

Performance Analysis of Single-Link System with Nonlinear Equivalent Capacity

Chaodit Aswakul and Javier Barria

Abstract—This letter presents a procedure to estimate the performance of a single-link system with *nonlinear equivalent capacity*. The idea is to convert the problems from the *nonlinear domain* into the *linear domain* so that the efficient numerical techniques in the *linear domain* can be employed. Based on the Kaufman–Roberts’s recursion [4], [6], the proposed procedure has the time complexity = $O(Ck)$ and the space complexity = $O(k)$, where C is the link capacity and k is the number of call types. The absolute accuracy [5, Sec. 4.3] of the proposed procedure in estimating the blocking probabilities is in the order of 10^{-3} – 10^{-2} for the reported results.

Index Terms—Asynchronous transfer mode, equivalent capacity, single-link system.

I. INTRODUCTION

IN CONVENTIONAL traffic engineering, various mathematical models and approximation techniques have been proposed (e.g., [8]) and successfully employed for performance analysis of conventional systems such as circuit switching. All these mathematical tools rely on an implicit assumption about the *linear* relationship between the number of connections n and the amount of link capacity $G(n)$ required for the n connections. In this letter, function $G(\cdot)$ will be referred to as the *equivalent capacity*.¹

A *linear* equivalent capacity (i.e., $G(n) \propto n$) can be a reasonable assumption when analyzing conventional systems. However, this linear relationship may not hold in cases such as asynchronous transfer mode (ATM) networks. For ATM networks, cell streams from different connections are *statistically multiplexed* in order to improve the utilization of ATM multiplexers. It has been found that the effect of this statistical multiplexing can be well captured by an equivalent capacity $G(n)$ that increases monotonically with decreasing slope as n increases, i.e., it is of *nonlinear* form [2], [3].

This letter is thus concerned with the performance analysis in the *nonlinear* domain of equivalent capacity. In particular, we consider a single finite-capacity ATM link accessed by many call streams. This system is assumed to have a *product-form so-*

lution and the *complete-sharing* call admission control (CAC) scheme. The probability of call blocking is considered as the main performance measure of interest [4]–[8]. To date, although the blocking probability expression is simple and well-known, there is no efficient recursive algorithm available that can calculate the blocking probability, except for the ordinary cases in which G is a linear function of n . Thus, to solve problems in the *nonlinear* domain, we propose here to first convert the problems into the *linear* domain so that the efficient numerical techniques in the linear domain (e.g., the Kaufman–Roberts’s recursion [4], [6]) can be employed.

II. MATHEMATICAL MODEL

Consider a link with capacity C and k call types. Let i index the call type ($i = 1, \dots, k$). Type- i calls arrive as a *Poisson* stream of rate λ_i and as i varies it, indexes *independent* Poisson streams. *Call holding times* are independent of each other and they are independent of call arrivals. The holding times of type- i calls are identically distributed with mean $1/\mu_i$. Denote the probability of blocking type- i calls by B_i .

Let $G_i(\cdot)$ be the equivalent capacity of type- i calls. That is, n_i type- i connections require link capacity of magnitude $G_i(n_i)$. Define a link-state vector $\underline{n} = (n_1, n_2, \dots, n_k)$, where n_i is the number of type- i connections. The *complete-sharing* CAC scheme admits a call as long as, after taking into account the new connection, \underline{n} is still contained in the state space S :

$$S = \left\{ \underline{n} \in I^k \mid \sum_{i=1}^k G_i(n_i) \leq C \right\} \quad (1)$$

where I is the set of nonnegative integers.

Let $N_i(t)$ be the number of type- i connections on the link at time t . The stochastic process $\{(N_1(t), \dots, N_k(t)), t \geq 0\}$ then has steady state distribution, $\pi(\underline{n}) \equiv \pi(n_1, \dots, n_k)$, given by a *product-form solution* [6]

$$\pi(\underline{n}) = \begin{cases} \frac{1}{K(S)} \prod_{i=1}^k \frac{\rho_i^{n_i}}{n_i!}, & \underline{n} \in S \\ 0, & \underline{n} \notin S \end{cases} \quad (2)$$

$$K(S) = \sum_{\underline{n} \in S} \left[\prod_{i=1}^k \frac{\rho_i^{n_i}}{n_i!} \right] \quad (3)$$

where $\rho_i \equiv (\lambda_i/\mu_i)$ is the *offered load* of type- i call stream. And, since the call arrival streams are Poisson,

$$B_i = \sum_{\{\underline{n} \in S \mid \underline{n} + \underline{e}_i \notin S\}} \pi(\underline{n}), \quad i = 1, \dots, k \quad (4)$$

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¹The terminology “*equivalent capacity*” was first introduced in [3] under the situation of a first-in-first-out ATM multiplexer accessed by 2-state fluid sources. However, we prefer using this terminology in a wider sense to generally define the amount of link capacity required for a given number of connections.

where \underline{e}_i is the k -dimensional vector consisting of only zeros except for a one in the i th component. Note that the form of equivalent capacity influences the blocking probabilities via the expression of the state space.

III. PROPOSED APPROXIMATION PROCEDURE

Consider the stochastic process $G_i(N_i(t))$ representing the link capacity utilized by type- i calls at time t . Define $\pi_{N_i}(n_i)$ as the steady-state marginal distribution of $N_i(t)$:

$$\pi_{N_i}(n_i) = \sum_{n_1} \cdots \sum_{n_{i-1}} \sum_{n_{i+1}} \cdots \sum_{n_k} \pi(n_1, \dots, n_i, \dots, n_k). \quad (5)$$

Since $\pi_{N_i}(n_i)$ typically takes its peak around $n_i = E(N_i)$, we propose here to approximate the nonlinear function $G_i(\cdot)$ by a linear function $G_{i(apx)}(\cdot)$ such that

$$\begin{aligned} G_{i(apx)}(\rho_{ci}^-) &= G_i(\rho_{ci}^-) \\ G_{i(apx)}(\rho_{ci}^+) &= G_i(\rho_{ci}^+) \end{aligned} \quad (6)$$

where ρ_{ci}^- denotes $\lfloor E(N_i) \rfloor$ and ρ_{ci}^+ denotes $\lceil E(N_i) \rceil + 1$. That is, the proposed approximation fits $G_i(\cdot)$ to a linear function at the two nearest integer points about the mean. This linear function can be written straightforwardly as

$$G_{i(apx)}(n_i) = b_i n_i + c_i \quad (7)$$

where

$$b_i = G_i(\rho_{ci}^+) - G_i(\rho_{ci}^-) \quad (8)$$

$$c_i = \rho_{ci}^+ G_i(\rho_{ci}^-) - \rho_{ci}^- G_i(\rho_{ci}^+). \quad (9)$$

Now, replacing $G_i(n_i)$ in (1) by $G_{i(apx)}(n_i)$ from (7), we then have the approximated state space

$$S_{(apx)} = \left\{ \underline{n} \in I^k \mid \sum_{i=1}^k b_i n_i \leq C' \right\} \quad (10)$$

where

$$C' = C - \sum_{i=1}^k c_i. \quad (11)$$

From (10) and (11), we may interpret b_i as the slope of a linear equivalent capacity and C' as a modified link capacity. Thus, (7)–(11) convert our problem from the nonlinear domain into the linear domain and we shall refer to this linearly approximated problem as *linear approximation model*. To solve for the blocking probability in the *linear approximation model*, we can apply any numerical techniques in the linear domain (e.g., [4], [6]). However, to formulate the *linear approximation model* above, we assume the knowledge of $E(N_i)$ ($i = 1, \dots, k$). This is not trivial because $E(N_i)$ is the carried load of type- i call stream, i.e.,

$$E(N_i) = \rho_i(1 - B_i) \quad (12)$$

and B_i is actually the output parameter to be computed. Hence, the linear approximation model cannot be formulated directly. To resolve this problem, a *fixed-point iteration* (e.g., [1]) can be applied as follows.

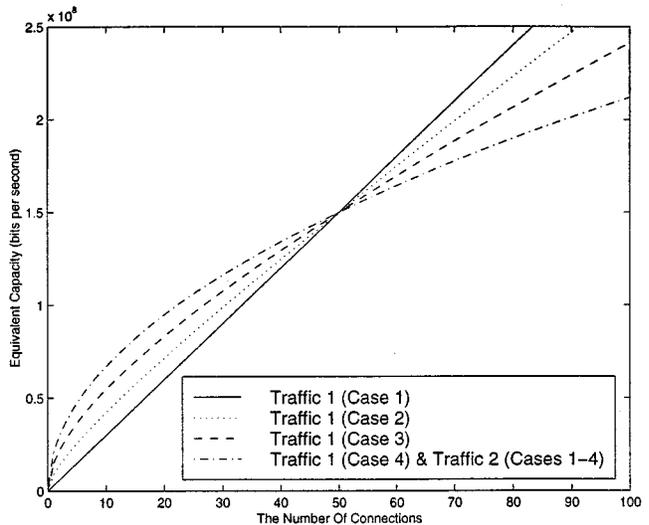


Fig. 1. Equivalent capacity of traffic 1 and traffic 2.

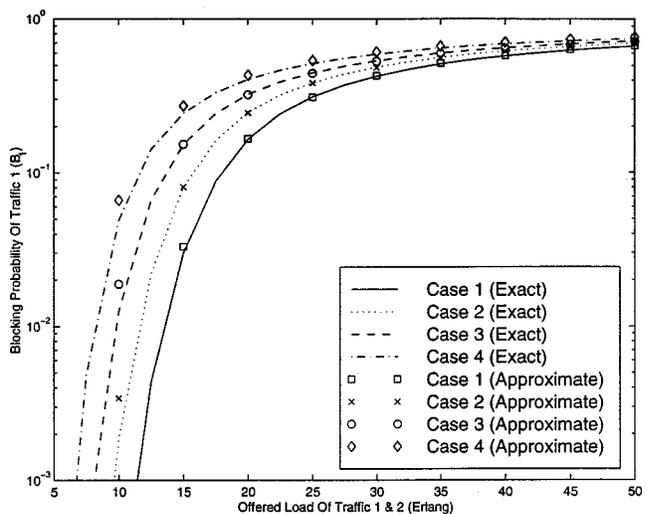


Fig. 2. Blocking probability of traffic 1.

Consider the blocking probability as the recursive parameter. Let $B_i(m)$ represent the value of B_i at the m th iteration for $m = 1, 2, 3, \dots$ and let $B_i(0)$ be the initial value for the recursion. Then, the fixed-point iteration procedure takes the following steps.

- Define $m = 0$.
- Calculate $E(N_i)$ for all i by (12) with B_i replaced by $B_i(m)$.
- Calculate $\rho_{ci}^- = \lfloor E(N_i) \rfloor$ and $\rho_{ci}^+ = \lceil E(N_i) \rceil + 1$ for all i .
- Calculate the parameters in the linear approximation model (i.e., b_i and C') by (8), (9), and (11).
- Calculate B_i for all i in the linear approximation model by the Kaufman–Roberts's recursion [4], [6].
- Update $B_i(m+1) = B_i$ for all i .
- If $(|B_i(m+1) - B_i(m)| / B_i(m+1)) < \tau$ (tolerance parameter) for all i , then stop the recursion. Otherwise, set $m = m+1$ and go back to step b).

From our numerical experience, this recursion always converges within a few iterations (generally less than 5). Thus, its computational complexity is in the same order as that of the

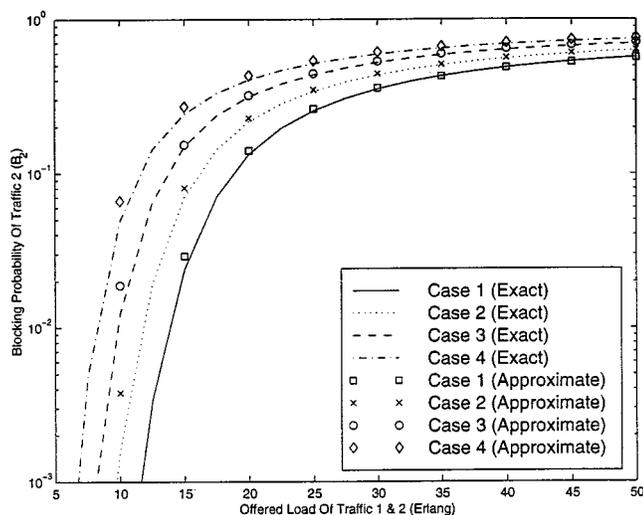


Fig. 3. Blocking probability of traffic 2.

Kaufman–Roberts’s recursion. That is, the time complexity is $O(Ck)$ and the space complexity is $O(k)$.

IV. NUMERICAL RESULTS

As an example, consider two call types. Suppose that type-1 call stream (*traffic 1*) and type-2 call stream (*traffic 2*) have the equivalent capacity $G(n)$ as depicted in Fig. 1. The form of $G(n) = \alpha n + \beta\sqrt{n}$ is adopted from [3] where $\alpha(\beta) = 0$ in the most (least) nonlinear case. The values of (α, β) in megabits per second (Mb/s) for *traffic 1* are (3,0), (2,7.0711), (1,14.1421) and (0,21.2132) in Cases 1–4, respectively. The values of (α, β) in Mb/s for *traffic 2* are (0,21.2132) in all the cases.

Figs. 2 and 3 depict the blocking probabilities computed from the linear approximation model, in comparison with their exact values. The exact values are obtained from directly computing the product-form solution and summing over the blocking-state occupancy probabilities. The offered loads of *traffic 1* and *traffic 2* are equally increased and the link capacity is fixed at 150 Mb/s. The figures indicate a good matching between the approximate values and the exact values over a practical range of offered loads. The absolute accuracy [5, Sec. 4.3] is

calculated, $\varepsilon_A = \sqrt{\sum(B - B^*)^2/M}$, where B^* denotes the approximation for B and M is the number of experiments. For Cases 1–4, $\varepsilon_A = (0.0039, 0.0054, 0.0072, 0.0192)$, respectively. That is, ε_A increases as the equivalent capacity is more nonlinear but it is still in an acceptable order (10^{-3} – 10^{-2}).

V. CONCLUSIONS

An approximation procedure, termed *linear approximation model*, is proposed in this letter to analyze a complete-sharing single-link system with *nonlinear* equivalent capacity. When employed with the Kaufman–Roberts’s recursion [4], [6], the *linear approximation model* requires the time complexity = $O(Ck)$ and the space complexity = $O(k)$. Based on the reported results, the absolute accuracy of the proposed procedure is in the acceptable order of 10^{-3} – 10^{-2} . Due to simplicity of the proposed approximation, one might expect that the degree of its accuracy could be vulnerable to extreme cases where the equivalent capacity is highly nonlinear. Although this high nonlinearity may not happen in most of practical cases, we are at present working on obtaining theoretical bounds for the accuracy of the procedure proposed here and the results will be reported in a forthcoming paper.

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