Chapter 3
Kinetics of Particles

3. Kinetics of Particles

- Introduction
  - Kinetics is the study of the relations between unbalanced force and the resulting changes in motion, i.e. **F** vs **r**, **v**, **a**.
- The three approaches
  - A. Direct Application or Force-Mass-Acceleration
  - B. Work and Energy
  - C. Impulse and Momentum
- Special Applications
  - Impact

3-1 Force, Mass, and Acceleration

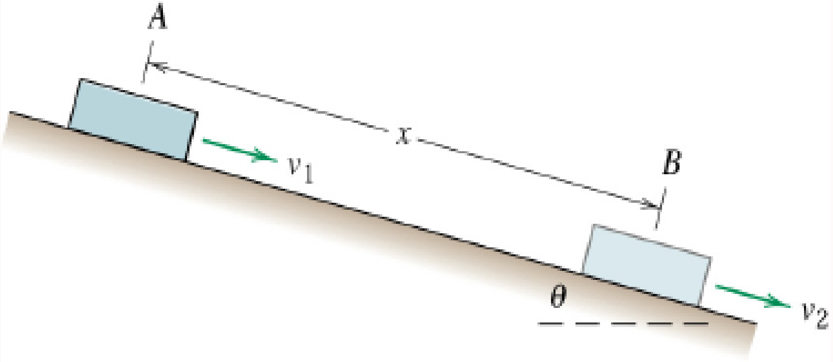
3-1. Force, Mass, and Acceleration

- The main equation is the Newton's second law.

$$\Sigma \vec{F} = m\vec{a}$$

- Combine it with coordinate systems studied in Chapter 2 to solve engineering problems
- Example

Suppose the block shown starts from rest at point A and slides down the incline due to the force of gravity. Find the speed of this block as a function of time, if θ = 15°.



3-1. Force, Mass, and Acceleration

Newton's second law

Newton's Second Law

$$\sum \vec{F} = m\vec{a}$$

- $\vec{a}$ must be an absolute acceleration.

Constrained and Unconstrained Motions

- Unconstrained motion: No mechanical guides or linkages to constrain its motion. Ex. airplanes, rockets, etc.
- Constrained motion: Motion is limited by some mechanical guide or linkages. Ex. mechanisms

3-1. Force, Mass, and Acceleration

Free Body Diagram

- A free-body diagram must be drawn to correctly evaluating all forces involved in Newton's second law.
- Procedures
  - Clearly draw an isolated body
  - Define coordinate system and their positive directions
  - Add all the forces (contact and non-contact) acting <u>on</u> that body

3-1. Force, Mass, and Acceleration

Rectilinear vs Curvilinear

$$\Sigma F_x = ma_x$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

Normal and Tangential Coordinates

$$\Sigma \vec{F}_n = m\vec{a}_n$$
$$\Sigma \vec{F}_t = m\vec{a}_t$$

where $a_n = \rho\dot{\beta}^2 = v^2/\rho = v\dot{\beta}$, $a_t = \dot{v}$, and $v = \rho\dot{\beta}$

Rectangular Coordinates

$$\Sigma \vec{F}_x = m\vec{a}_x$$
$$\Sigma \vec{F}_y = m\vec{a}_y$$

Polar Coordinates

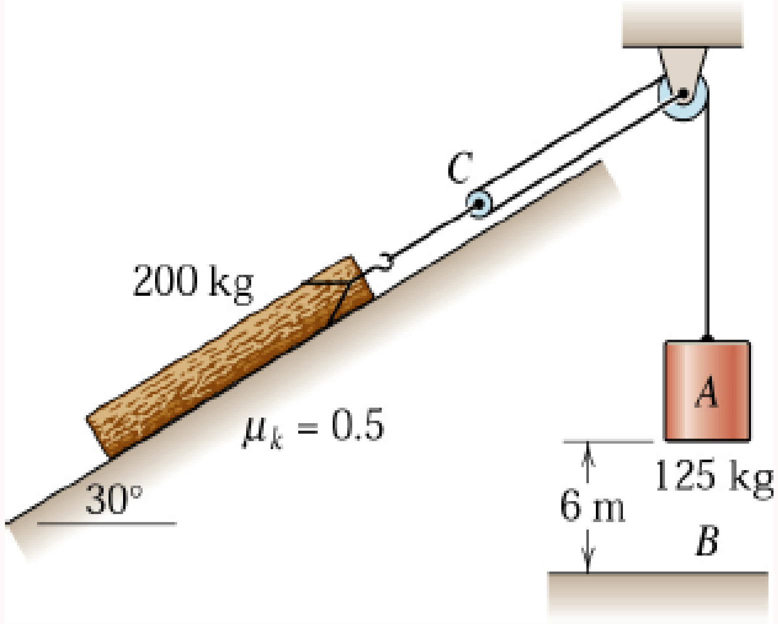
$$\Sigma \vec{F}_r = m\vec{a}_r$$
$$\Sigma \vec{F}_\theta = m\vec{a}_\theta$$

where $a_r = \ddot{r} - r\dot{\theta}^2$ and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

3-1. Force, Mass, and Acceleration

Example 1: A log and a pulley

The 125-kg concrete block *A* is released from rest in the position shown and pulls the 200-kg log up the 30° ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at *B*.

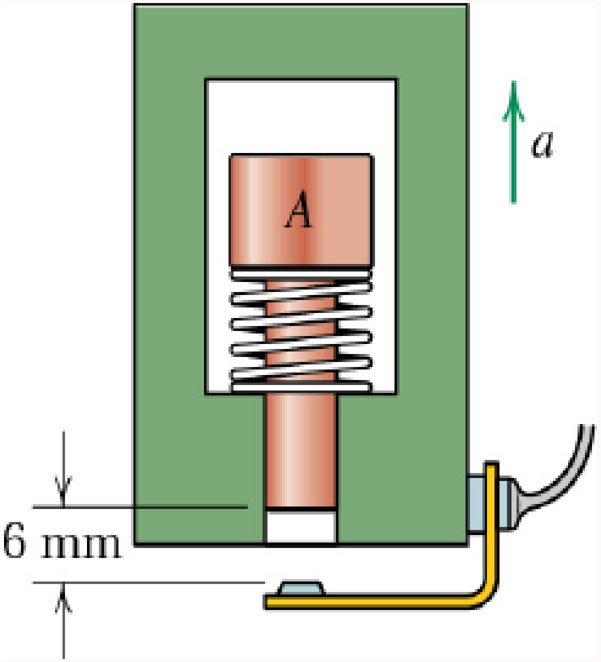


Ans: 4.62 m/s

3-1. Force, Mass, and Acceleration

Example 2: An accelerometer

The device shown is used as an accelerometer and consists of a 100-g plunger *A* which deflects the spring as the housing of the unit is given an upward acceleration *a*.



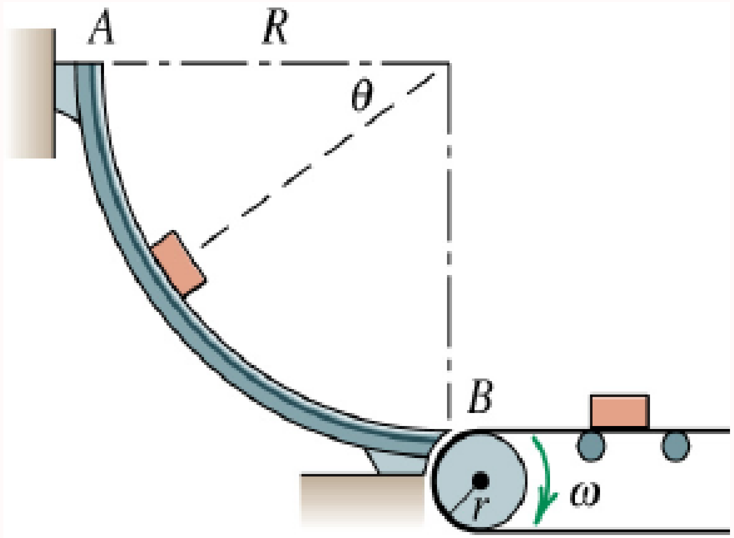
Specify the neccessary spring stiffness *k* which will permit the plunger to deflect 6 mm beyond the equilibrium position and touch the electical contact when the steadily but slowly increasing upward acceleration reaches 5g. Friction may be neglected.

Ans: 818 N/m

3-1. Force, Mass, and Acceleration

Example 3: A Conveyor

Small objects are released from rest at A and slide down the smooth circular surface of radius R to a conveyor B. Determine the expression for the normal contact force N between the guide and each object in terms of θ and specify the correct angular velocity ω of the conveyor pulley of radius r to prevent any sliding on the belt as the objects transfer to the conveyor.

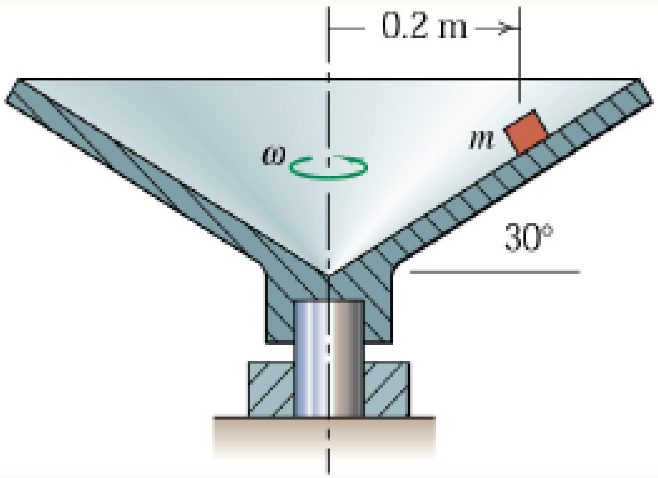


Ans: $N = 3mg\sin(\theta)$, $\omega=\sqrt{2gR}/r$

3-1. Force, Mass, and Acceleration

Example 4: A Conical dish

The small object is placed on the inner surface of the conical dish at the radius shown. If the coefficient of static friction between the object and the conical surface is 0.30, for what range of angular velocities ω about the vertical axis will the block remain on the dish without slipping? Assume that speed changes are made slowly so that any angular acceleration may be neglected.

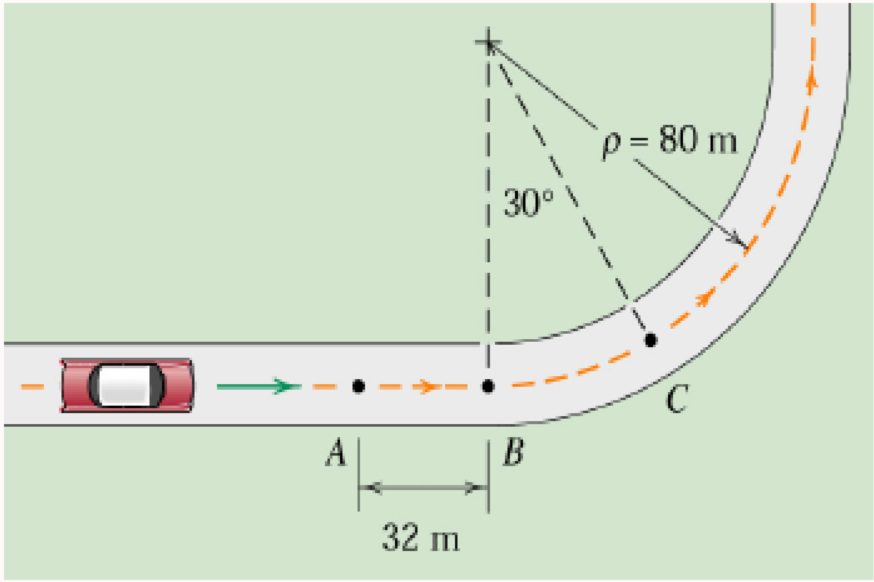


Ans: $3.41 \leq \omega \leq 7.21$ rad/s

3-1. Force, Mass, and Acceleration

Example 5: A car on a curve

The 1500-kg car is traveling at 100 km/h on the straight portion of the road, and then its speed is reduced uniformly from *A* to *C*, at which point it comes to rest. Compute the magnitude *F* of the total friction force exerted by the road on the car (a) just before it passes point *B*, (b) just after it passes point *B*, and (c) just before it stops at point *C*.



Ans: 7.83 kN, 11.34 kN, 7.83 kN