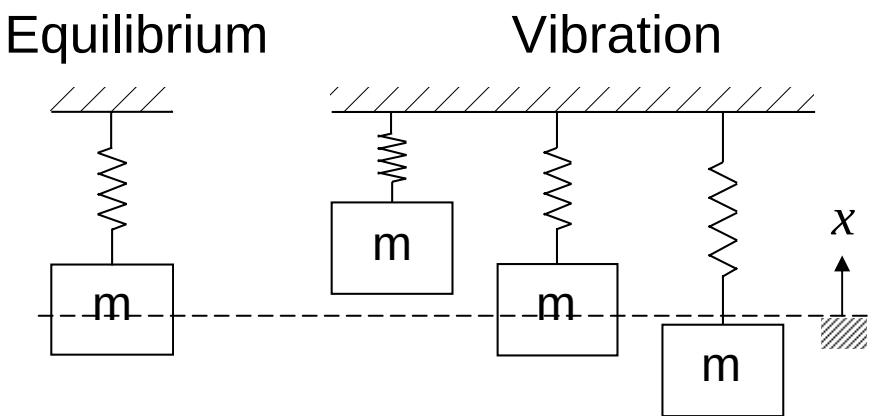
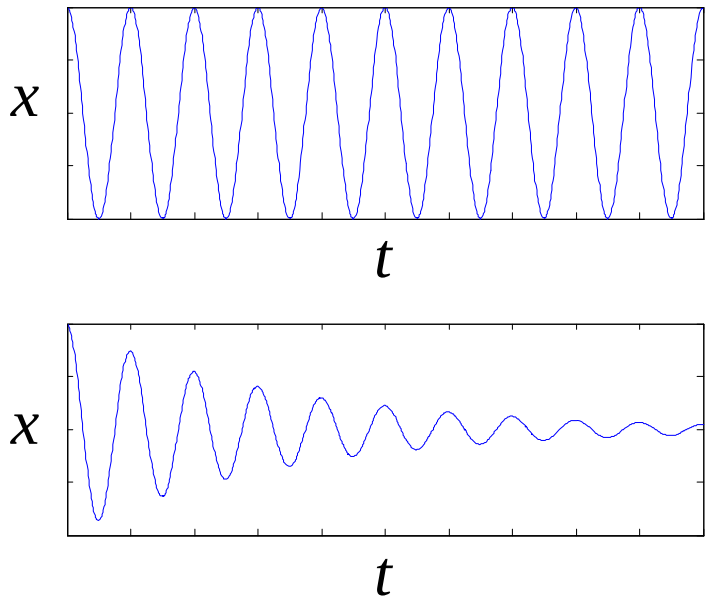
Basic Concepts of Vibration

What is Vibration?

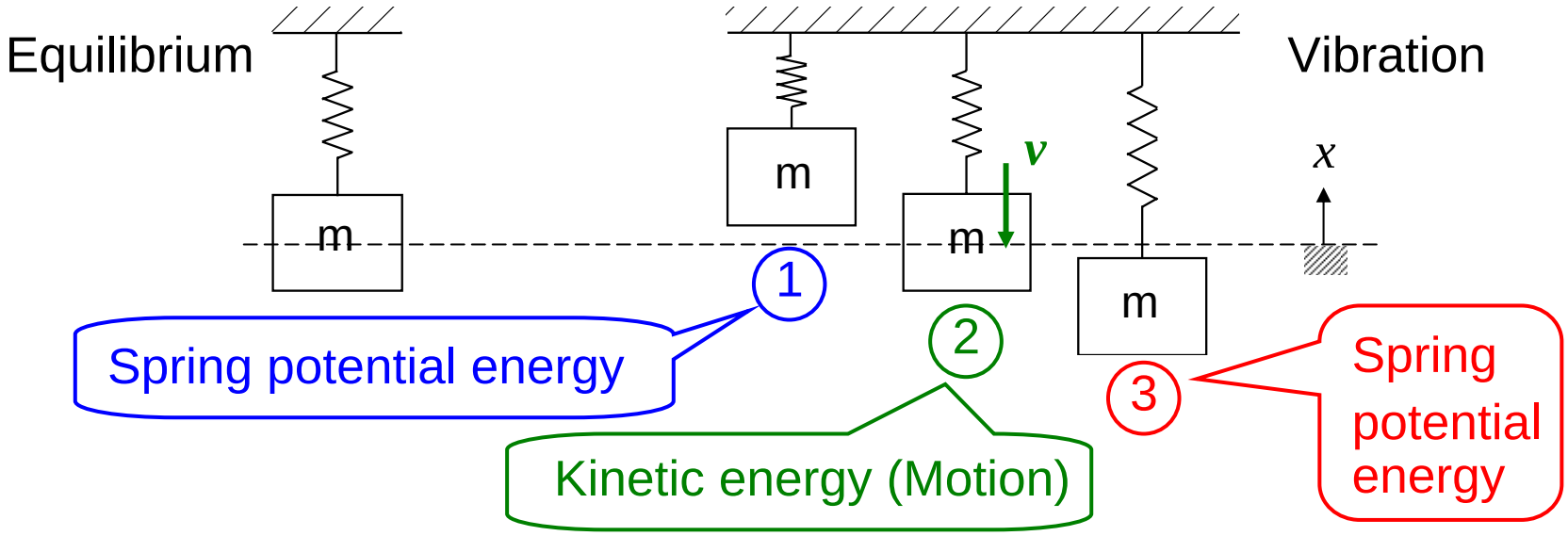
Vibration is the study of the repetitive motion of objects relative to a stationary frame of reference or nominal position.

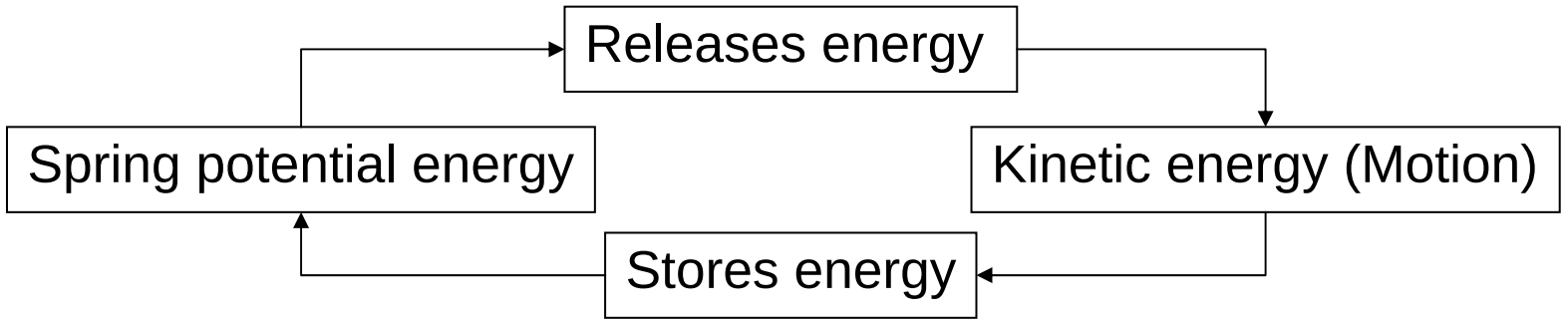


vibration waveforms



Physical Explanation





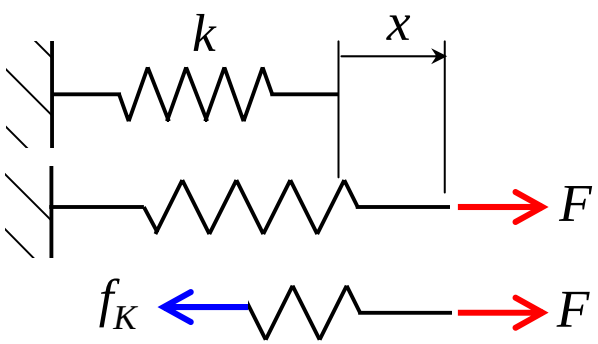
A component that stores and release potential energy is required.

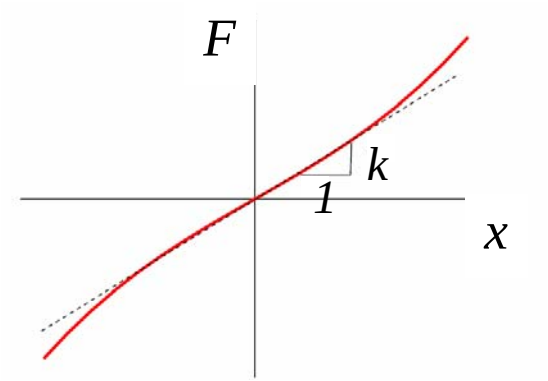
Elementary parts of vibrating systems (1)

1. Elastic components

Elastic components store or release potential (strain) energy as displacements increase or decrease.

e.g., helical spring, elastic bar & beam.





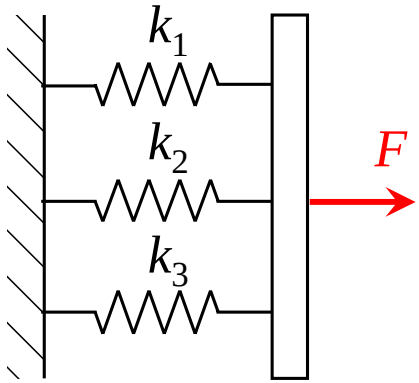
Restoring force $f_k = -F = -kx$

Potential energy $V = \int_0^x kx dx = \frac{1}{2} kx^2$

Elementary parts of vibrating systems (2)

Combination of springs

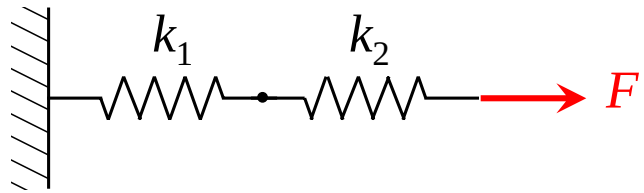
Parallel



$$k_{eq} = k_1 + k_2 + k_3$$

$$k_{eq} = \sum_{i=1}^{n} k_i \qquad n \text{ springs}$$

Series



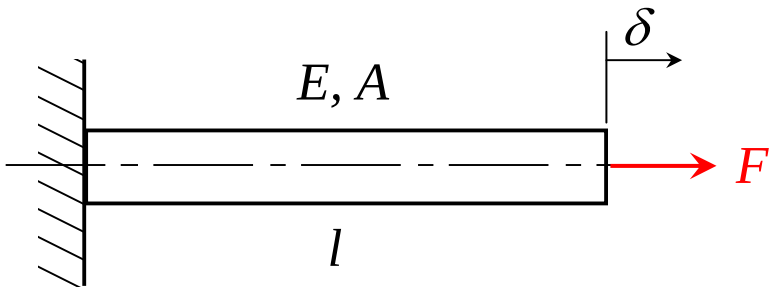
$$k_{eq} = \left(\frac{1}{k_1} + \frac{1}{k_2} \right)^{-1}$$

$$k_{eq} = \left(\sum_{i=1}^{n} \frac{1}{k_i} \right)^{-1} \qquad n \text{ springs}$$

Elementary parts of vibrating systems (3)

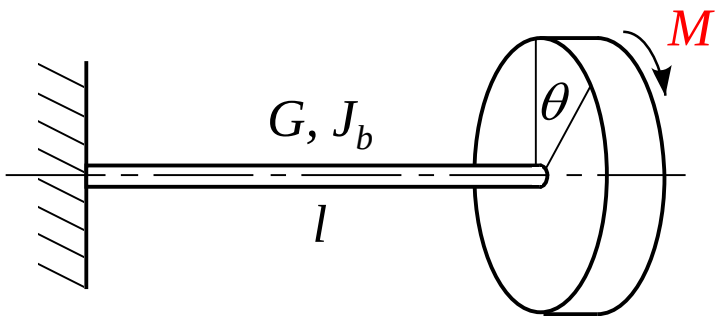
Elastic elements as springs

1. Thin rod



$$k_{eq} = \frac{F}{\delta} = \frac{EA}{l}$$

2. Torsional bar

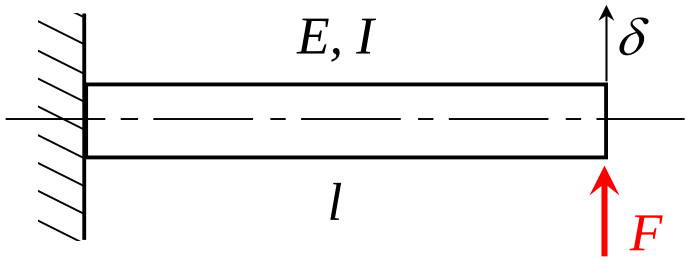


$$k_{eq} = \frac{M}{\theta} = \frac{GJ_b}{l}$$

Elementary parts of vibrating systems (4)

Elastic elements as springs

3. Cantilever beam



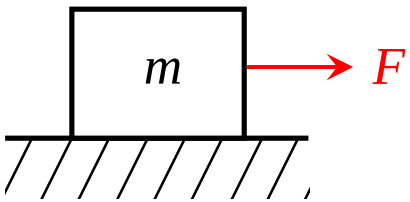
$$k_{eq} = \frac{F}{\delta} = \frac{3EI}{l^3}$$

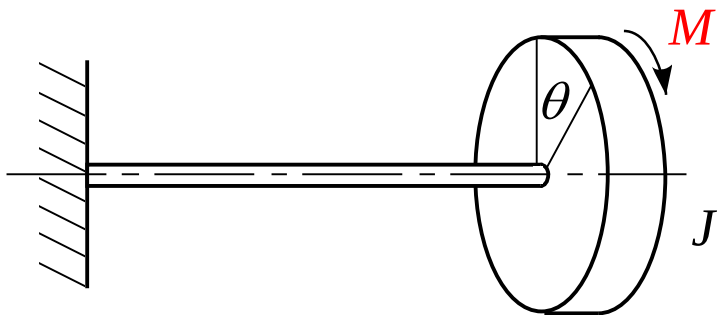
Elementary parts of vibrating systems (5)

2. Inertia (mass) components

Inertia components store or release kinetic energy as velocities increase or decrease.

e.g., mass (translation), mass moment of inertia (rotation)





Kinetic energy (translation)

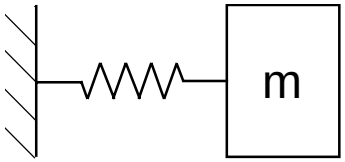
$$T = \frac{1}{2} m\dot{x}^2$$

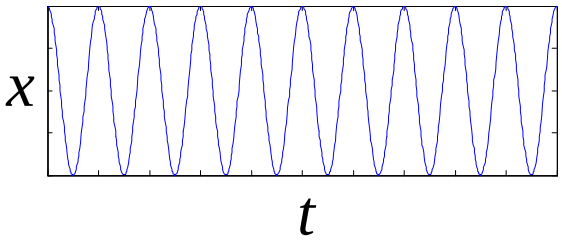
Kinetic energy (rotation)

$$T = \frac{1}{2} J\dot{\theta}^2$$

Elementary parts of vibrating systems (6)

Vibration of the spring-mass system

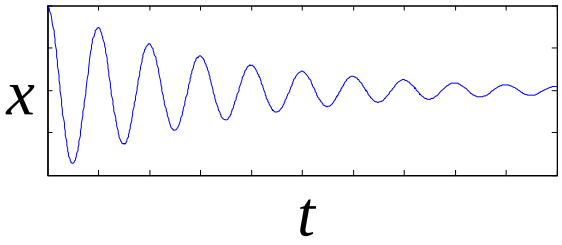




Ideal system

There is no energy loss during vibration.

The system will oscillate indefinitely.



Real system

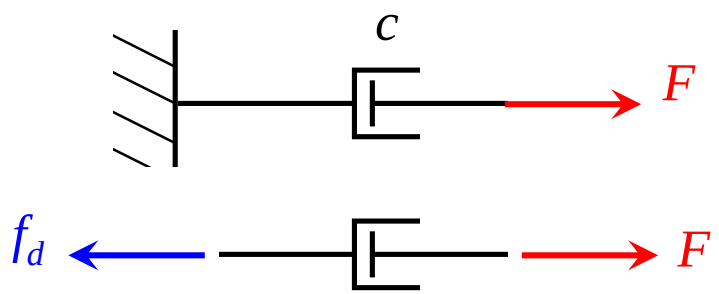
Oscillating systems eventually die out and reduce to zero motion.

There is a component that dissipates energy.

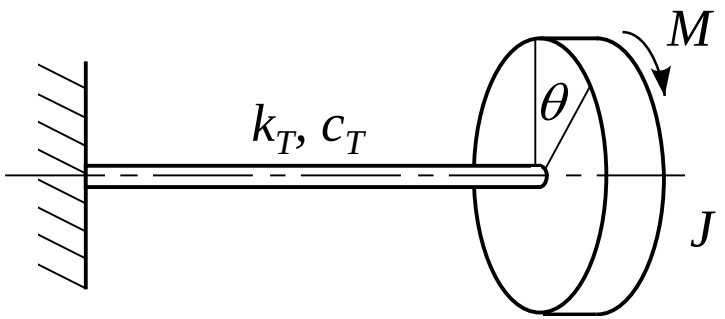
Elementary parts of vibrating systems (7)

3. Viscous damper

Viscous damper or dashpot dissipates energy. Energy is converted to heat or sound.



$$f_d = -F = -c\dot{x}$$



k, m, c for rotational motion

$$F_k = kx \qquad M_k = k_T\theta$$

$$F_d = c\dot{x} \qquad M_d = c_T\dot{\theta}$$

$$F = m\ddot{x} \qquad M = J\ddot{\theta}$$

Combination of springs (Example)

Find the equivalent single stiffness representation of the five-spring system shown in the figure.