2/5 Normal and Tangential Coordinate (*n*-*t*)

2. Normal And Tangential Coordinate (*n-t*)

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- Acceleration
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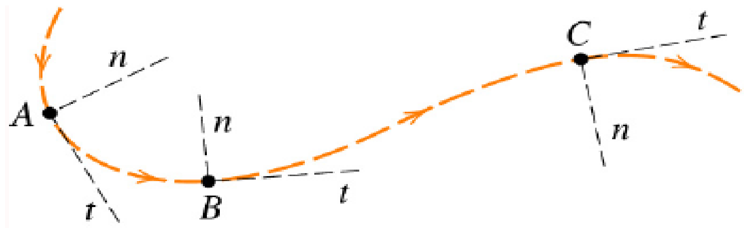
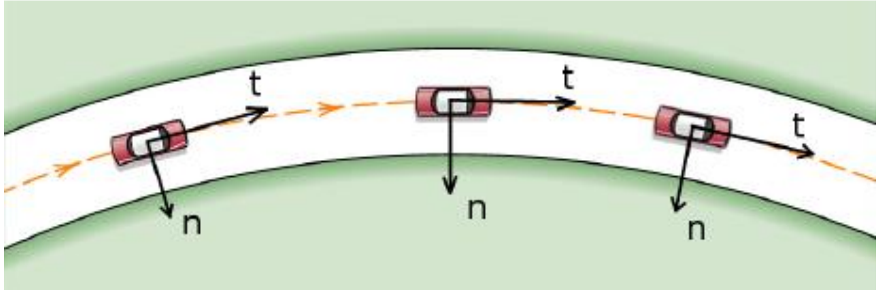
- Examples

2. Normal And Tangential Coordinate (*n*-*t*)



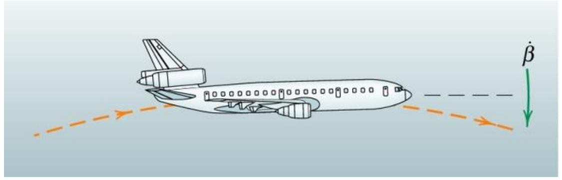
- Most convenient when position, velocity, and acceleration are described relative to the path of the particle itself
- Origin of this coordinate moves with the particle (Position vector is zero)
- The coordinate axes rotate along the path
  - *t* coordinate axis is **tangential** to the path and points to the direction of **positive** velocity.
  - *n* coordinate axis is **normal** to the path and points **toward center** of curvature of the path.

2. Normal And Tangential Coordinate (*n-t*)



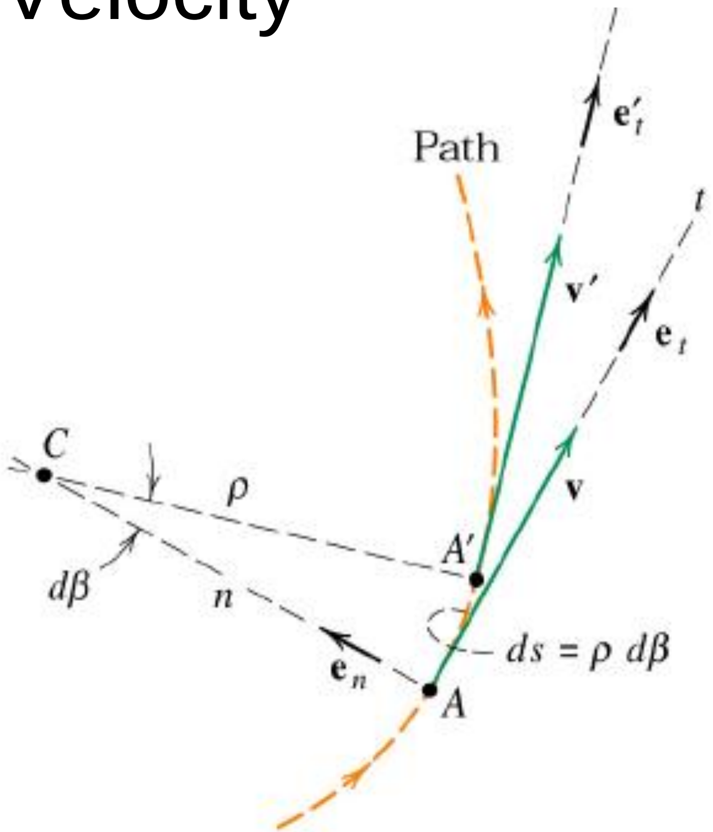
Applications

- Moving car
  - Forward/backward velocity and forward/backward/lateral acceleration make more sense to the driver.
  - Brake and acceleration forces are often more convenient to describe relative to the car (in the *t* direction)
  - Turning (side) force also easier to describe relative to the car (in the *n* direction)



2. Normal And Tangential Coordinate (*n*-*t*)

Velocity



- For a short period of time, *dt*
- Path from *A* to *A′* can be approximated as an arc of a circle
- The center of the circle is at *C*, the **center of curvature**.
- The radius of this circle is call the **radius of curvature**, ρ

Notes:

- The center of curvature *C* can move
- Radius of curvature ρ is not constant

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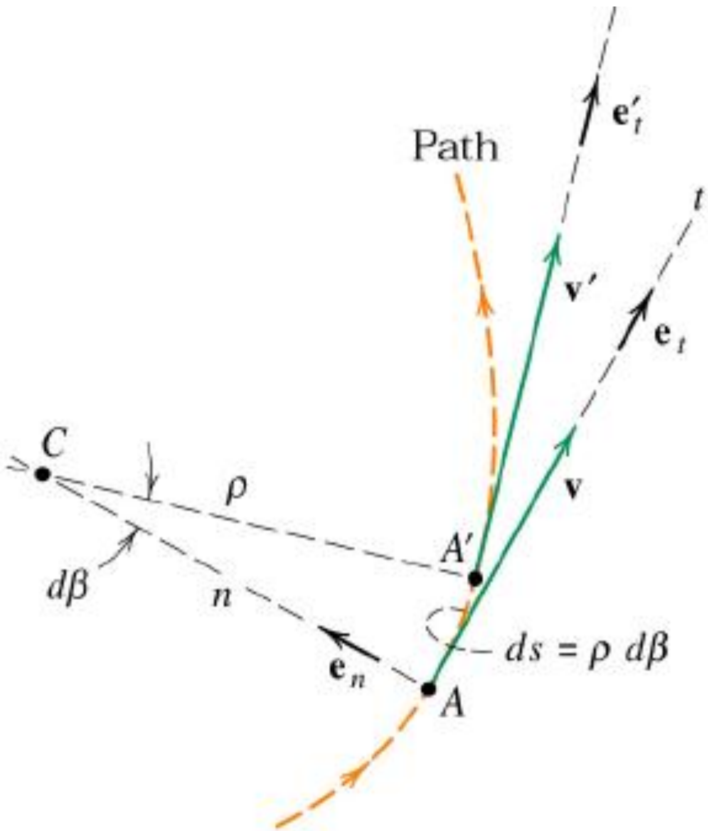
Velocity



- During dt, $\hat{e}_n$ rotated $d\beta$
- Distance travelled is $ds = \rho d\beta$
- Recall that $\vec{v}$ is tangent to the path and that $v = ds/dt$

Velocity (*n*-*t*)

$$\vec{v} = v\,\hat{e}_t = \rho\dot{\beta}\,\hat{e}_t$$

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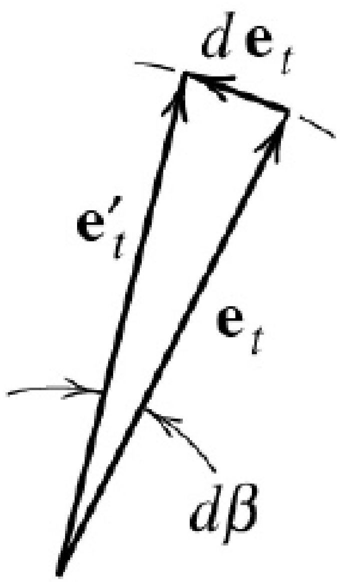
Acceleration



- $\vec{a} = d\vec{v}/dt \;= v\dfrac{d\,\hat{e}_t}{dt} + \dot{v}\,\hat{e}_t$
- Now we need $\frac{d\,\hat{e}_t}{dt}$
- From the figure, $\hat{e}_t$ changes $d\beta$ in dt

$$d\,\hat{e}_t = |\hat{e}_t| \times d\beta\hat{e}_n$$

Derivative of $\hat{e}_t$

$$\frac{d\hat{e}_t}{dt} = \dot{\beta}\,\hat{e}_n$$

Acceleration (*n-t*)

$$\vec{a} = \frac{v^2}{\rho}\,\hat{e}_n + \dot{v}\,\hat{e}_t$$

$$\begin{aligned} a_n &= \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta} > 0 \\ a_t &= \dot{v} = \ddot{s} \\ a &= \sqrt{a_n^2 + a_t^2} \end{aligned}$$

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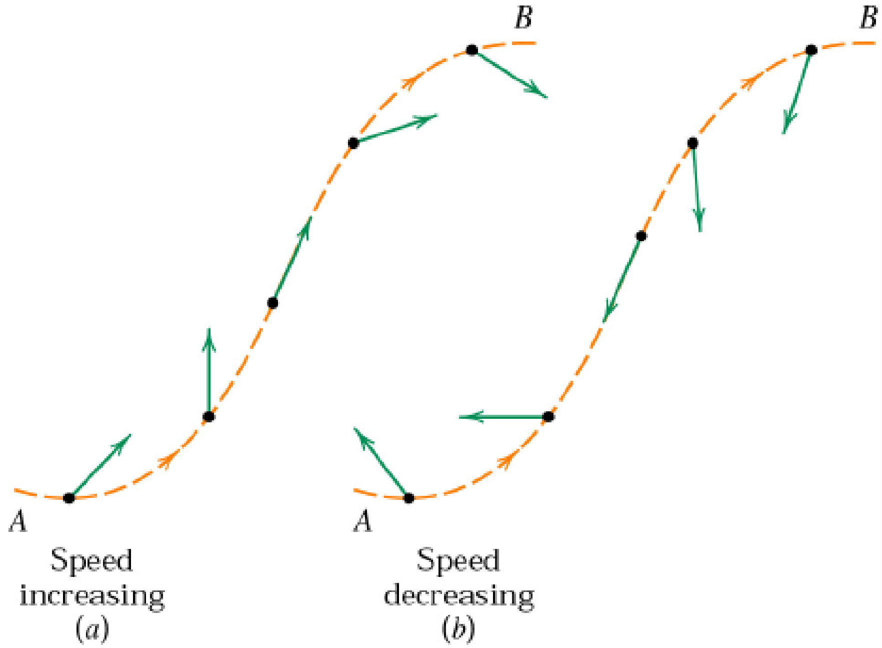
Acceleration

Directions of a_t and a_n



The arrows show the **acceleration** of a particle is moving from A to B

If speed is increasing a_t // **v** // $\mathbf{e}_t$
If speed is decreasing a_t // - **v** // - $\mathbf{e}_t$

a_n is always directed **toward** the center of curvature

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v and a_t

The formula for the velocity/acceleration in the *t* direction is the same as those of rectilinear motion.

$$a_t = \dot{v} = \ddot{s}$$ = change in speed

$$v = \frac{ds}{dt}$$

$$a_t = \frac{dv}{dt}$$

$$a_t\, ds = v\, dv$$

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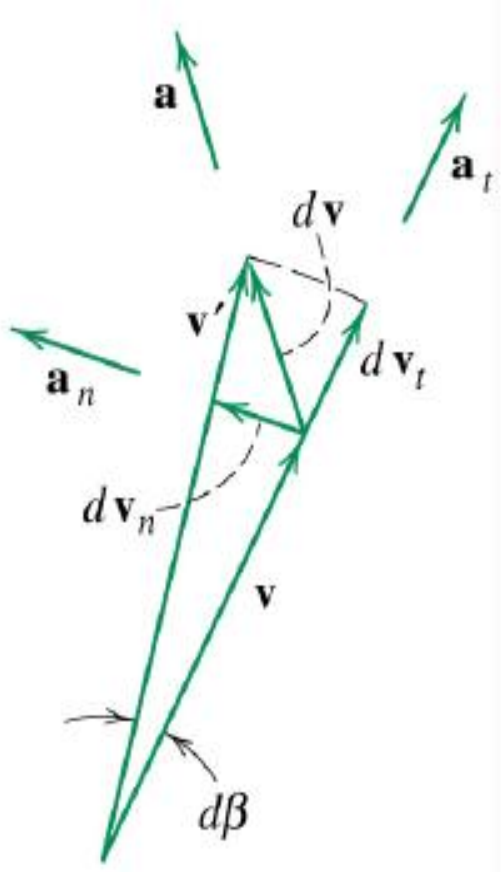
Geometric representation



- a_n is a result of change in the magnitude of $\vec{v}$
- a_t is a result of change in the direction of $\vec{v}$

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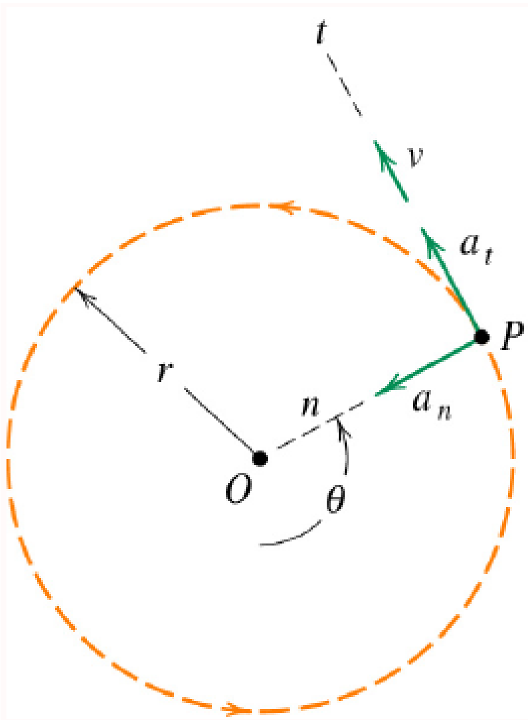
Special Case: Circular motion



- Radius of curvature ρ becomes constant radius r
- β is an angle θ from any reference to $\overline{OP}$

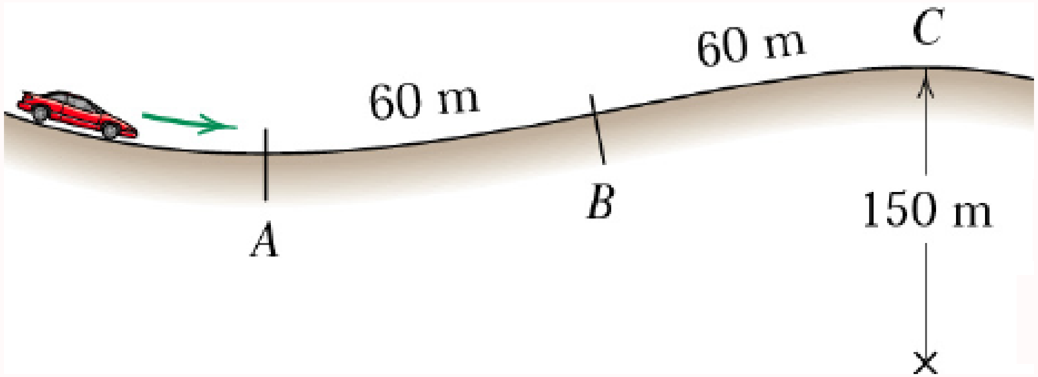
Circular Motion (*n-t*)

$$
\begin{aligned}
v &= r\dot{\theta} \\
a_n &= v^2/r = r\dot{\theta}^2 = v\dot{\theta} \\
a_t &= \dot{v} = r\ddot{\theta}
\end{aligned}
$$

2. Normal And Tangential Coordinate (*n*-*t*)

Example 1: Car on a hill

To anticipate the dip and hump in the road, the driver of a car applies her brakes to produces a uniform deceleration. Her speed is 100 km/h at the bottom A of the dip and 50 km/h at the top C of the hump, which is 120 m along the road from A. If the passengers experience a total acceleration of 3 m/s^2 at A and if the radius of curvature of the hump at C is 150 m, calculate (a) the radius of curvature ρ at A, (b) the acceleration at the inflection point B, and (c) the total acceleration at C.



Ans: $\rho_A = 432\,\text{m}$

$a_B = 2.41\,\text{m/s}^2$ in -t direction

$\vec{a} = 1.286\,\hat{e}_n - 2.41\,\hat{e}_t\ \text{m/s}^2$

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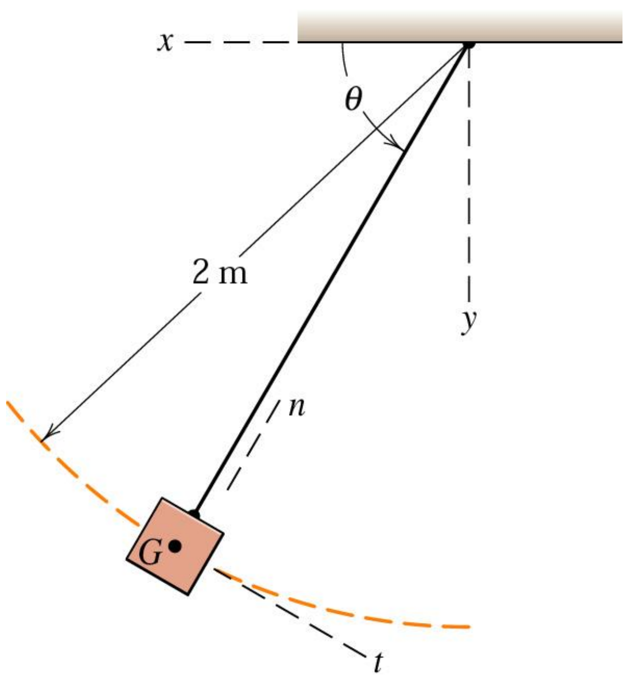
Example 2: Pendulum

Write the vector expression of the acceleration **a** of the mass center G of the simple pendulum in both *n*-*t* and *x*-*y* when $\theta = 60^\circ, \dot{\theta} = 2 \text{ rad/s and } \ddot{\theta} = 2.45 \text{ rad/s}^2$



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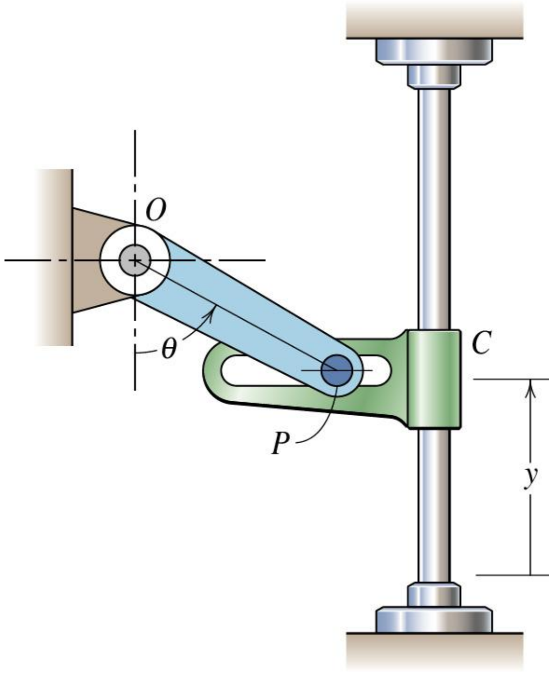
Example 3: Crank and Slot

Pin P in the crank PO engages the horizontal slot in the guide C and controls its motion on the fixed vertical rod. Determine the velocity and the acceleration of the guide C if a) $\dot{\theta} = \omega \quad \ddot{\theta} = 0$

b) $\dot{\theta} = 0 \quad \ddot{\theta} = \alpha$



Ans: a) $\dot{y} = r\omega \sin\theta \qquad \ddot{y} = r\omega^2 \cos\theta$

b) $\dot{y} = 0 \qquad \ddot{y} = r\alpha \sin\theta$

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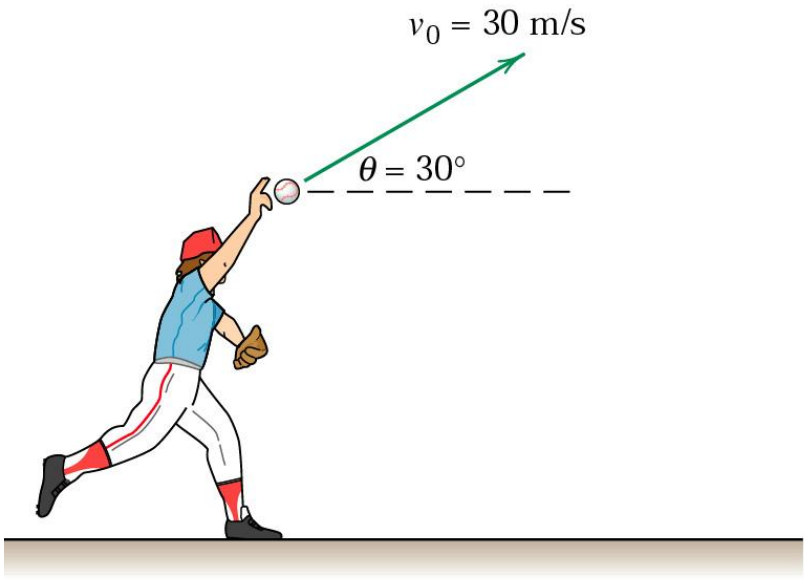
Example 4: Baseball

A baseball player releases a ball with the initial conditions shown. Determine the radius of curvature of the trajectory a) just after release and b) at the apex. For each case, compute the time rate of change of the speed.



Ans: a) 105.9 m, -4.91 m/s^2

b) 68.8 m, 0 m/s^2