient Restrictions

Wald Test:			
Equation: Untitled			
Null Hypothesis:		C(2)=0	
		C(3)=0	
F-statistic	0.861298	Probability	0.425694
Chi-square	1.722595	Probability	0.422613

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## Regress empnum on K36, K610, invsize and years



Dependent Variable: EMPNUM				
Sample: 1 154				
Included observations: 106				
Excluded observations: 48				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	967.899	305.2325	3.171022	0.002
K36	-359.283	327.6044	-1.096698	0.2754
K610	-397.298	302.7533	-1.312282	0.1924
INVSIZE	0.14796	0.016836	8.788151	0
YEARS	-125.994	33.28816	-3.784936	0.0003
R-squared	0.436065	Mean dependent var	622.8517	
Adjusted R-squared	0.413731	S.D. dependent var	997.0641	
S.E. of regression	763.4342	Akaike info criterion	16.15955	
Sum squared resid	58866008	Schwarz criterion	16.28519	
Log likelihood	-851.456	F-statistic	19.52466	
Durbin-Watson stat	1.256433	Prob(F-statistic)	0	

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## Wage differences among four our groups

*marrmale* = 1, =0 otherwise

*marrfem* = 1, =0 otherwise

*singfem* = 1, =0 otherwise

*singmale* - base group

$$\log(\text{wage}) = \delta_0 + \delta_1 \text{marrmale} + \delta_2 \text{marrfem} + \delta_3 \text{singfem} + \beta_1 \text{educ} + \beta_2 \text{exper} + \beta_3 \text{exper}^2 + \beta_4 \text{tenure} + \beta_5 \text{tenure}^2 + u$$

obs	WAGE	FEMALE	MARRIED	MARRMALE	MARRFEM	SINGFEM
1	3.1	1	0	0	0	1
2	3.24	1	1	0	1	0
3	3	0	0	0	0	0
4	6	0	1	1	0	0
5	5.3	0	1	1	0	0
..	..	..	..	..	..	..
..	..	..	..	..	..	..

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## Regress log(wage) on *marrmale*, *marrfem*, *singfem*, *educ*, *exper*, *exper*<sup>2</sup>, *tenure*, and *tenure*<sup>2</sup>



Dependent Variable: LOG(WAGE)				
Included observations: 526				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.321378	0.100009	3.213492	0.0014
MARRMALE	0.212676	0.055357	3.841881	0.0001
MARRFEM	-0.198268	0.057835	-3.428132	0.0007
SINGFEM	-0.11035	0.055742	-1.979658	0.0483
EDUC	0.07891	0.006694	11.78733	0
EXPER	0.026801	0.005243	5.111835	0
EXPER^2	-0.000535	0.00011	-4.847105	0
TENURE	0.029088	0.006762	4.301614	0
TENURE^2	-0.000533	0.000231	-2.305553	0.0215
R-squared	0.460877	Mean dependent var	1.623268	
Adjusted R-squared	0.452535	S.D. dependent var	0.531538	

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Example: Estimated wage differential between groups

$$\log(\widehat{wage}) = \beta_0 + \delta_1 marrmale + \delta_2 marrfem + \delta_3 singfem + \dots$$

$$\log(\widehat{wage}) = .321 + .213marrmale - .198marrfem - .110singfem + \dots$$

s.e.	(.100)	(.055)	(.578)	(.056)
t-stat	[3.21]	[3.84]	[-3.42]	[-1.98]

n=526, R<sup>2</sup>=.460877 R<sup>2</sup>-bar=.452535



### Interpretation

- 1) What is the base category?
- 2) Interpret  $\hat{\delta}_1$ ,  $\hat{\delta}_2$ , and  $\hat{\delta}_3$
- 3) What is the proportionate difference between single and married women (the new base group)?
- 4) Test whether the proportionate difference between single women and married women (the base group) is statistically significant.

$$\log(\widehat{wage}) = \beta_0 + \delta_1 marrmale + \delta_2 marrfem + \delta_3 singfem + \beta_1 educ + \beta_2 exper + \beta_3 exper^2 + \beta_4 tenure + \beta_5 tenure^2$$

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## Summary: four groups

- Intercept for the base group – the overall intercept in the model.
- Dummy variable coefficient – the estimated proportional differential in wages between the group of interest and the **base group**.

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## IV. Interactions Involving Dummy Variables



Model: interaction terms between two dummies

$$\log(\widehat{wage}) = \beta_0 + \delta_1 female + \delta_2 married + \delta_3 female*married + \beta_1 educ + \beta_2 exper + \beta_3 exper^2 + \beta_4 tenure + \beta_5 tenure^2 + u$$

- Interaction term : female\*married

$$\Delta \log(\widehat{wage}) / \Delta married = \delta_2 + \delta_3 female$$

$$\Delta \log(\widehat{wage}) / \Delta female = \delta_1 + \delta_3 married$$

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## Interactions



Example:

$$\log(\widehat{wage}) = .321 - .110female + .213married - .301female*married + \dots$$

s.e.	(0.100)	(0.056)	(0.055)	(0.072)
p-value	{.0014}	{.0483}	{.0001}	{0}

Does married premium depend on gender?

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Regress  $\log(\text{wage})$  on *female, married, female\*married educ, exper, exper^2, tenure and tenure^2*



**Dependent Variable: LOG(WAGE)**

Sample: 1 526  
Included observations: 526

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.321378	0.100009	3.213492	0.0014
FEMALE	-0.11035	0.055742	-1.979658	0.0483
MARRIED	0.212676	0.055357	3.841881	0.0001
FEMALE*MARRIED	-0.30059	0.071767	-4.188461	0
EDUC	0.07891	0.006694	11.78733	0
EXPER	0.026801	0.005243	5.111835	0
EXPER^2	-0.00054	0.00011	-4.847105	0
TENURE	0.029088	0.006762	4.301614	0
TENURE^2	-0.00053	0.000231	-2.305553	0.0215
R-squared	0.460877	Mean dependent var		1.623268
Adjusted R-squared	0.452535	S.D. dependent var		0.531538

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**Comparing Models**

(\*)  $\log(\widehat{wage}) = .321 - .110\text{female} + .213\text{married} - .301\text{female*married} + \dots$   
 (\*\*)  $\log(\widehat{wage}) = .321 + .213\text{marrmale} - .198\text{marrfem} - .110\text{singfem} + \dots$



**Interpretation:**

- 1) In Model (\*), set female=1 and married=1. Is there any relationship to Model (\*\*)?
- 2) What is the intercept,  $\hat{\beta}_0$ ?
- 3) What is the estimated proportional wage differential between married female and single male?
- 4) Test whether the proportional wage differential between married female and single men is statistically significant.

$$\log(\widehat{wage}) = \hat{\beta}_0 + \hat{\delta}_1\text{female} + \hat{\delta}_2\text{married} + \hat{\delta}_3\text{female*married} + \hat{\beta}_1\text{educ} + \hat{\beta}_2\text{exper} + \hat{\beta}_3\text{exper}^2 + \hat{\beta}_4\text{tenure} + \hat{\beta}_5\text{tenure}^2$$

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**Interaction between binary and quantitative variables: allowing for different slope and Intercept**



- Consider the model

$$\text{wage} = \beta_0 + \delta_0\text{female} + \beta_1\text{educ} + \delta_1\text{female*educ} + u$$

For female: intercept =  $\beta_0 + \delta_0$   
 slope =  $\beta_1 + \delta_1$

For male: Intercept =  $\beta_0$   
 Slope =  $\beta_1$

- Interpretation

$\delta_0$  : the difference in intercepts between women and men.

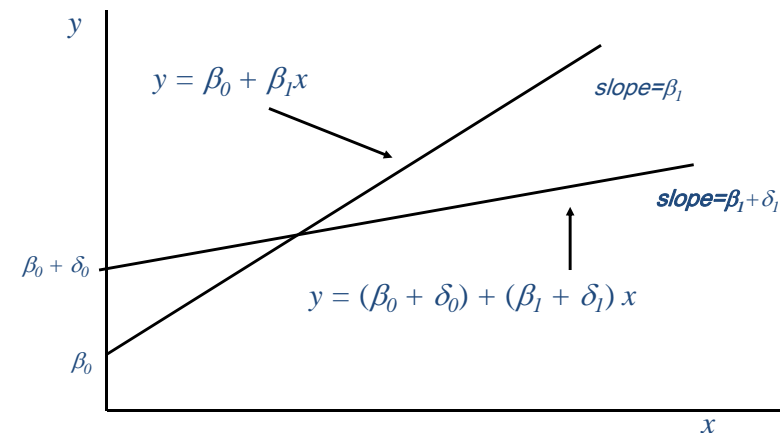
$\delta_1$  : the difference in slopes between women and men

Graphs: Case 1:  $\delta_0 > 0$  ,  $\delta_1 < 0$

Case 2:  $\delta_0 < 0$  ,  $\delta_1 < 0$

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**Example of  $\delta_0 > 0$  and  $\delta_1 < 0$**

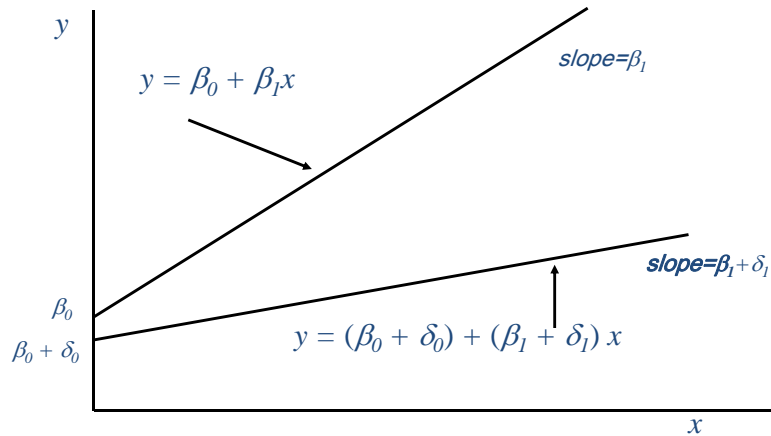


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## Example of $\delta_0 < 0$ and $\delta_1 < 0$



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Example:  $wage = \beta_0 + \delta_0 female + \beta_1 educ + \delta_1 female * educ + u$   
 $\widehat{wage} = .2004 - 1.19 female + .539 educ - .086 female * educ$   
 s.e. (.844) (1.33) (.064) (.1036)  
 t-stat [.238] [-.90] [8.40] [-.83]  
 p-value {.812} {.366} {.000} {.407}

### Interpretation:

- 1) What is the estimated return to education for men?
- 2) Interpret  $\hat{\delta}_0$  and Test.
- 3) Interpret  $\hat{\delta}_1$  and Test.
- 4) Test whether there are differences in regression functions between male and female.

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## Regress wage on female educ and female\*educ



Dependent Variable: WAGE				
Sample: 1 526				
Included observations: 526				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.200496	0.843562	0.237678	0.8122
FEMALE	-1.19852	1.32504	-0.904518	0.3661
EDUC	0.539476	0.064223	8.400054	0
FEMALE*EDUC	-0.086	0.103639	-0.829795	0.407
R-squared	0.259796	Mean dependent var	5.896103	
Adjusted R-squared	0.255542	S.D. dependent var	3.693086	
S.E. of regression	3.186469	Akaike info criterion	5.163279	
Sum squared resid	5300.17	Schwarz criterion	5.195715	
Log likelihood	-1353.94	F-statistic	61.07022	
Durbin-Watson stat	1.825925	Prob(F-statistic)	0	

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### View/Coefficient Tests/Redundant Variables

Redundant Variables: FEMALE FEMALE*EDUC			
F-statistic	33.51095	Probability	0
Log likelihood ratio	63.53858	Probability	0

### View/Coefficient Tests/Wald – Coefficient Restrictions

Wald Test:			
Equation: Untitled			
Null Hypothesis:		C(2)=0	
		C(4)=0	
F-statistic	33.51095	Probability	0
Chi-square	67.0219	Probability	0

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## V. Chow test: a test for differences in Regression Function



(0) Male :  $wage = \beta_0 + \beta_1 educ + u_M$   $n_M = 274$   
 (1) Female :  $wage = \alpha_0 + \alpha_1 educ + u_F$   $n_F = 252$   
 (P) All :  $wage = b_0 + b_1 educ + u$   $n = 526$

Male:	$\widehat{wage} = 0.200 + 0.539educ$	
	t-stat [1.97] [6.97]	SSR <sub>M</sub> = 4009.931
Female:	$\widehat{wage} = .998 + .453educ$	
	t-stat [-1.37] [7.82]	SSR <sub>F</sub> = 1290.239
Pooled:	$\widehat{wage} = -.905 + 0.541educ$	
	t-stat [-1.32] [10.17]	SSR = 5980.62

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## Data for Chow Test



Obser	Female	educ	wage
1	0	11	3
2	0	8	6
3	0	12	5.3
..	..	..	..
273	0	15	4.67
274	0	16	11.56
275	1	11	3.1
276	1	12	3.24
..	..	..	..
525	1	10	2.27
526	1	14	3.5

Chow Breakpoint is the observation 275

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## Pooled Data: Regress wage on educ



Dependent Variable: WAGE				
Method: Least Squares				
Sample: 1 526				
Included observations: 526				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.90485	0.684968	-1.321013	0.1871
EDUC	0.541359	0.053248	10.16675	0
R-squared	0.164758	Mean dependent var	5.896103	
Adjusted R-squared	0.163164	S.D. dependent var	3.693086	
S.E. of regression	3.37839	Akaike info criterion	5.27647	
Sum squared resid	5980.682	Schwarz criterion	5.292688	
Log likelihood	-1385.71	F-statistic	103.3627	
Durbin-Watson stat	1.724873	Prob(F-statistic)	0	

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## Male: Regress wage on educ



Dependent Variable: WAGE				
Method: Least Squares				
Sample: 1 274				
Included observations: 274				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.200496	1.016462	0.197249	0.8438
EDUC	0.539476	0.077386	6.971203	0
R-squared	0.151585	Mean dependent var	7.099489	
Adjusted R-squared	0.148465	S.D. dependent var	4.160858	
S.E. of regression	3.839582	Akaike info criterion	5.535877	
Sum squared resid	4009.931	Schwarz criterion	5.56225	
Log likelihood	-756.415	F-statistic	48.59767	
Durbin-Watson stat	1.918787	Prob(F-statistic)	0	

Choose Estimate.  
In the "Equation Specification" box,  
replace with observations between 1-274.

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## Female: Regress wage on educ



Dependent Variable: WAGE	Choose Estimate.			
Method: Least Squares	In the "Equation Specification" box,			
Sample: 275 526	replace with observations between 275-526.			
<b>Included observations: 252</b>				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.99803	0.728507	-1.369962	0.1719
EDUC	0.453477	0.057992	7.819667	0
R-squared	0.196522	Mean dependent var		4.587659
Adjusted R-squared	0.193308	S.D. dependent var		2.529363
S.E. of regression	2.271774	Akaike info criterion		4.486904
<b>Sum squared resid</b>	<b>1290.239</b>	Schwarz criterion		4.514915
Log likelihood	-563.35	F-statistic		61.14719
Durbin-Watson stat	1.983793	Prob(F-statistic)		0

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Assumptions:  $u_M \sim N(0, \sigma^2)$   
 $u_F \sim N(0, \sigma^2)$   
 $u_M$  and  $u_F$  are independently distributed



Chow Test is basically F-test, a relative increase in SSR when moving from unrestricted to restricted model.

**Step 1:** Restricted model: The pooled data restrict the slopes and intercepts are equal ( $\beta_0 = \alpha_0$  and  $\beta_1 = \alpha_1$ )

$$SSR_R = 5980.682 \quad DF_R = n - k - 1 = 524$$

**Step 2:** Unrestricted model. SSR is found by the sum of SSRs from two models

$$SSR_U = SSR_M + SSR_F = 4009.9 + 1290.2 = 5300$$

$$DF_U = (n_M - k - 1) + (n_F - k - 1) = n_M + n_F - 2(k + 1) = n - 2(k + 1) = 522$$

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F-statistic:

$$F = (5981 - 5300) / 2 \text{ divided by } 5300 / 522 = 33.5$$



**Step 3:**

$$F = (SSR_R - SSR_U) / (DF_R - DF_U) \text{ divided by } SSR_U / DF_U$$

$$F = \frac{(SSR_R - SSR_U) / (k + 1)}{SSR_U / (n - 2(k + 1))}$$

- Rejection Rule  $F > C$

$$C = F_{2,522} \text{ (table G3)} \quad C = 3.00 \text{ } (\alpha=5\%)$$

$$= 4.61 \text{ } (\alpha=1\%)$$

**Conclusion:** Male and female regressions are different.

- Chow Test may tell us that the regression functions of male and female are different, but it does not tell us whether it is due to the intercept or slope.

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## Steps for Chow Test



**Step 1:** Obtain the original equation for both groups (Male: 1-274) Female (275-526)

$$\text{Pooled: } \widehat{wage} = -.905 + 0.541educ$$

**Step 2:** In the "equation" window,

- Choose View/Stability Tests/Chow Breakpoint Test
- Pick the breakpoint observation: 275

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## Step 1: Pooled Data: Regress wage on educ



Dependent Variable: WAGE				
Method: Least Squares				
Sample: 1 526				
Included observations: 526				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.90485	0.684968	-1.321013	0.1871
EDUC	0.541359	0.053248	10.16675	0
R-squared	0.164758	Mean dependent var	5.896103	
Adjusted R-squared	0.163164	S.D. dependent var	3.693086	
S.E. of regression	3.37839	Akaike info criterion	5.27647	
Sum squared resid	5980.682	Schwarz criterion	5.292688	
Log likelihood	-1385.71	F-statistic	103.3627	
Durbin-Watson stat	1.724873	Prob(F-statistic)	0	

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**Step 2:** Eviews In the “equation” window, choose View/Stability Tests/Chow Breakpoint Test



Chow Breakpoint Test: 275			
F-statistic	33.51095	Probability	0
Log likelihood ratio	63.53861	Probability	0

Pick the breakpoint observation: 275

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## Chow Test versus Dummy Variables



- Dummy Variables:

$$wage = \beta_0 + \delta_0 female + \beta_1 educ + \delta_1 female * educ + u$$

- t-Test:  $H_0 : \delta_0 = 0$
- t-Test:  $H_0 : \delta_1 = 0$
- F-Test:  $H_0 : \delta_0 = 0, \delta_1 = 0$

- Better features of using dummies than Chow:

- (1) We can test certain slope or intercept whether the interested coefficient is significant.
- (2) There is only one model equation with dummies.

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## Recap of Binary Variable



- Describing Qualitative Information
- A Single Dummy Independent Variable
- Using Dummy Variable for Multiple Categories
- Interactions Involving Dummy Variables
- Chow Test

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