Chapter 6
Plane Kinetics of Rigid Bodies

6. Plane Kinetics

- Introduction
- 6.1 Force, Mass, and Acceleration (Newton's laws of motion)
- 6.2 Work and Energy
- 6.3 Impulse and Momentum

6. Plane Kinetics

Introduction

- Relations between forces and motion of rigid body will be studied.
- For rigid bodies, translational and rotational motions must be considered (for particles, only translational motion is).
- We will consider only <u>plane</u> motion (2-D motion)
  - Translation
    - Rectilinear
    - Curvilinear
  - Fixed Axis Rotation
  - General Plane Motion

6-1 Force, Mass, and Acceleration

6-1 Force, Mass, and Acceleration

- 1. Introduction
- 2. Force Equation
- 3. Moment Equation (about G)
- 4. Kinetic Diagram
- 5. Moment Equation about Other Point
- 6. Translation
  - Rectilinear
  - Curvilinear
- 7. Fixed Axis Rotation
- 8. General Plane Motion

6-1 Force, Mass, and Acceleration

1. Introduction

- A free body diagram is required.
- Three Newton's laws of Motion are used.
- The second law has two equations,
  - force equation
  - moment equation

  both applies simultaneously.

  Proofs are in Chapter 4: Systems of Particles.

$$\Sigma\vec{F} = m\vec{a}_G$$
$$\Sigma\vec{M}_G = \dot{\vec{H}}_G$$

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2. Force Equation

Newton's Second Law (Rigid Body)

$$\Sigma \vec{F} = m\vec{a}_G$$

- $\vec{F}$ = forces acting on the rigid body,
- m = mass of the body,
- $\vec{a}_G$ = acceleration of the center of mass, G

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3. Moment Equation (about G)

The Moment Equation (Rigid Body)

$$\Sigma M_G = I_G \alpha$$

- $\vec{M}_G$ = moment (of external force) about G
- $\vec{H}_G$ = Angular momentum of the system about G

- Hence, for plane motion

$$H_G = \bar{I}\omega = I_G\omega$$

- Then, (since I_G is a constant)

$$\dot{H}_G = \bar{I}\dot{\omega} = I_G\dot{\omega}$$

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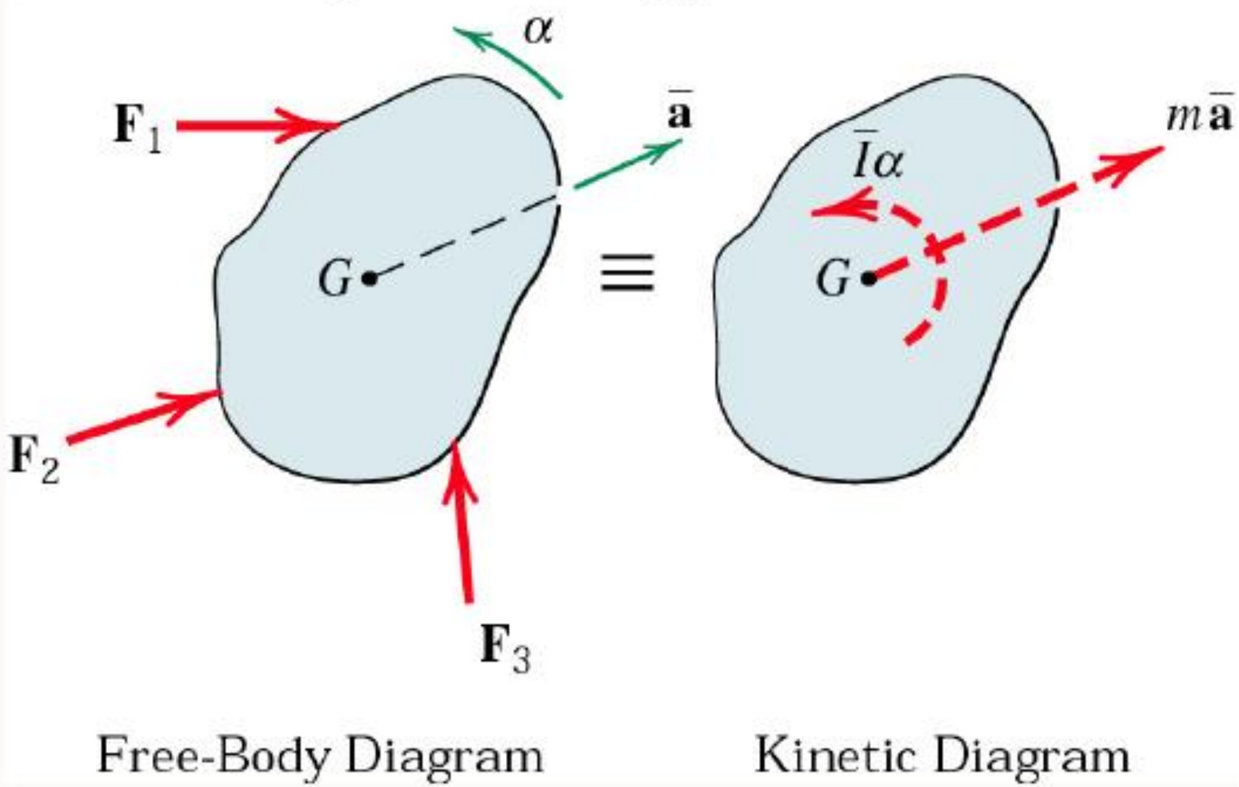
4. Kinetic Diagram

- A Kinetic Diagram is strongly recommended.



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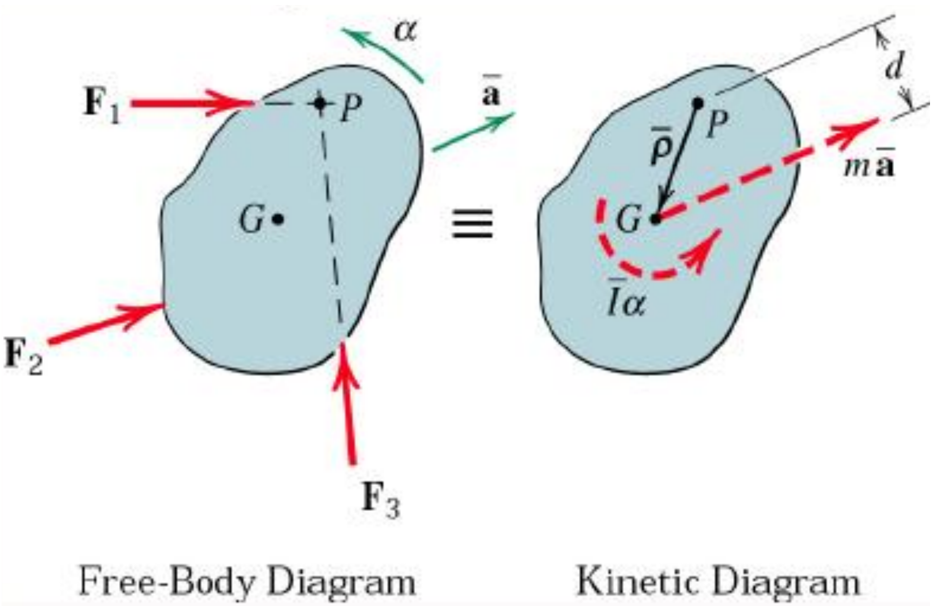
5. Moment Equation about Other Point

Alternative Moment Equation

$$\Sigma \vec{M}_P = I_G \vec{\alpha} + \vec{\rho}_G \times m\vec{a}_G$$

- M_P = moment about some point P
- ρ_G = vector from P to the mass center G



$$\Sigma M_P = I_G \alpha + d m a_G$$

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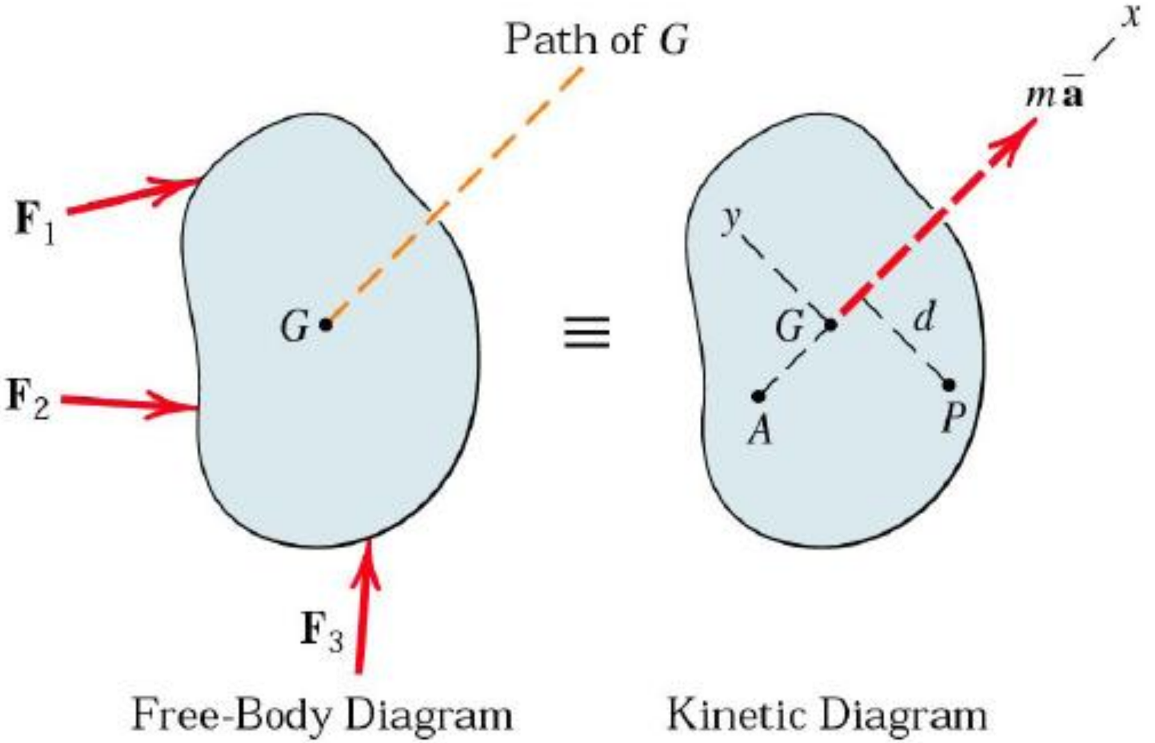
6.1 Rectilinear Translation



(a) Rectilinear Translation
($\alpha = 0$, $\omega = 0$)

- $\Sigma\vec{F} = m\vec{a}_G$, $\Sigma\vec{M}_G = 0$

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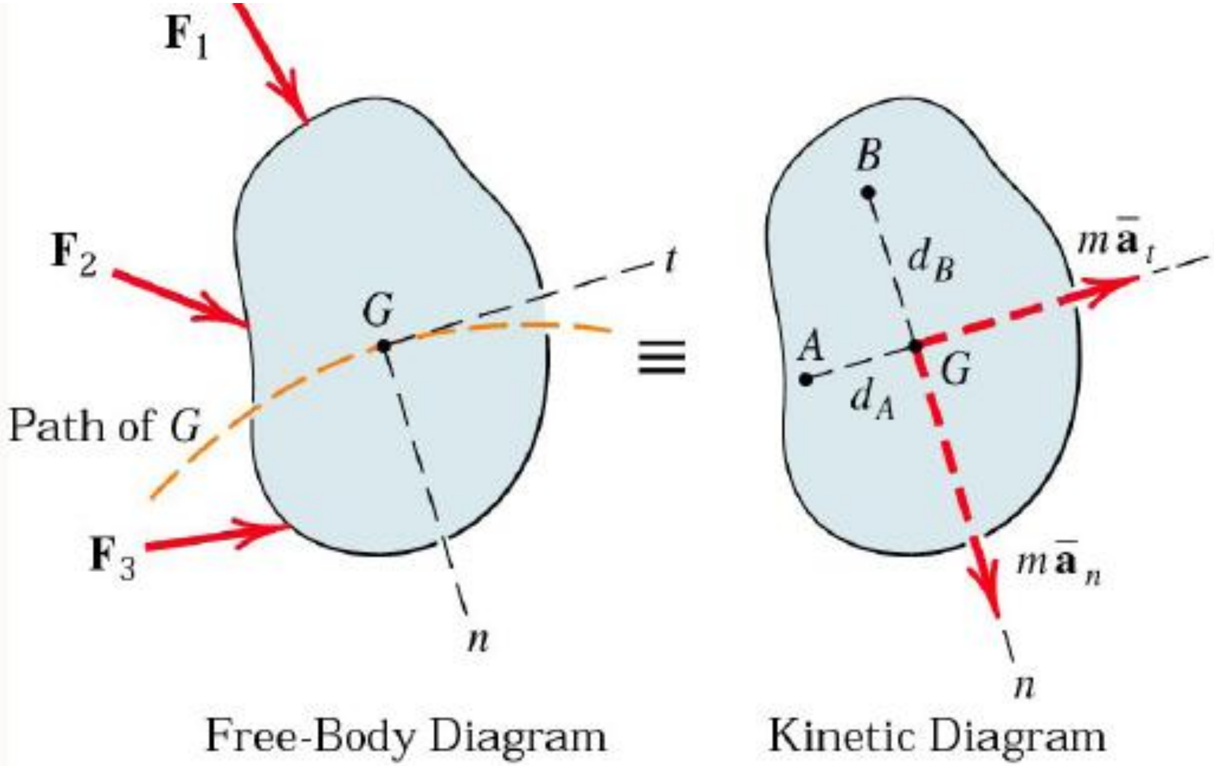
6.2 Curvilinear Translation



(b) Curvilinear Translation
$(\alpha = 0,\ \omega = 0)$

- $\Sigma\vec{F} = m\vec{a}_G,\ \Sigma\vec{M}_G = 0$

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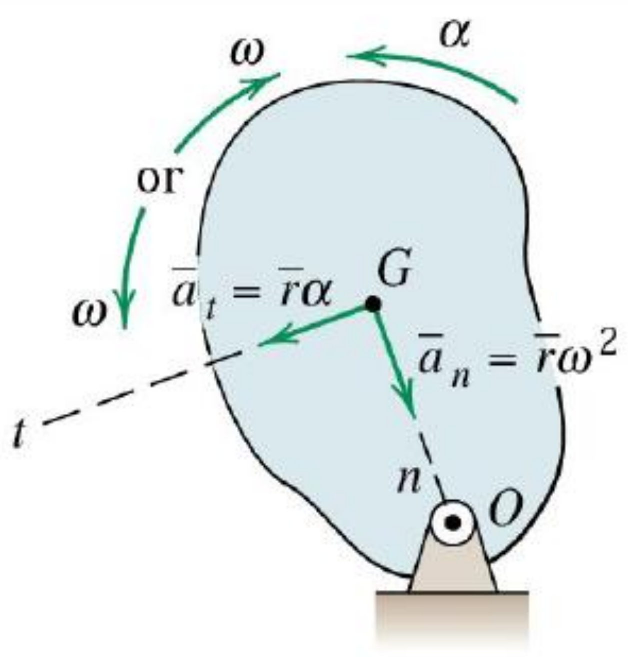
7. Fixed Axis Rotation



Fixed-Axis Rotation

Kinematics:

- $(\vec{a}_G)_t = \vec{\alpha}_{OG} \times \vec{r}_{O\to G}$
- $(\vec{a}_G)_n = \vec{\omega}_{OG} \times (\vec{\omega}_{OG} \times \vec{r}_{O\to G})$

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7. Fixed Axis Rotation

EOM: General Motion

$$\Sigma \vec{F} = m\vec{a}_G$$
$$\Sigma M_G = I_G \alpha$$

and

$$\Sigma M_O = I_O \alpha$$

- Rigid body rotates about O
- M_O = moment of forces about O
- I_O = mass moment of inertia about point O

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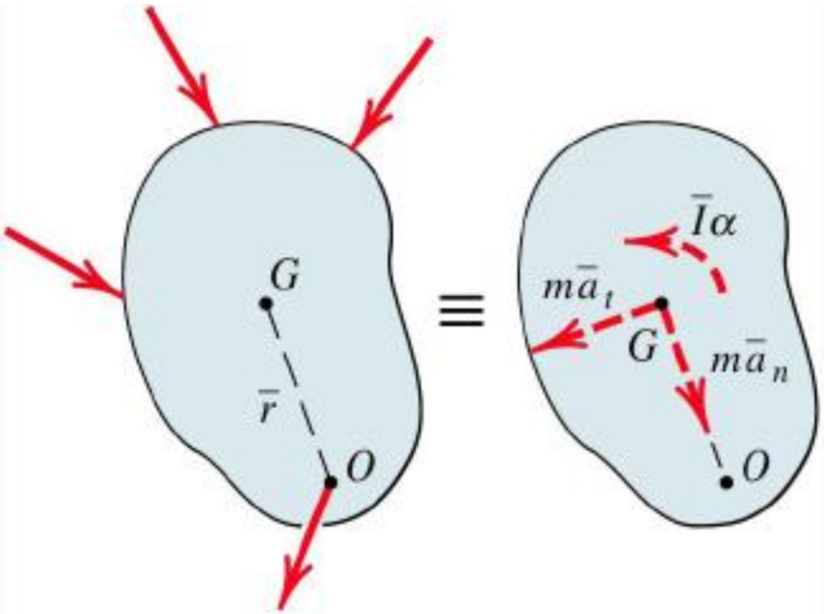
7. Fixed Axis Rotation

Proof:

Moment Equation about point O

$$\Sigma M_O = I_O \alpha$$



Free-Body Diagram Kinetic Diagram

- $\Sigma M_O = I_G \alpha + m a_{Gt} \bar{r}$
- But, $a_{Gt} = \alpha \bar{r}$
- Then,

$$\Sigma M_O = I_G \alpha + m\bar{r}^2 \alpha$$

$$\Sigma M_O = (I_G + m\bar{r}^2)\alpha$$

- We know that

$$I_O = I_G + m\bar{r}^2$$

↑ Parallel Axis Theorem

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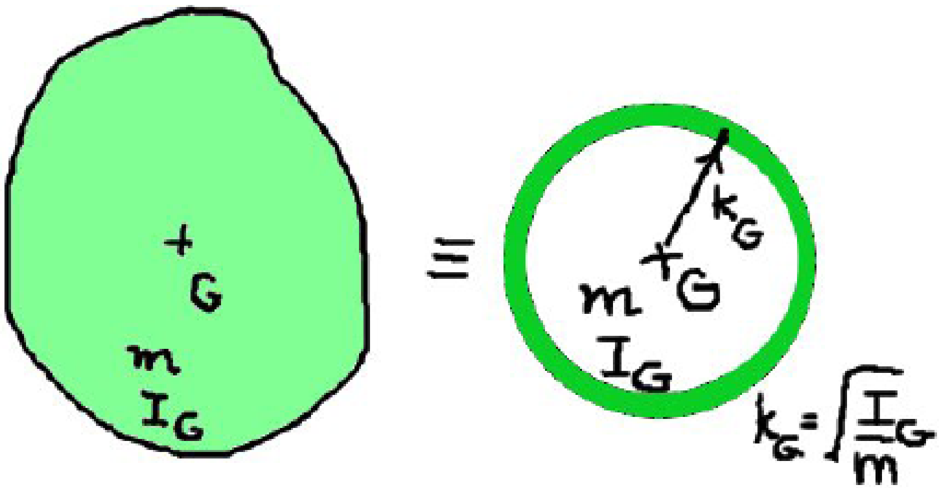
7. Fixed Axis Rotation

Radius of Gyration

- Radius of gyration is often used to specify the mass moment of inertia of a rigid body.
- Given mass m and the radius of gyration k_P (about point P), we have

$$I_P = mk_P^2$$

- Usually point P is G or the fixed point O
- Imagine mass concentrated at radius k_G



6-1 Force, Mass, and Acceleration

8. General Plane Motion

Same as before

EOM: General Motion

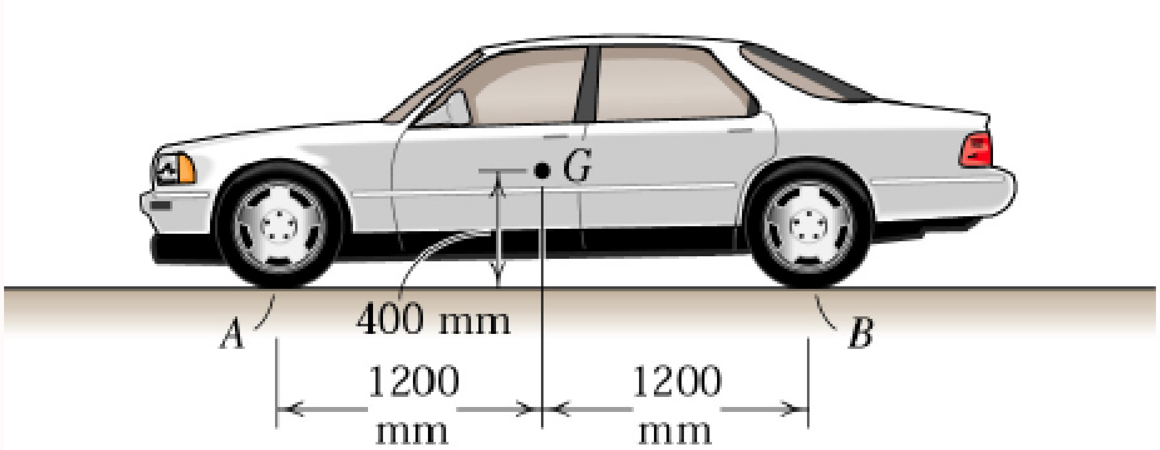
$$\Sigma \vec{F} = m\vec{a}_G$$

$$\Sigma M_G = I_G \alpha$$

6-1 Force, Mass, and Acceleration

Example 1: A Moving Car

The 1600-kg car has its mass center at G. Calculate the normal force N_A and N_B between the road and the front and rear pairs of wheels when acceleration of the car is 2 m/s^2. The mass of the wheels are small compared to the mass of the car.



- Do this: If the coefficient of **static** friction between the tire and the ground is 0.8, what is the maximum possible acceleration of this car if
  - it is a front wheel drive car,
  - it is a rear wheel drive car.

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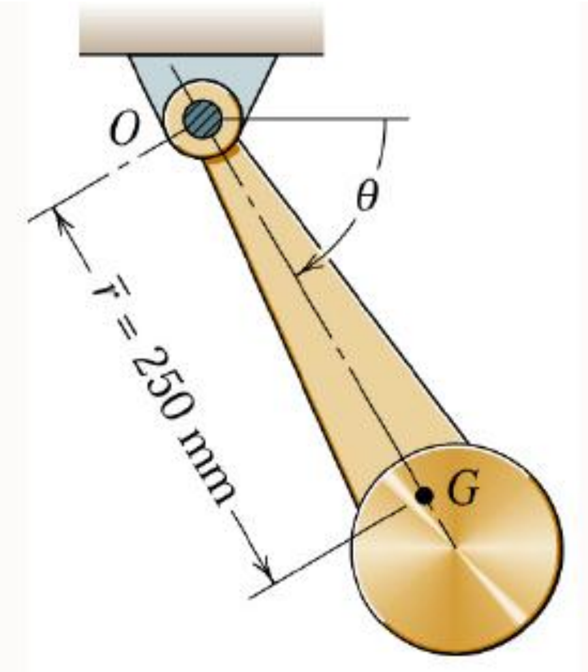
Example 2: Swinging Pendulum

The pendulum has a mass of 7.5 kg with center of mass at G and has a radius of gyration about the pivot O of 295 mm. If the pendulum is released from rest at $\theta = 0$, determine the total force supported by the bearing at the instant when $\theta = 60°$. Friction in the bearing is negligible.



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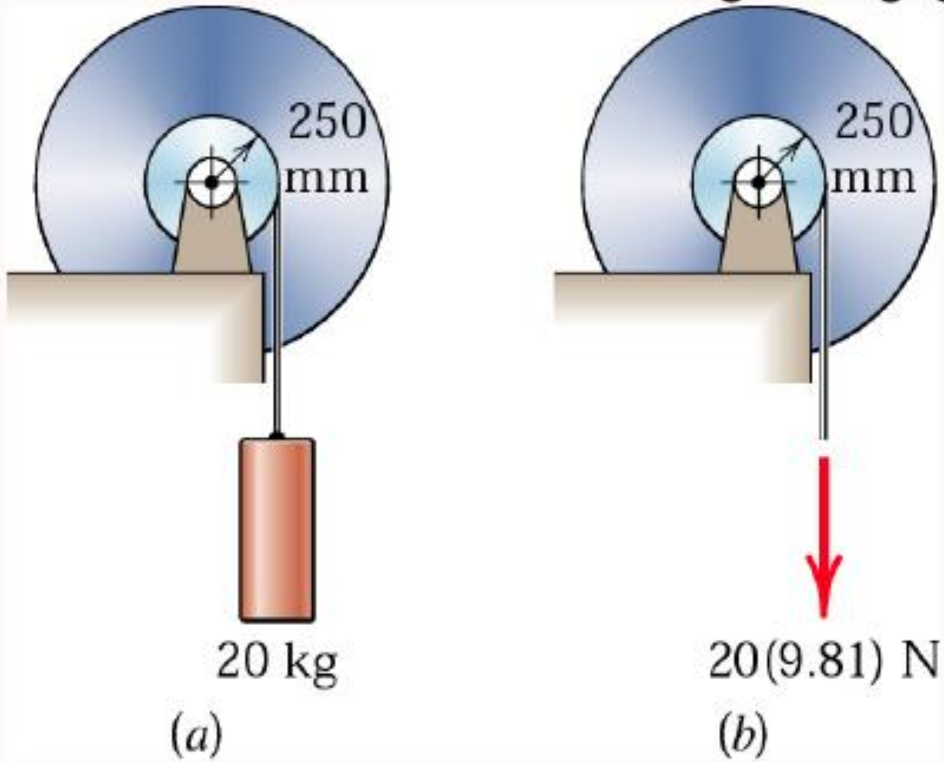
Example 3: Concept Test

Each of the two drums and connected hubs of 250 mm radius has a mass of 100 kg and has a radius of gyration about its center of 375 mm. Calculation the angular acceleration of each drum. Friction in each bearing is negligible.



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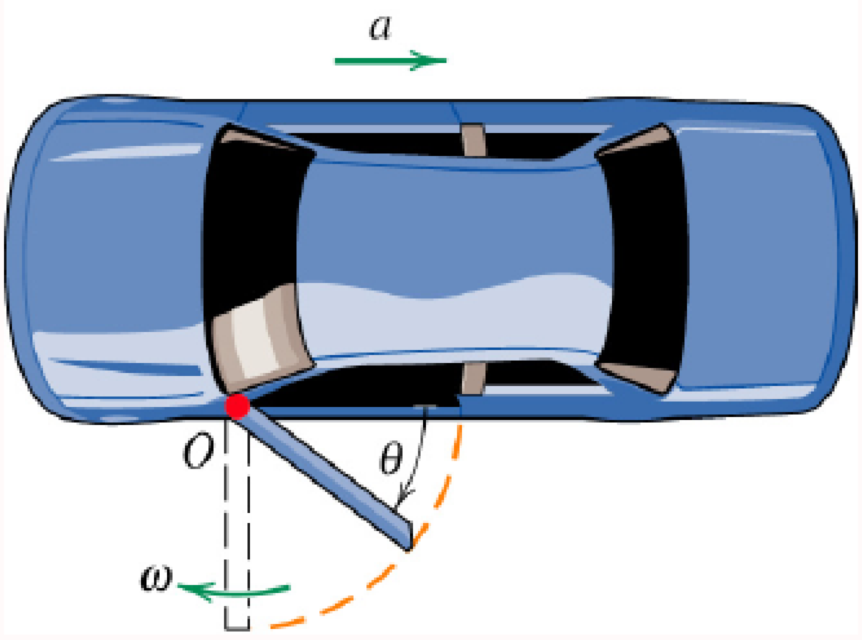
Example 4: Car's Door

A car door swings open when the car brakes with acceleration a. Derive expressions for the angular velocity of the door as it swing pass the 90° and the forces at O for any value of θ. The door has mass m, mass center is located at $\bar{r}$ from O, and the radius of gyration about O is k_O.



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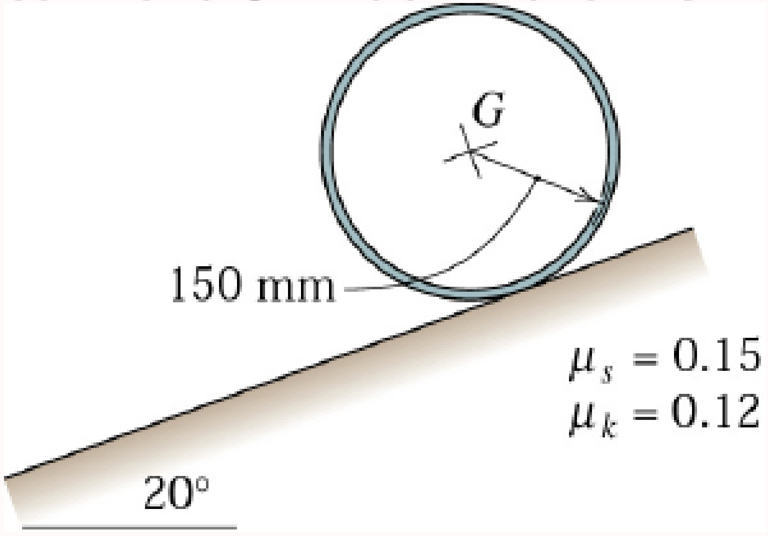
Example 5: Rolling Loop

A metal hoop with a radius $r = 150$ mm is released from rest on the 20° incline. If $\mu_s = 0.15$ and $\mu_k = 0.12$, determine the angular acceleration α of the hoop and the time t for the hoop to move 3 m down the incline.



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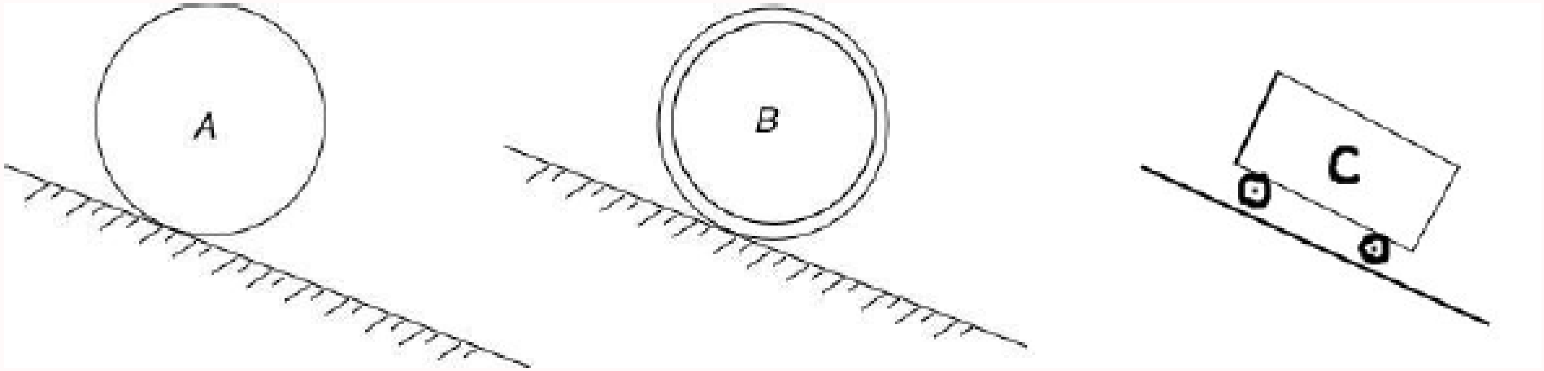
Example 6: Which one is the fastest?

Which one roll down the incline the fastest? (same mass, noslip)



(No friction for case C)