Chapter 5

Plane Kinematics of Rigid Bodies

5. Plane Kinematics

- Introduction
- 5.1 Rotation
- 5.2 Absolute Motion
- 5.3 Relative Velocity
- 5.4 Relative Acceleration

5. Plane Kinematics

1.1 Introduction

	Particle	Rigid Body
Size	Small & Not important	Big & Important
Motion	Translation only	Translation and Rotation

- Rigid body

  = a body with negligible deformation

  = distance between any two points in a rigid body is constant

5. Plane Kinematics

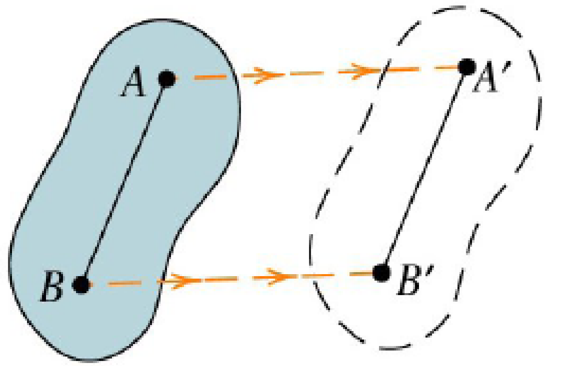
1.2 Motions of a Rigid Body

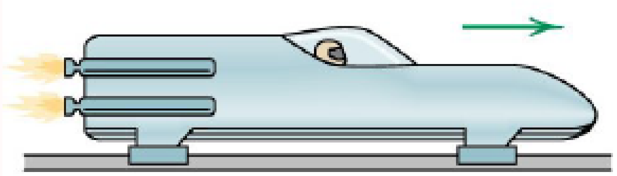
- 1. In space = three dimensions
- 2. In plane = two dimensions
  - Translation
    - Rectilinear
    - Curvilinear
  - Rotation

5. Plane Kinematics

1.3 Plane Motions of a Rigid Body

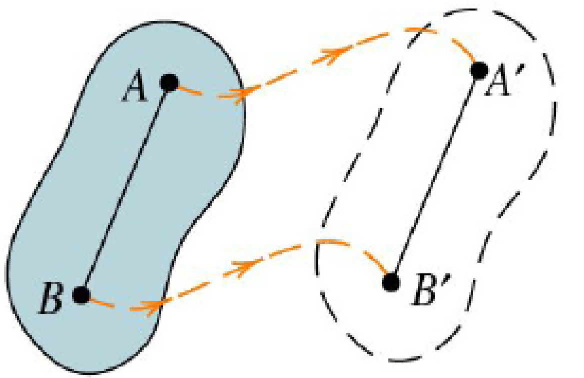
- Translation
  - Rectilinear Translation

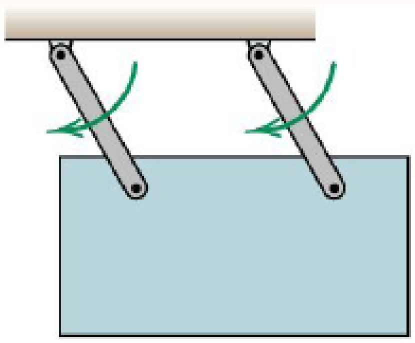




Rocket test sled

  - Curviliear translation

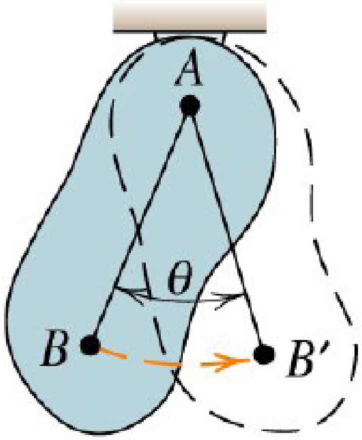


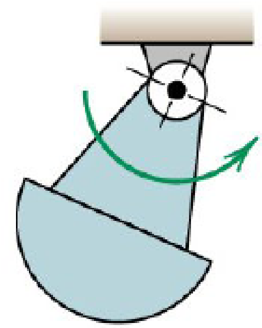


Parallel-link swinging plate

1.3 Plane Motions of a Rigid Body

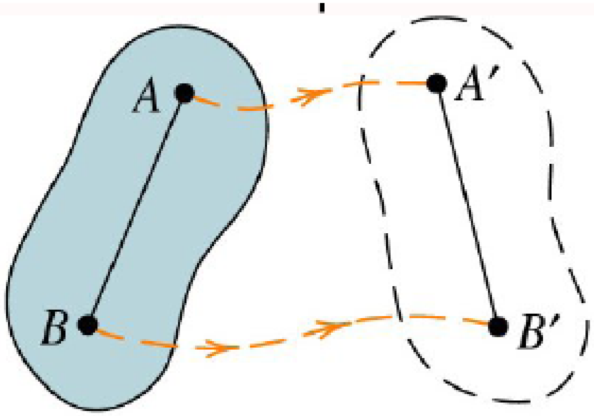
- Rotation
  - Fixed-axis rotation

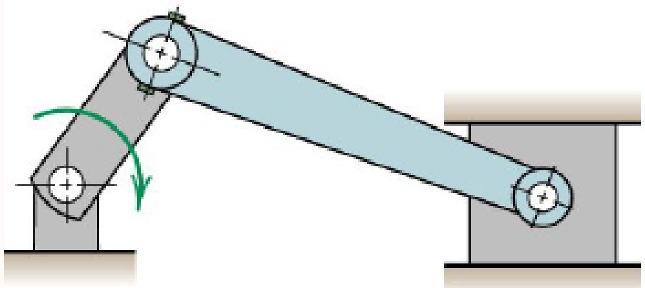




Compound pendulum

- General Plane Motion = Translation + Rotation



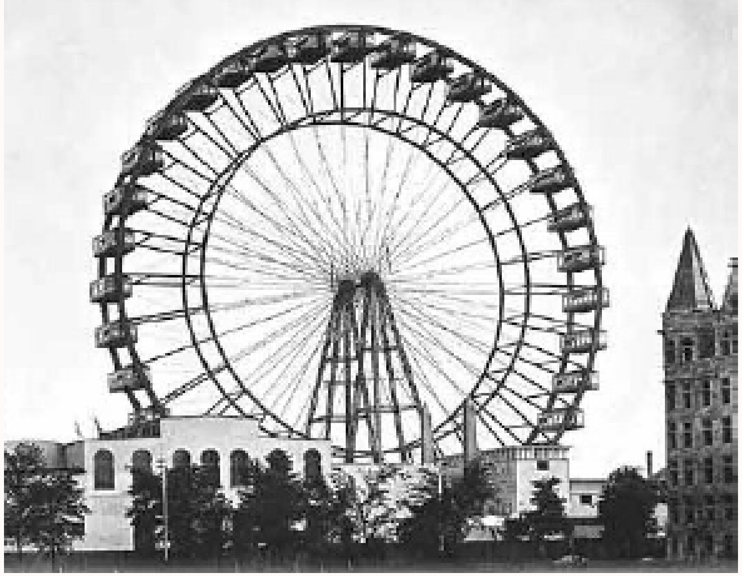


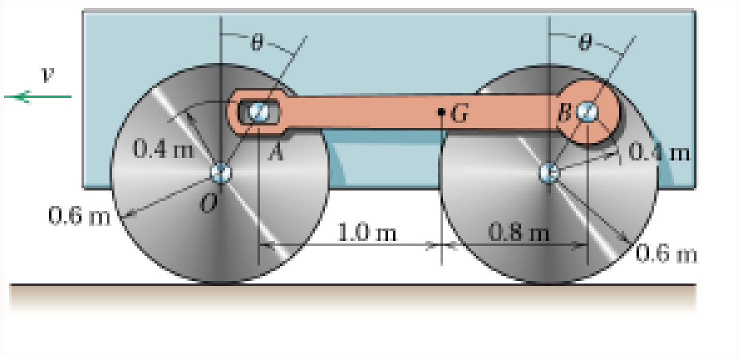
Connecting rod in a reciprocating engine

5. Plane Kinematics

1.3 Plane Motions of a Rigid Body

What is the type of motion of these bodies?



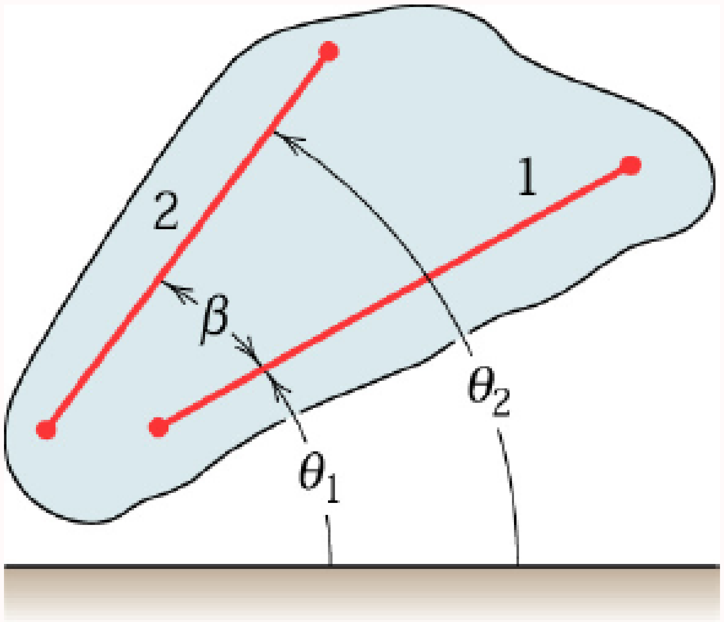


- Wheel?, Car?, Link AB?
- Ferris wheel: the wheel?, the car?

5-1 Rotation

5-1 Rotation

- How to describe rotation of a rigid body?



- **Angle between any line on a body and a reference line can be used to measure rotation of the body.**
- $\theta_2 = \theta_1 + \beta$
- For a rigid body, β = constant.
- Angular velocity $\dot{\theta}_2 = \dot{\theta}_1$
- Angular acceleration $\ddot{\theta}_2 = \ddot{\theta}_1$

- ω as well as α is the same for every point

5-1 Rotation

Rotation

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\omega\, d\omega = \alpha\, d\theta$$

- For constant angular acceleration (α=constant), we have

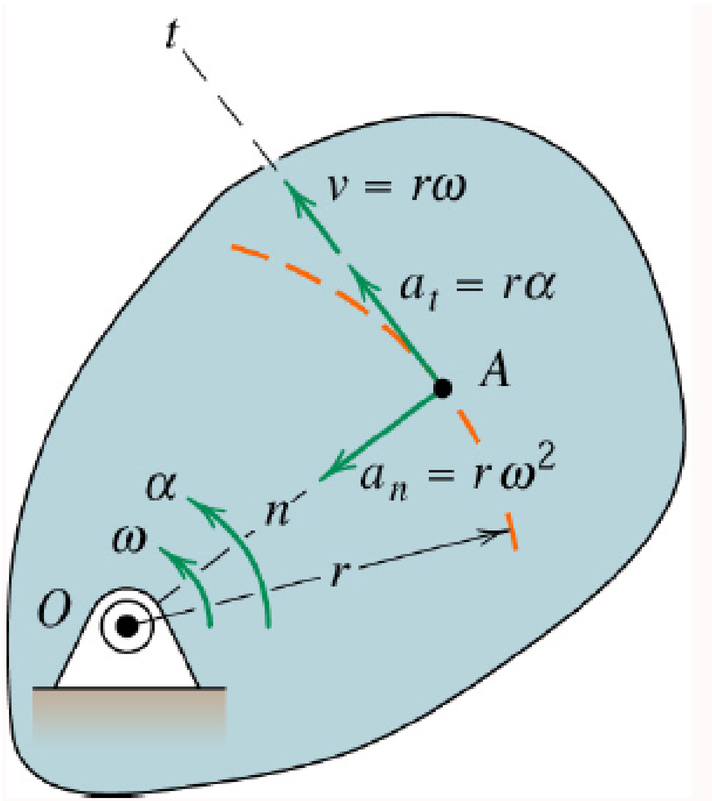
$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

5-1 Rotation

1. Rotation about a Fixed Axis



- Any point in the body moves in circular motion
- For Point A

Circular Motion

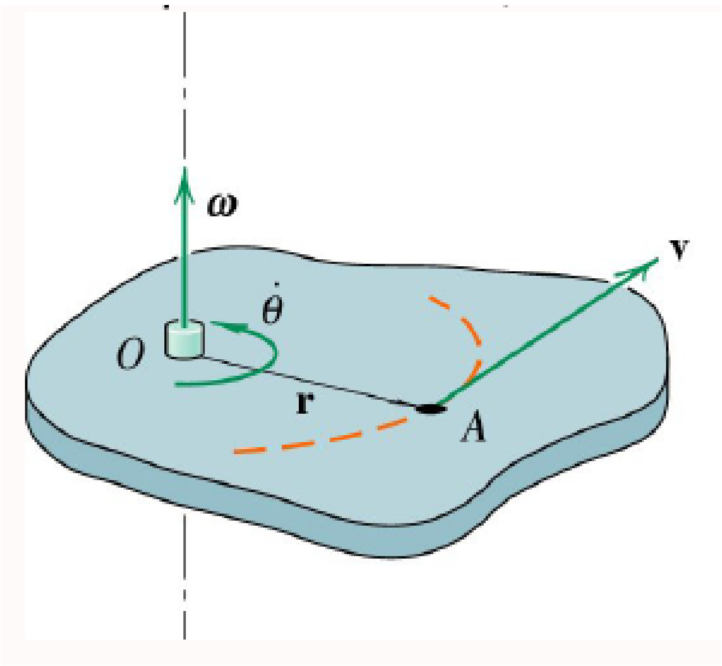
$$v = r\omega$$
$$a_n = r\omega^2 = v^2/r = v\omega$$
$$a_t = r\alpha$$

- Note: *v* and *a* of other points are different because of different *r* (ω and α are the same)

5-1 Rotation

1. Rotation about a Fixed Axis

Velocity



- The equations can be rewritten in a vector form (for plane motion)
- Direction of ω is given using the right-hand rule.

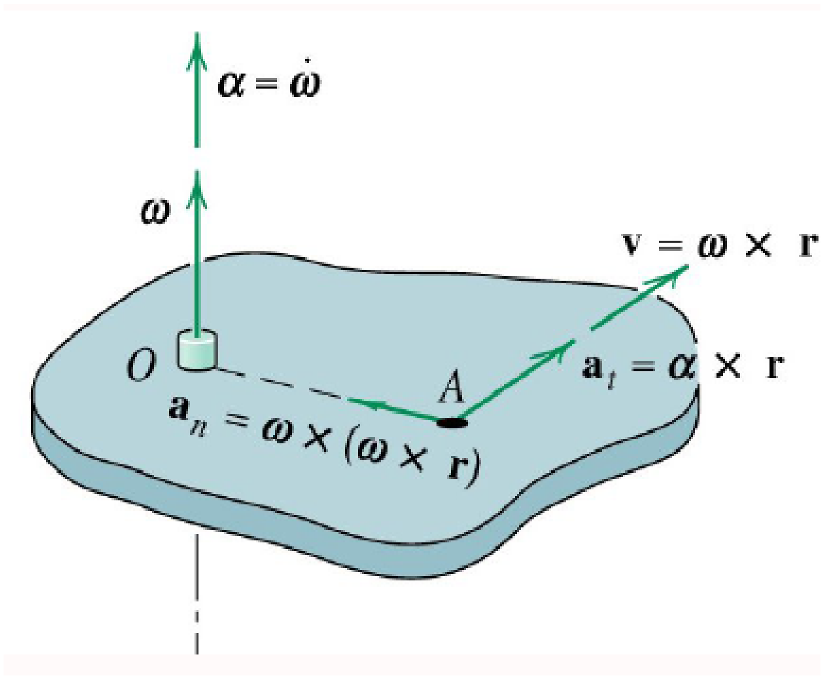
Velocity (Pure Rotation)

$$\vec{V} = \vec{\omega} \times \vec{r}$$

5-1 Rotation

1. Rotation about a Fixed Axis

Acceleration



- Direction of α is given using the right-hand rule.

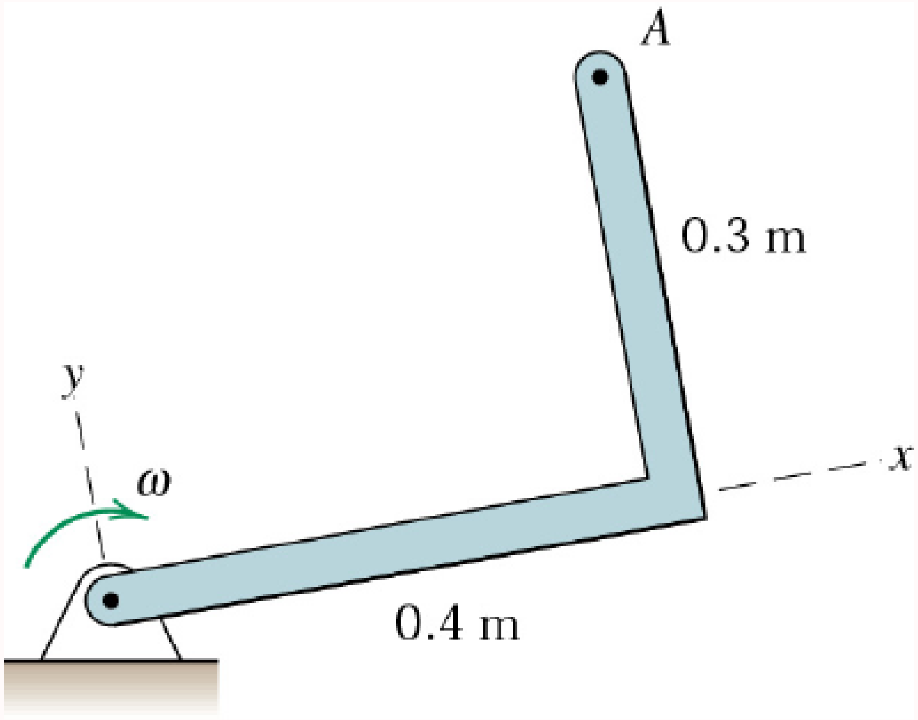
Acceleration (Pure Rotation)

$$\vec{a}_n = \vec{\omega} \times (\vec{\omega} \times \vec{r})$$
$$\vec{a}_t = \vec{\alpha} \times \vec{r}$$

5-1 Rotation

Example 1: L-shaped bar

The right-angle bar rotates clockwise with an angular velocity which is decreasing at the rate of 4 rad/s^2. Write the vector expression for the velocity and acceleration of point *A* when ω = 2 rad/s.



5-1 Rotation

Example 2:

Starting from rest when $s = 0$, pulley A is given a constant angular acceleration of 6 rad/s^2. Determine the speed of block B when it has risen 6 m.