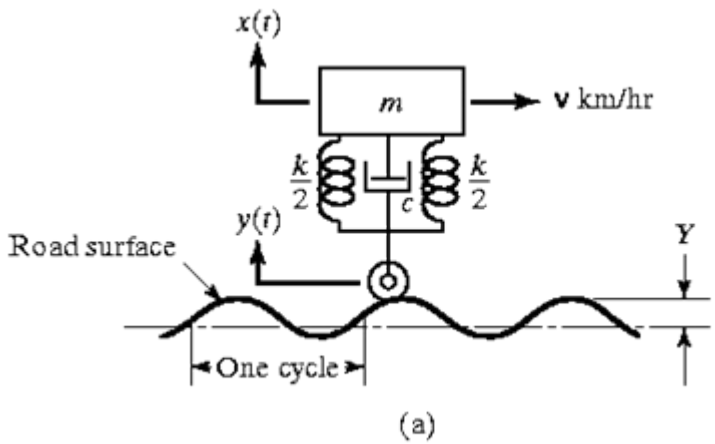
3.2 Applications

3.2.2 Base Excitation

Examples

- Sensitive equipment placed on a vibrating foundation
- Building subjected to earthquakes
- Car's suspension



(a)

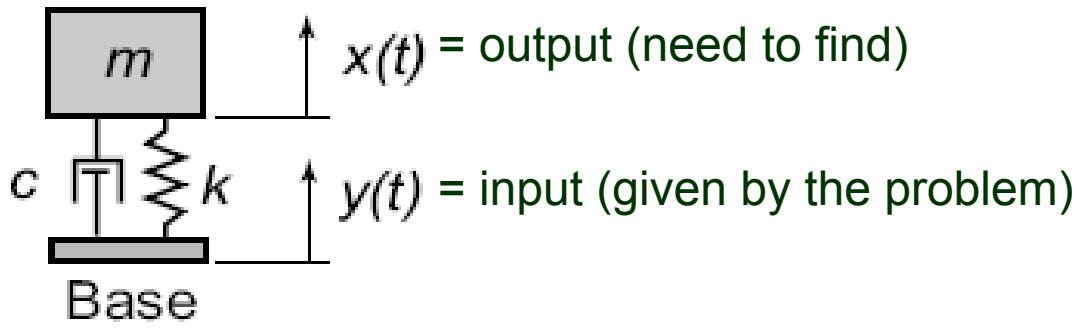
Notes: This is different than harmonic force excitation because this case

- the excitation is at the base not the mass and
- the excitation is given in terms of displacement not force

3.2 Applications

3.2.2 Base Excitation

Consider a system attached to the base via a spring and a damper. The base is excited and moved by the displacement input function $y(t)$. We need to determine how the mass moves $x(t)$.

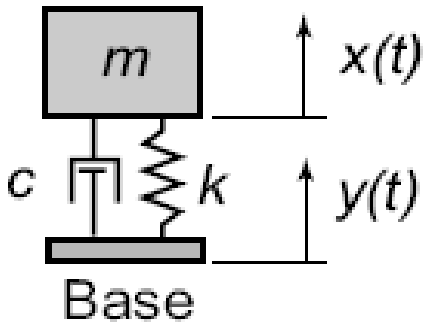


EOM is $$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

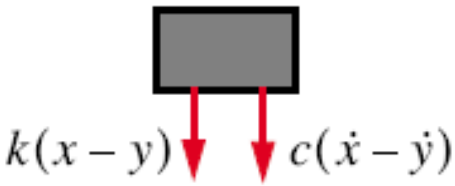
3.2 Applications

3.2.2 Base Excitation

Model:



Free-body diagram



$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$ ←EOM

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 2\zeta\omega_n\dot{y} + \omega_n^2 y$$

$$x(t) = \mathrm{Re}[z(t)] \qquad y(t) = Y\cos(\omega t) = Y\,\mathrm{Re}(e^{i\omega t})$$

$$\ddot{z} + 2\zeta\omega_n\dot{z} + \omega_n^2 z = 2\zeta\omega_n\frac{d(Ye^{i\omega t})}{dt} + \omega_n^2 Ye^{i\omega t}$$

$$z(t) = Ze^{i\omega t}$$

$$Z = \frac{\omega_n^2 + 2\zeta\omega\omega_n i}{-\omega^2 + \omega_n^2 + 2\zeta\omega\omega_n i}Y = \frac{1 + 2\zeta r i}{1 - r^2 + 2\zeta r i}Y$$

$$x = Y\left|H(\omega)\right|\cos(\omega t + \theta)$$

$$H(\omega) = \frac{1 + 2\zeta r i}{1 - r^2 + 2\zeta r i}$$

3.2 Applications

3.2.2 Base Excitation

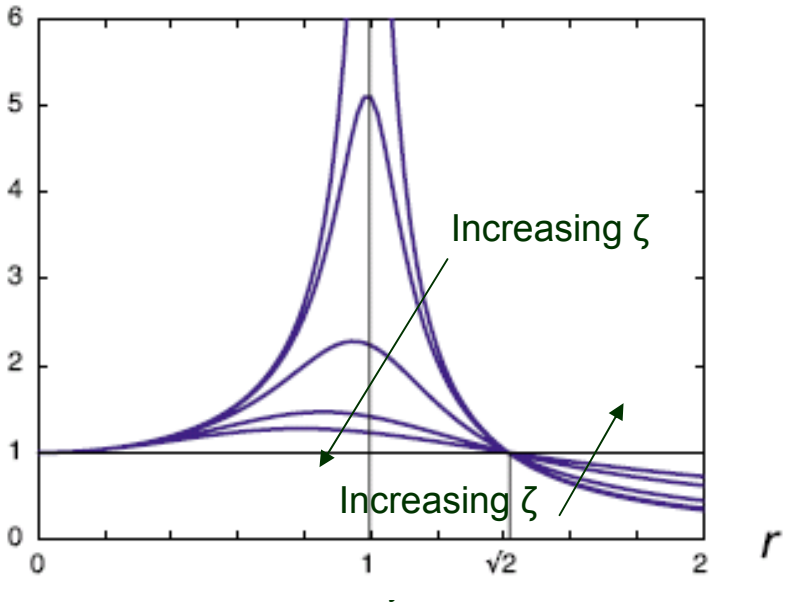
Displacement transmissibility:

$$H(\omega) = \frac{\text{output displacement}}{\text{input displacement}} = \frac{Z}{Y} = \frac{1+2\zeta r i}{1-r^2+2\zeta r i}$$

$$\downarrow$$

$$|H(\omega)| = \left|\frac{Z}{Y}\right| = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

←displacement transmissibility

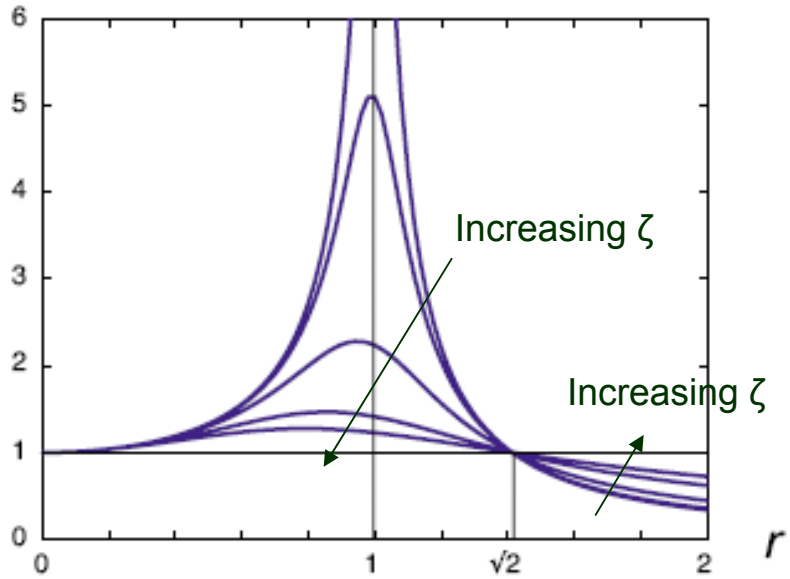


tells how motion is transmitted from the base to the mass at various driving frequencies

3.2 Applications

3.2.2 Base Excitation

Displacement transmissibility:



Notes:

1. The value of $|H|$ is unity at $r = 0$ and close to unity for small values of r.
2. For an undamped system ($\zeta = 0$), $|H| \rightarrow \infty$ at resonance ($r = 1$).
3. The value of $|H|$ is less than unity ($|H| < 1$) for values of $r > \sqrt{2}$ (for any amount of damping ζ).
4. The value of $|H| = 1$ for all values of ζ at $r = \sqrt{2}$.
5. For $r < \sqrt{2}$, smaller damping ratios lead to larger values of $|H|$. On the other hand, for $r > \sqrt{2}$, smaller values of damping ratio lead to smaller values of $|H|$

3.2 Applications

3.2.2 Base Excitation

Force transmissibility:

$$F = k(x-y) + c(\dot{x}-\dot{y}) = -m\ddot{x}$$

$$\downarrow$$

$$F = m\omega^2 X\cos(\omega t+\theta) = m\omega^2 Y\left|H(\omega)\right|\cos(\omega t+\theta) = F_T\cos(\omega t+\theta)$$

$$\downarrow$$

$$\frac{F_T}{kY} = r^2\sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

←Force transmissibility

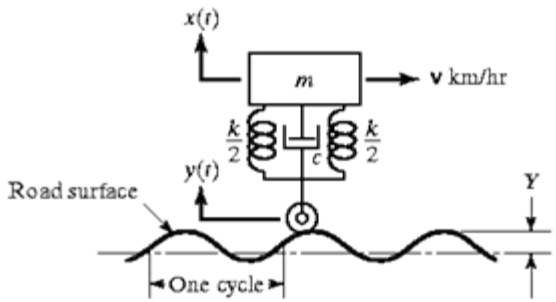
tells how much force is transmitted from the base to the mass at various driving frequencies

3.2 Applications

3.2.2 Base Excitation

Example: Vehicle Moving on a Rough Road

The figure shows a simple model of a motor vehicle that can vibrate in the vertical direction while traveling over a rough road. The vehicle has a mass of 1200 kg. The suspension system has a spring constant of 400 kN/m and a damping ratio of $\zeta = 0.5$. If the vehicle speed is 20 km/hr, determine the displacement amplitude of the vehicle. The road surface varies sinusoidally with an amplitude of $Y = 0.05$ m and a wavelength of 6 m.

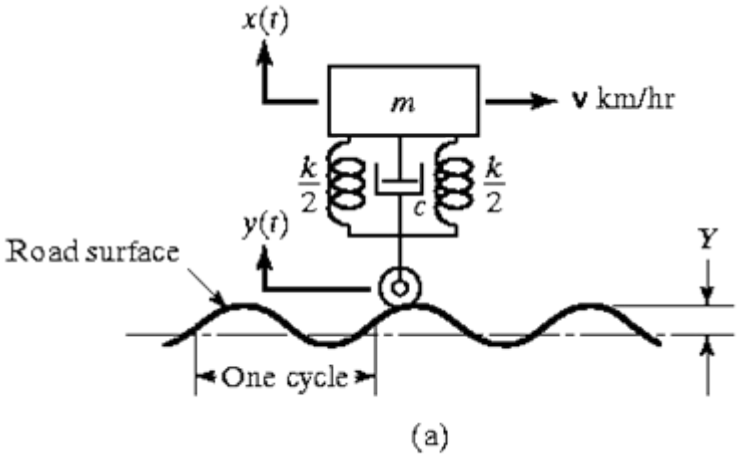


(a)

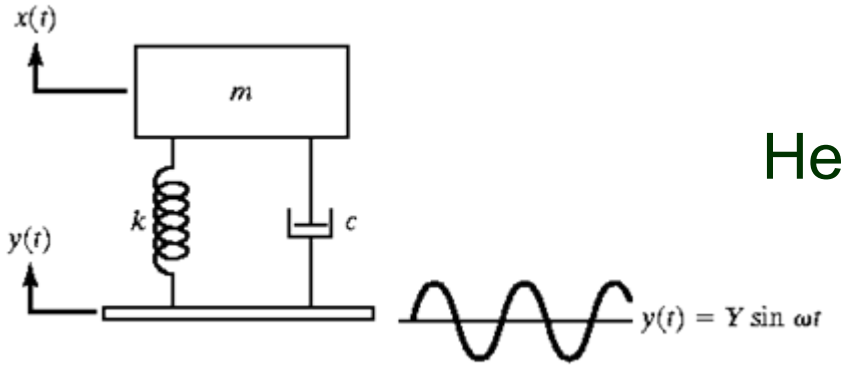
3.2 Applications

3.2.2 Base Excitation

Solution:



(a)



(b)

The frequency can be found by

$$\omega = 2\pi f = 2\pi\left(\frac{v\times 1000}{3600}\right)\frac{1}{6} = 0.291v \text{ rad/s}$$

For v = 20 km/hr, ω = 5.82 rad/s.

The natural frequency is given by,

$$\omega_n = \sqrt{\frac{k}{m}} = \left(\frac{400\times 10^3}{1200}\right)^{1/2} = 18.3 \text{ rad/s}$$

Hence, the frequency ratio is

$$r = \frac{\omega}{\omega_n} = \frac{5.81778}{18.2574} = 0.319$$

3.2 Applications

3.2.2 Base Excitation

Solution:

The amplitude ratio:

$$\left|\frac{X}{Y}\right| = \left\{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right\}^{1/2} = \left\{\frac{1+(2\times 0.5\times 0.318653)^2}{(1-0.318653^2)^2+(2\times 0.5\times 0.318653)^2}\right\}^{1/2}$$

$$= 1.10$$

Thus, the displacement amplitude of the vehicle is given by

$$X = 1.10Y = 1.10(0.05) = 0.055\ \text{m}$$

This indicates that a 5 cm bump in the road is transmitted as a 5.5 cm bump to the chassis and the passengers of the car.