Chapter 2 Kinematics of Particles

Kinematics of Particles

What is Kinematics of Particles?

- Study of **motion** of bodies (assumed as particles) **without** reference to **forces**
- Kinematics of Particles "describes" motion of particle, generally, the **relations between**
  - position (displacement)
  - velocity
  - Acceleration
- Example: A car
  - Given the velocity as a function of time, how far did the car moved?
  - Given the velocity of the car as a function of time, what is the acceleration at each point in time?

Rectilinear Motion

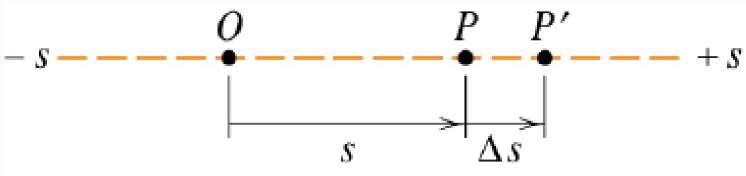
Rectilinear Motion

1. Displacement and Instantaneous Velocity
2. Instantaneous Acceleration
3. Graphical Interpretation
4. Special Case: Constant Acceleration
5. Examples

Rectilinear Motion

1. Displacement and Instantaneous Velocity

- For a straight motion of a particle;



- Position of P is specified by the displacement *s* (scalar) measured from some fixed point *O*.
- During Δt sec, P moved Δs m
- Average speed, $v_{av} = \Delta s / \Delta t$ m/s
- $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$

Instantaneous Velocity

$$v = \frac{ds}{dt} = \dot{s}$$

Rectilinear Motion

2. Instantaneous Acceleration

- Similarly, we can define instantaneous acceleration
- At time t_1 the velocity is v_1, at time t_2 the velocity is v_2
- So the average acceleration is

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

- Again, taking the limit as $\Delta t \to 0$ or $t_2 \to t_1$,

Instantaneous Acceleration

$$a = \frac{dv}{dt} = \dot{v} \qquad \text{or} \qquad a = \frac{d^2 s}{dt^2} = \ddot{s}$$

Rectilinear Motion

2. Instantaneous Acceleration

- Using the equations, we have

$$vdt = ds \qquad \text{and} \qquad adt = dv$$

- Eliminating dt, we have

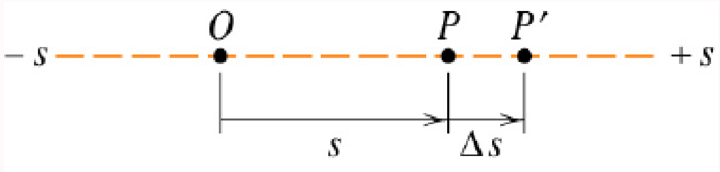
Instantaneous Acceleration

$$vdv = ads$$

Rectilinear Motion

Notes on directions

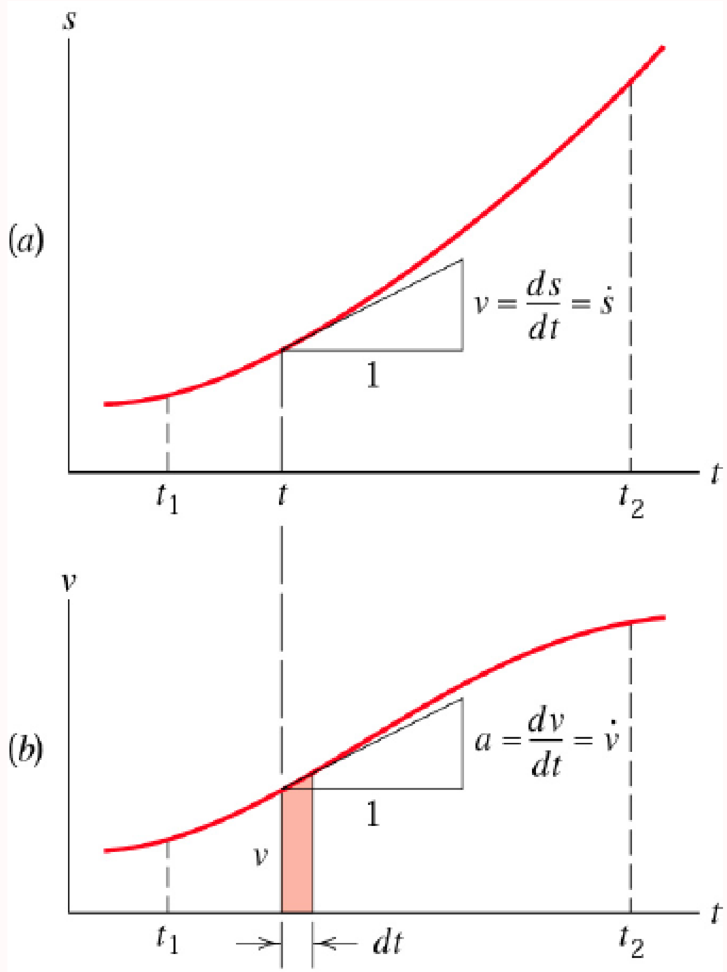
- Positive direction of *a*, *v*, and *s* must be the same!



- If we defind $+s$ to the right
- $\vec{v}$ and $\vec{a}$ pointing to the right are positive.
- Positve *v* means *s* is increasing (since *ds* is positive).
- Similarly, positive *a* means *v* is increasing.

Rectilinear Motion

3. Graphical Interpretation

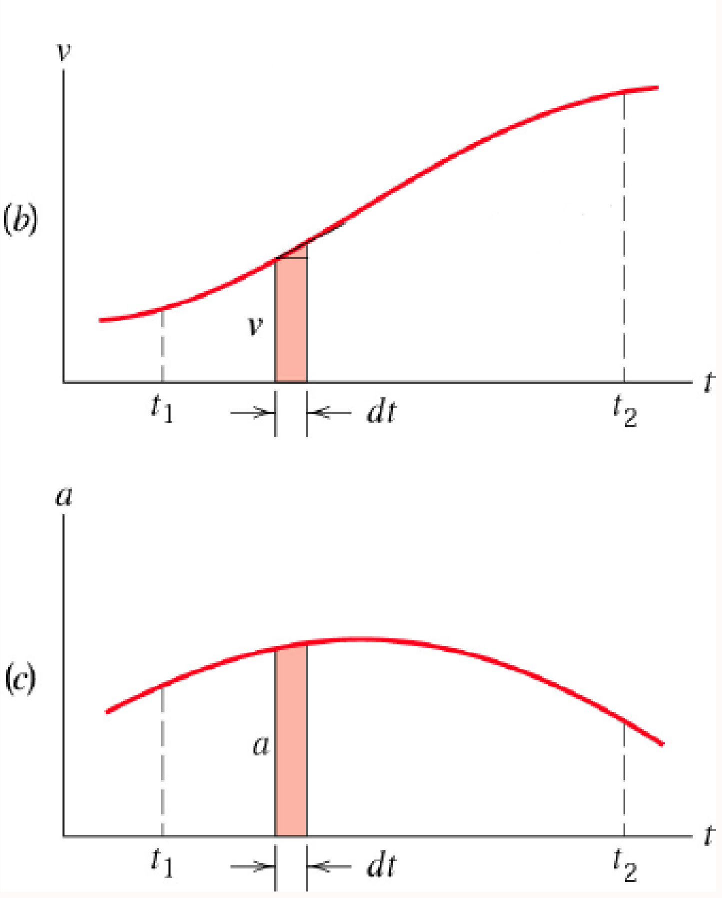


- Slope of *s*-*t* curve = velocity
- Slope of *v*-*t* curve = acceleration

Rectilinear Motion

3. Graphical Interpretation

- The usual interpretations: Area under curves



- Area under v-t curve = (changes in) displacement

$$\int_{t_1}^{t_2} v\,dt = \int_{s_1}^{s_2} ds = s_2 - s_1$$

- Area under a-t curve = (changes in) velocity

$$\int_{t_1}^{t_2} a\,dt = \int_{v_1}^{v_2} dv = v_2 - v_1$$

Rectilinear Motion

4. Special Case: Constant Acceleration

Constant Acceleration: $v(t)$

$$v(t) = v_1 + a(t - t_1)$$

Constant Acceleration: $v(s)$

$$v^2(s) = v_1^2 + 2a(s - s_1)$$

Constant Acceleration: $s(t)$

$$s = s_1 + v_1(t - t_1) + \frac{a}{2}(t - t_1)^2$$

Rectilinear Motion

Examples 1: Viscosity

Let’s set up an experiment to determine the viscosity of a fluid. By dropping the ball into a column of fluid, the acceleration of the ball is then obtained as

$$a = g - \frac{c}{m}v^2$$

where *c* depends on the fluid viscosity.

Find

a) the terminal velocity of the ball (the column is high enough)

b) the distance needed to achieve 95% of this terminal velocity

Rectilinear Motion

Examples 2:

A body moves in a straight line with the velocity's square as shown. Determine the displacement of the body during the last 2 seconds before arrival at *B*.