1. Price discrimination requires the ability to sort customers and the ability to prevent arbitrage. Explain how the following can function as price discrimination schemes and discuss both sorting and arbitrage:

   a. **Requiring airline travelers to spend at least one Saturday night away from home to qualify for a low fare.**

      The requirement of staying over Saturday night separates business travelers, who prefer to return home for the weekend, from tourists, who travel on the weekend. Arbitrage is not possible when the ticket specifies the name of the traveler.

   b. **Insisting on delivering cement to buyers and basing prices on buyers’ locations.**

      By basing prices on the buyer’s location, customers are sorted by geography. Prices may then include transportation charges, which the customer pays for whether delivery is received at the buyer’s location or at the cement plant. Since cement is heavy and bulky, transportation charges may be large. Note that this pricing strategy sometimes leads to what is called “basing-point” pricing, where all cement producers use the same base point and calculate transportation charges from that base point. Every seller then quotes individual customers the same price. This pricing system is often viewed as a method to facilitate collusion among sellers. For example, in *FTC v. Cement Institute*, 333 U.S. 683 [1948], the Court found that sealed bids by eleven companies for a 6,000-barrel government order in 1936 all quoted $3.286854 per barrel.

   c. **Selling food processors along with coupons that can be sent to the manufacturer for a $10 rebate.**

      Rebate coupons for food processors separate consumers into two groups: (1) customers who are less price sensitive (those who have a lower elasticity of demand) and do not fill out the forms necessary to request the rebate; and (2) customers who are more price sensitive (those who have a higher demand elasticity) and do the paperwork to request the rebate. The latter group could buy the food processors, send in the rebate coupons, and resell the processors at a price just below the retail price without the rebate. To prevent this type of arbitrage, sellers could limit the number of rebates per household.

   d. **Offering temporary price cuts on bathroom tissue.**

      A temporary price cut on bathroom tissue is a form of intertemporal price discrimination. During the price cut, price-sensitive consumers buy greater quantities of tissue than they would otherwise and store it for later use. Non-price-sensitive consumers buy the same amount of tissue that they would buy without the price cut. Arbitrage is possible, but the profits on reselling bathroom tissue probably are so small that they do not compensate for the cost of storage, transportation, and resale.
e. Charging high-income patients more than low-income patients for plastic surgery.

The plastic surgeon might not be able to separate high-income patients from low-income patients, but he or she can guess. One strategy is to quote a high price initially, observe the patient’s reaction, and then negotiate the final price. Many medical insurance policies do not cover elective plastic surgery. Since plastic surgery cannot be transferred from low-income patients to high-income patients, arbitrage does not present a problem.

8. Sal’s satellite company broadcasts TV to subscribers in Los Angeles and New York. The demand functions for each of these two groups are

\[ Q_{NY} = 60 - 0.25P_{NY} \]
\[ Q_{LA} = 100 - 0.50P_{LA} \]

where \( Q \) is in thousands of subscriptions per year and \( P \) is the subscription price per year. The cost of providing \( Q \) units of service is given by

\[ C = 1000 + 40Q \]

where \( Q = Q_{NY} + Q_{LA} \).

a. What are the profit-maximizing prices and quantities for the New York and Los Angeles markets?

Sal should pick quantities in each market so that the marginal revenues are equal to one another and equal to marginal cost. To determine marginal revenues in each market, first solve for price as a function of quantity:

\[ P_{NY} = 240 - 4Q_{NY}, \text{ and} \]
\[ P_{LA} = 200 - 2Q_{LA}. \]

Since marginal revenue curves have twice the slope of their demand curves, the marginal revenue curves for the respective markets are:

\[ MR_{NY} = 240 - 8Q_{NY}, \text{ and} \]
\[ MR_{LA} = 200 - 4Q_{LA}. \]

Set each marginal revenue equal to marginal cost, which is $40, and determine the profit-maximizing quantity in each submarket:

\[ 40 = 240 - 8Q_{NY}, \text{ or } Q_{NY} = 25 \text{ thousand, and} \]
\[ 40 = 200 - 4Q_{LA}, \text{ or } Q_{LA} = 40 \text{ thousand.} \]

Determine the price in each submarket by substituting the profit-maximizing quantity into the respective demand equation:

\[ P_{NY} = 240 - 4(25) = $140, \text{ and} \]
\[ P_{LA} = 200 - 2(40) = $120. \]

b. As a consequence of a new satellite that the Pentagon recently deployed, people in Los Angeles receive Sal’s New York broadcasts, and people in New York receive Sal’s Los Angeles broadcasts. As a result, anyone in New York or Los Angeles can receive Sal’s broadcasts by subscribing in either city. Thus Sal can charge only a single price. What price should he charge, and what quantities will he sell in New York and Los Angeles?

Sal’s combined demand function is the horizontal summation of the \( LA \) and \( NY \) demand functions. Above a price of $200 (the vertical intercept of the \( LA \) demand function), the total
demand is just the New York demand function, whereas below a price of $200, we add the two demands:

\[ QT = 60 - 0.25P + 100 - 0.50P, \text{ or } QT = 160 - 0.75P. \]

Solving for price gives the inverse demand function:

\[ P = 213.33 - 1.333Q, \]

and therefore, \( MR = 213.33 - 2.667Q. \)

Setting marginal revenue equal to marginal cost:

\[ 213.33 - 2.667Q = 40, \text{ or } Q = 65 \text{ thousand}. \]

Although a price of $126.67 is charged in both markets, different quantities are purchased in each market.

\[ Q_{NY} = 60 - 0.25(126.67) = 28.3 \text{ thousand and} \]
\[ Q_{LA} = 100 - 0.50(126.67) = 36.7 \text{ thousand}. \]

Together, 65 thousand subscriptions are purchased at a price of $126.67 each.

c. In which of the above situations, a or b, is Sal better off? In terms of consumer surplus, which situation do people in New York prefer and which do people in Los Angeles prefer? Why?

Sal is better off in the situation with the highest profit, which occurs in part a with price discrimination. Under price discrimination, profit is equal to:

\[ \pi = P_N Y Q_{NY} + P_{LA} Q_{LA} - [1000 + 40(Q_{NY} + Q_{LA})], \text{ or} \]
\[ \pi = 140(25) + 120(40) - [1000 + 40(25 + 40)] = 4700 \text{ thousand}. \]

Under the market conditions in b, profit is:

\[ \pi = P Q_T - [1000 + 40Q_T], \text{ or} \]
\[ \pi = 126.67(65) - [1000 + 40(65)] = 4633.33 \text{ thousand}. \]

Therefore, Sal is better off when the two markets are separated.

Under the market conditions in a, the consumer surpluses in the two cities are:

\[ CS_{NY} = (0.5)(25)(240 - 140) = 1250 \text{ thousand, and} \]
\[ CS_{LA} = (0.5)(40)(200 - 120) = 1600 \text{ thousand.} \]

Under the market conditions in b, the respective consumer surpluses are:

\[ CS_{NY} = (0.5)(28.3)(240 - 126.67) = 1603.67 \text{ thousand, and} \]
\[ CS_{LA} = (0.5)(36.7)(200 - 126.67) = 1345.67 \text{ thousand.} \]

New Yorkers prefer b because their price is $126.67 instead of $140, giving them a higher consumer surplus. Customers in Los Angeles prefer a because their price is $120 instead of $126.67, and their consumer surplus is greater in a.
10. As the owner of the only tennis club in an isolated wealthy community, you must decide on membership dues and fees for court time. There are two types of tennis players. “Serious” players have demand

\[ Q_1 = 10 - P \]

where \( Q_1 \) is court hours per week and \( P \) is the fee per hour for each individual player. There are also “occasional” players with demand

\[ Q_2 = 4 - 0.25P. \]

Assume that there are 1000 players of each type. Because you have plenty of courts, the marginal cost of court time is zero. You have fixed costs of $10,000 per week. Serious and occasional players look alike, so you must charge them the same prices.

a. Suppose that to maintain a “professional” atmosphere, you want to limit membership to serious players. How should you set the annual membership dues and court fees (assume 52 weeks per year) to maximize profits, keeping in mind the constraint that only serious players choose to join? What would profits be (per week)?

In order to limit membership to serious players, the club owner should charge an entry fee, \( T \), equal to the total consumer surplus of serious players and a usage fee \( P \) equal to marginal cost of zero. With individual demands of \( Q_1 = 10 - P \), individual consumer surplus is equal to:

\[ (0.5)(10 - 0)(10 - 0) = $50, \text{ or} \]
\[ (50)(52) = $2600 \text{ per year}. \]

An entry fee of $2600 maximizes profits by capturing all consumer surpluses. The profit-maximizing court fee is set to zero, because marginal cost is equal to zero. The entry fee of $2600 is higher than the occasional players are willing to pay (higher than their consumer surplus at a court fee of zero); therefore, this strategy will limit membership to the serious players. Weekly profits would be

\[ \pi = (50)(1000) - 10,000 = $40,000. \]

b. A friend tells you that you could make greater profits by encouraging both types of players to join. Is your friend right? What annual dues and court fees would maximize weekly profits? What would these profits be?

When there are two classes of customers, serious and occasional players, the club owner maximizes profits by charging court fees above marginal cost and by setting the entry fee (annual dues) equal to the remaining consumer surplus of the consumer with the lesser demand, in this case, the occasional player. The entry fee, \( T \), equals the consumer surplus remaining after the court fee \( P \) is assessed:

\[ T = 0.5Q_2(16 - P), \text{ where} \]
\[ Q_2 = 4 - 0.25P. \]

Therefore,

\[ T = 0.5(4 - 0.25P)(16 - P) = 32 - 4P + 0.125P^2. \]

Total entry fees paid by all players would be

\[ 2000T = 2000(32 - 4P + 0.125P^2) = 64,000 - 8000P + 250P^2. \]

Revenues from court fees equal
\[ P(1000Q_1 + 1000Q_2) = P[1000(10 - P) + 1000(4 - 0.25P)] = 14,000P - 1250P^2. \]

Therefore, total revenue from entry fees and court fees is
\[ TR = 64,000 + 6000P - 1000P^2. \]

Marginal cost is zero, so we want to maximize total revenue. To do this, differentiate total revenue with respect to price and set the derivative to zero:
\[ \frac{dTR}{dP} = 6000 - 2000P = 0. \]

Solving for the optimal court fee, \( P = $3.00 \) per hour. Serious players will play \( 10 - 3 = 7 \) hours per week, and occasional players will demand \( 4 - 0.25(3) = 3.25 \) hours of court time per week. Total revenue is then \( 64,000 + 6000(3) - 1000(3)^2 = $73,000 \) per week. So profit is \( 73,000 - 10,000 = $63,000 \) per week, which is greater than the $40,000 profit when only serious players become members. Therefore, your friend is right; it is more profitable to encourage both types of players to join.

c. Suppose that over the years young, upwardly mobile professionals move to your community, all of whom are serious players. You believe there are now 3000 serious players and 1000 occasional players. Would it still be profitable to cater to the occasional player? What would be the profit-maximizing annual dues and court fees? What would profits be per week?

An entry fee of $50 per week would attract only serious players. With 3000 serious players, total revenues would be $150,000 and profits would be $140,000 per week. With both serious and occasional players, we may follow the same procedure as in part b. Entry fees would be equal to 4000 times the consumer surplus of the occasional player:
\[ T = 4000(32 - 4P + 0.125P^2) = 128,000 - 16,000P + 500P^2 \]

Court fees are
\[ P(3000Q_1 + 1000Q_2) = P[3000(10 - P) + 1000(4 - 0.25P)] = 34,000P - 3250P^2, \]
\[ TR = 128,000 + 18,000P - 2750P^2, \]
\[ \frac{dTR}{dP} = 18,000 - 5500P = 0, \text{ so } P = $3.27 \text{ per hour.} \]

With a court fee of $3.27 per hour, total revenue is \( 128,000 + 18,000(3.27) - 2750(3.27)^2 = $157,455 \) per week. Profit is \( 157,455 - 10,000 = $147,455 \) per week, which is more than the $140,000 with serious players only. So you should set the entry fee and court fee to attract both types of players. The annual dues (i.e., the entry fee) should equal 52 times the weekly consumer surplus of the occasional player, which is \( 52[32 - 4(3.27) + 0.125(3.27)^2] = $1053. \) The club’s annual profit will be \( 52(147,455) = $7.67 \text{ million per year.} \)