Economic Risk and Decision Analysis for Oil and Gas Industry

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Presented by
Dr. Thitisak Boonpramote

Department of Mining and Petroleum Engineering, Chulalongkorn University

Review Basic Probability Concept

Basic Concepts in Probability

- A **phenomenon** is **random** if individual outcomes are uncertain but there is a **regular distribution of outcomes** in **a large number of repetitions**.
- **Probability** of an **outcome of a random phenomenon** is the proportion of time it would occur in a **very long series of repetitions**.

Probability for Equally Likely Outcomes

Number of ways **event** can occur

Probability of an event = f / N

Total number of **possible outcomes**

Definitions Used

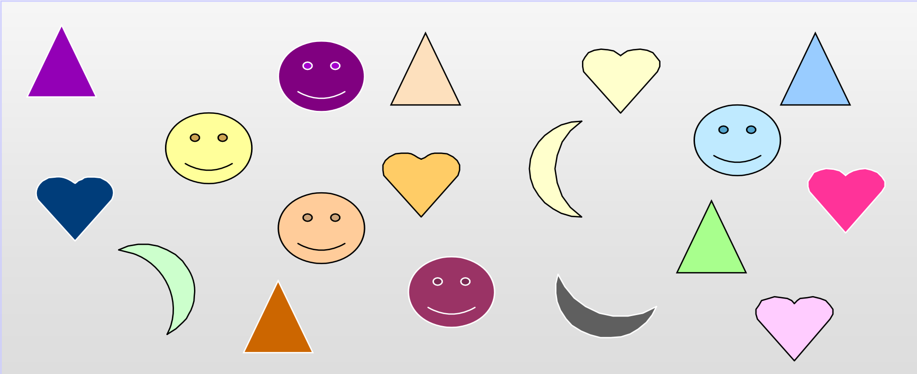
- **Random trial** or **event**
  - An action, event or operation that can produce **any result or outcome**
  - Every possibility has **equal chance** of being chosen.
- **Elementary event**
  - **Each** of the possible results in a **single trial** or experiment. E.g., <u>**each**</u> coin toss
- **Sample space**
  - A **complete set of elementary events** representing all possible outcomes from an experiment. E.g., <u>**all**</u> coin tosses
- **Event set**
  - Subset of the sample space

Events

- **Simple event**
  - **Outcome** from a sample space with **one characteristic**
  - e.g.: A **red card** from a deck of cards
- **Joint events**
  - Involves **two outcomes simultaneously**
  - e.g.: An **ace that is also red** from a deck of cards

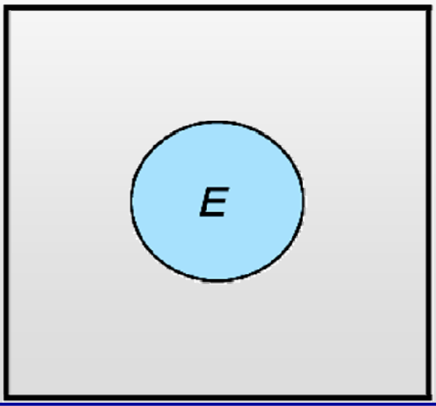
Simple Event

The Event of a Triangle



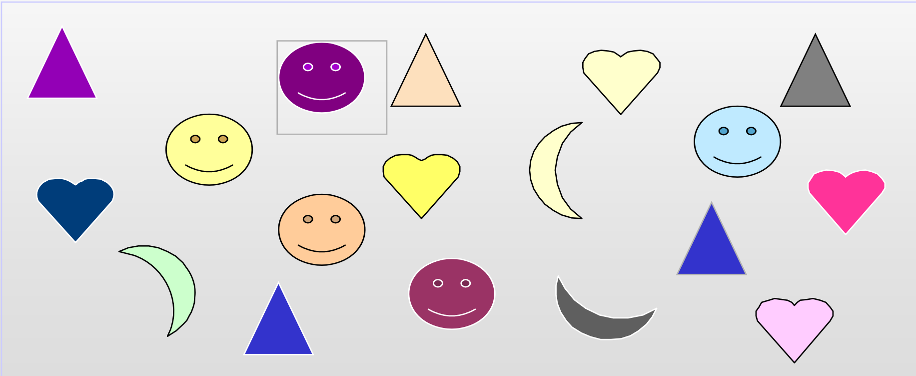
There are 5 triangles in this collection of 18 objects

Venn diagram for event E



Joint Events

The event of a **triangle** ***AND*** **blue**



Two triangles that are blue

Contingency Table: Joint events

A Deck of 52 Cards

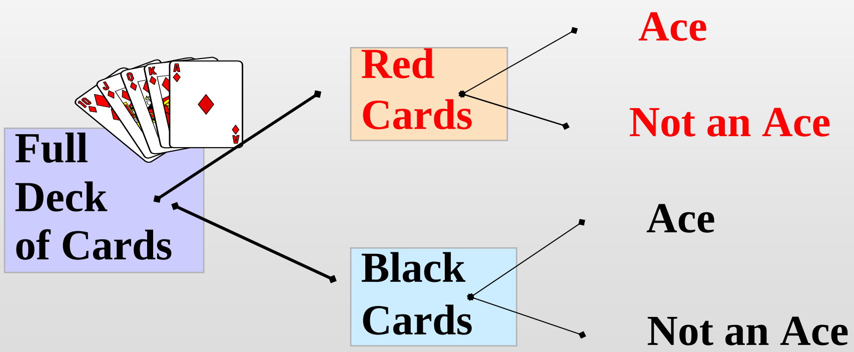
Red Ace

	Ace	Not an Ace	Total
Red	2	24	26
Black	2	24	26
Total	4	48	52

Sample Space

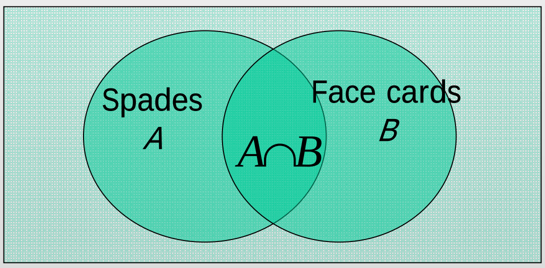
Tree Diagram: Joint events

Event Possibilities



Venn Diagrams: Joint events

- **Partially overlapping events**
  - Part of one event and part of another event can occur together



Operations on Event Sets

- **Union of two events**
  - Event set that consists of all the outcomes (sample points) that belong to either *A* or *B* or both;
  - denoted by ***AUB***
- **Intersection of two events**
  - Event set that consists of all the outcomes (sample points) the two event sets *A* and *B* have in common;
  - denoted by ***A∩B***
- **Complement of an event set**
  - Set of all sample points in sample space *S* not contained in *A*;
  - denoted by $\bar{A}$

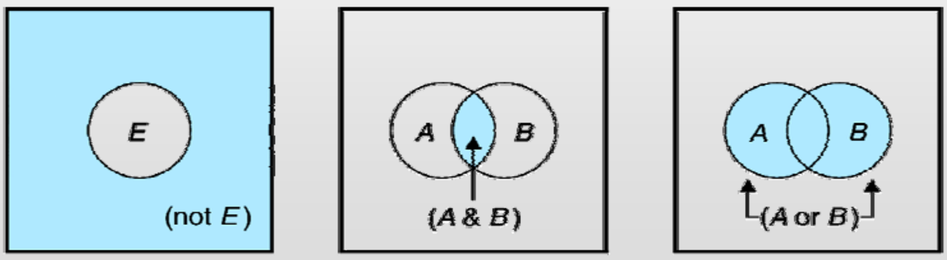
Venn diagram for event

Venn diagrams for

(a) event (not E) : The event "E does not occur"

(b) event (A & B) : The event "both A and B occur."

(c) event (A or B) : the event "either A or B or both occur."



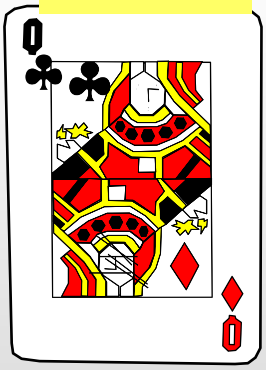
(a) (b) (c)

Special Events

- **Impossible event**

  e.g.: **Club & diamond** on one card draw

Null Event



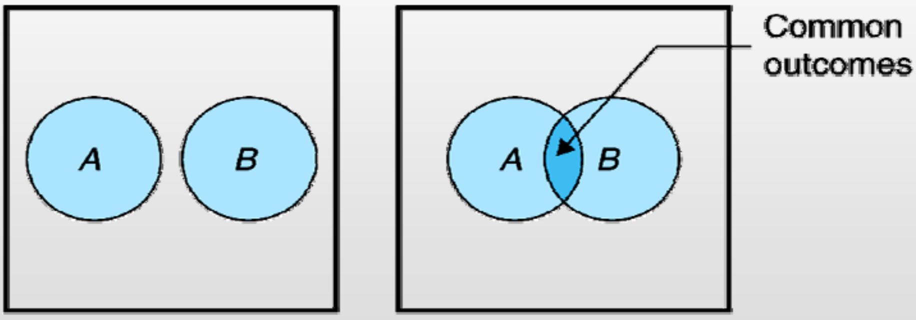
Special Events

(continued)

- **Mutually exclusive events**
  - **Two events cannot occur together**
  - e.g. -- A: queen of diamonds; B: queen of clubs
    - Events A and B are mutually exclusive

- **Collectively exhaustive events**
  - **One of the events must occur**
  - **The set of events covers the whole sample space**
  - e.g. -- A: all the aces; B: all the black cards; C: all the diamonds; D: all the hearts
    - Events A, B, C and D are collectively exhaustive
    - Events B, C and D are also collectively exhaustive

M.E. /Non-M.E. events

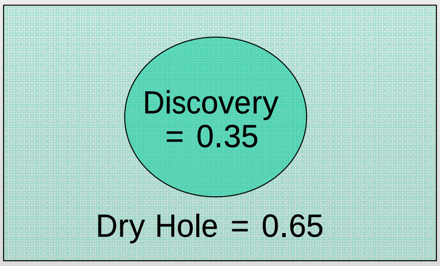


(a) (b)

Example: Venn Diagrams for M.E.

Mutually exclusive events

- Two or more events that cannot occur together



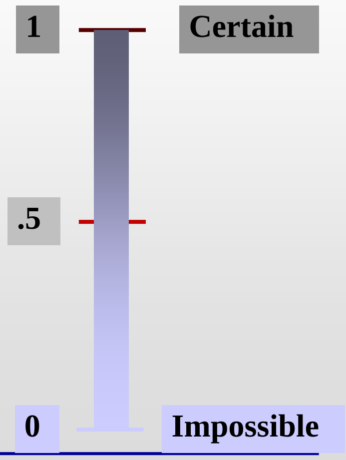
Classical vs. Empirical Approach

Classical Approach Assumptions

- **Events** are **equally likely, collectively exhaustive, and mutually exclusive**:
  - **Equally likely** – no outcome more or less likely than any other
  - **Collectively exhaustive** – A set of events that accounts for **all of the elementary events in the sample space**. The sum of all favorable and unfavorable outcomes equals total number of outcomes
  - **Mutually exclusive** – different outcomes **cannot** occur simultaneously in a single event

Properties or Axioms

- **Probability** is the **numerical measure** of the likelihood that an event will occur
- A **number between 0 and 1**, called **probability of that event**, is associated with each **event set**
- **Sum of the probabilities** of all **mutually exclusive and collectively exhaustive events is 1**
- **Sum of probabilities of all simple events** (or sample points) constituting the sample space is equal to **1**
- **Probability of compound event** is **sum of probabilities of simple events** comprising the compound event



More Definitions

- **Simple probability**
  - Probability of occurrence that is ***independent*** of the occurrence of another event (**unconditional**);
  - denoted by ***P(A)*** or ***P(B)***, where A and B are events

- **Conditional probability**
  - Probability of occurrence that ***depends*** on another event;
  - denoted by ***P(A|B)***, read as '**probability of A, given that B has occurred**'

Simple Probability

- Approach based on ***a priori*** or **abstract reasoning**
- **Probability of occurrence of event** expressed as ***P*(*A*)**

$$P(A) = \frac{m}{n}$$

Favorable cases (m)

All possible cases (n)

Computing Probabilities

- The **probability of an event E**:

$$P(E) = \frac{\text{number of event outcomes}}{\text{total number of possible outcomes in the sample space}} = \frac{X}{T}$$

- Each of the outcomes in the sample space is equally likely to occur

Classical Approach

- Probability of **event not happening,** $P(\overline{A})$
  where $\overline{A}$ is **complement of** A

$$P(\overline{A}) = \frac{n-m}{n}$$

Same concept stated as law of proportion or law of chance

More Definitions

- **Joint probability**
  - Probability of **more than one event occurring simultaneously** or **in succession**;
  - denoted by ***P(AB)***, interpreted as the **probability of both A and B**

- **Marginal probability**
  - Sum of joint probabilities

Empirical Approach

- Approach based on **experiments** and **observations** which are random
- **No bias** in favor of any outcome, so all elements have the same chance at selection
- **Large number of trials** required to establish chance of an event
- **Probability of event** represents **proportion of times** (under identical conditions) event can be expected to occur

Empirical Approach Definition

- 'If experiment is **repeated a large number of times**, under essentially **identical conditions**, then the limiting value of the **ratio** of the number of times the **Event A** happens to the **total number of trials** of the experiment as the number of trials increases indefinitely is called the **probability of the occurrence of A**.'

$$P(A) = \lim_{n \to \infty} \frac{m}{n}$$

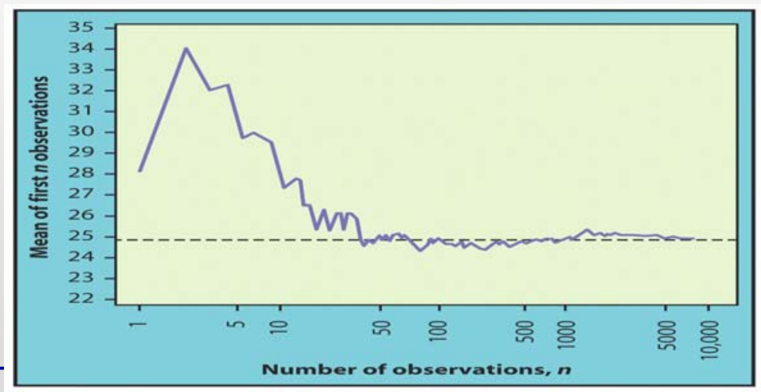
Probability: Long-Run Relative Frequency

Probability = *proportion of time it occurs over the long run*

- Can be applied when situation can be **repeated numerous times** and outcome observed each time.
- **Relative frequency** should settle down to **constant value over long run**, which is the **probability**.
- Does not apply to situations where outcome one time is **influenced** by or influences outcome the next time.
- Cannot be used to determine whether outcome will occur on a **single occasion** but can be used to **predict long-term proportion of times** the outcome will occur.

LAW OF LARGE NUMBERS

Draw independent observations at random from any population with finite mean μ. As the number of observations drawn increases, the mean $\bar{x}$ of the observed values gets closer and closer to the mean μ of the population.



Dependent/Independent Events

- Occurrence of **independent events** in no way affects the occurrence of the other(s)
  - Coin toss always has same two outcomes

- **Dependent events** are affected by previous trials
  - Removing a card from a deck changes number of cards remaining (outcomes) *unless card is replaced* before next draw

When Will It Happen?

Probability an **outcome will occur** on any given instance is p.
Probability the **outcome will not occur** is $(1 - p)$.
Outcome each time is *independent* of outcome all other times.

Probability it ***doesn't* occur on 1st try but *does* occur on 2nd try** is $(1 - p)p$.

Try on Which the Outcome First Happens	Probability
1	p
2	$(1 - p)p$
3	$(1 - p)(1 - p)p = (1 - p)^2 p$
4	$(1 - p)(1 - p)(1 - p)p = (1 - p)^3 p$
5	$(1 - p)(1 - p)(1 - p)(1 - p)p = (1 - p)^4 p$

Rule of Probability

Complementation Rule

Rule 1: If there are **only two possible outcomes** in an uncertain situation, then their probabilities **must add to 1**.

Since an event either occurs or does not occur, and since the sum of collectively exhaustive events must equal one, then

$$P(A) + P(\overline{A}) = 1$$

Marginal probability of Event A

Marginal probability of no Event A

$$P(\overline{A}) = 1 - P(A)$$

Example : If probability of a single birth resulting in a boy is 0.51, then the probability of it resulting in a girl is 0.49.

Addition Rule and Mutually Exclusive

Rule 2: If two outcomes cannot happen simultaneously, they are said to be **mutually exclusive.**
The **probability of one or the other** of **two mutually exclusive** outcomes happening is the **sum** of their individual probabilities.

$$P(A \text{ or } B) = P(A) + P(B)$$

Example : If you think probability of getting an A in this class is 50% and probability of getting a B is 30%, then probability of getting either **an A or a B** is 80%. Thus, probability of getting C or less is 20% (using Rule 1).

Addition Rule: Compound event

- For N **mutually exclusive events**

$$P(A \text{ or } B \text{ or } C \ldots \text{ or } N) = P(A) + P(B) + P(C) + \ldots + P(N)$$

- For events that are **not** **mutually exclusive**

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

Addition Rule: Compound event

When two or more events will happen at the same time, and the events are ***not*** **mutually exclusive**, then:

P(X or Y) = P(X) + P(Y) **- P(X and Y)**

For example, what is the probability that a card chosen at random from a deck of cards will either be **a king or a heart**?

P(King or Heart) = P(X or Y) = 4/52 + 13/52 - 1/52 = 30.77%

Computing Compound Probability

- Probability of a **compound event**, **A or B**:

$$P(A \text{ or } B) = P(A \cup B)$$

$$= \frac{\text{number of outcomes from either A or B or both}}{\text{total number of outcomes in sample space}}$$

E.g. $P(\text{Red Card or Ace})$

$$= \frac{\text{4 Aces + 26 Red Cards - 2 Red Aces}}{\text{52 total number of cards}}$$

$$= \frac{28}{52} = \frac{7}{13}$$

Addition Rule and Compound Probability

$P(A_1 \text{ or } B_1) = P(A_1) + P(B_1) - P(A_1 \text{ and } B_1)$

Event	Event		Total
	B_1	B_2	
A_1	$P(A_1 \text{ and } B_1)$	$P(A_1 \text{ and } B_2)$	$P(A_1)$
A_2	$P(A_2 \text{ and } B_1)$	$P(A_2 \text{ and } B_2)$	$P(A_2)$
Total	$P(B_1)$	$P(B_2)$	1

For Mutually Exclusive Events: $P(A \text{ or } B) = P(A) + P(B)$

Addition Rule and M.E.

- Addition rule.

$$P = P_a + P_b.$$

Deal with several outcomes such as the question of whether oil or gas will be the dominant phase in the prospect being evaluated.

Multiplication Rule and Independent

Rule 3: If two events ***do not influence*** each other, and if knowledge about one doesn't help with knowledge of the probability of the other, the events are said to be **independent** of each other.
If **two events are independent**, the **probability that they both happen** is found by **multiplying** their individual probabilities.

Multiplication Rule and Joint Probability

- when two or more events will happen at the same time, and the events *are* **independent**, then the **special rule of multiplication law** is used to find the **joint probability**:

  P(X and Y) = P(X) x P(Y)

- when two or more events will happen at the same time, and the events *are* **dependent**, then the **general rule of multiplication law** is used to find the **joint probability**:

  P(X and Y) = P(X) x P(Y|X)

Multiplication Rule

- For two **independent events**

$$P(AB) = P(A \text{ and } B) = P(A) \times P(B)$$

- For *n* **independent events**

$$P(ABC\ldots N) = P(A \text{ and } B \text{ and } C\ldots \text{ and } N)$$
$$= P(A) \times P(B) \times P(C) \times \ldots \times P(N)$$

Multiplication Rule for Independent events

- Multiplication rule.

$$P = P_a \times P_b \times P_c \times P_d.$$

Used when estimating the probability of discovery for mapped prospect.

Prospect probability is a product of several independent factors (such as reservoir, trap, charge and retention).

Joint Probabilities

Probability of obtaining a particular combination of events.

- E.g., probability of flipping a coin twice and getting heads both times.
  - **P (A and B)** = n(A and B) / n(S)

Example : Woman will have two children.
Assume outcome of 2nd birth independent of 1st and probability birth results in boy is 0.51.
Then **probability of a boy followed by a girl** is (0.51)(0.49) = 0.2499.
About a 25% chance a woman will have **a boy and then a girl**.

Computing Joint Probability

- The probability of a **joint event, A and B**:

$$P(A \text{ and } B) = P(A \cap B)$$

$$= \frac{\text{number of outcomes from both A and B}}{\text{total number of possible outcomes in sample space}}$$

$$\text{E.g. } P(\text{Red Card and Ace})$$

$$= \frac{\text{2 Red Aces}}{\text{52 Total Number of Cards}} = \frac{1}{26}$$

Joint Probability Using Contingency Table

Event	Event		Total
	B_1	B_2	
A_1	$P(A_1 \text{ and } B_1)$	$P(A_1 \text{ and } B_2)$	$P(A_1)$
A_2	$P(A_2 \text{ and } B_1)$	$P(A_2 \text{ and } B_2)$	$P(A_2)$
Total	$P(B_1)$	$P(B_2)$	1

Joint Probability

Marginal (Simple) Probability

Multiplication Rule and Conditional Probability

- For **dependent events** (when the **probability of A occurring given that B has occurred** is different from the marginal probability of A)

$$P(AB) = P(A \text{ and } B)$$
$$= P(B) \times P(A|B) \text{ or } P(A) \times P(B|A)$$

Conditional Probability

The probability that event *B* occurs given that event *A* has occurred is called a **conditional probability.** It is denoted by the symbol ***P*(*B* | *A*),** which is read "the probability of *B* given *A*." We call *A* the **given event.**

Conditional Probability

Definition of conditional probability in terms of unconditional probabilities

P(B| A) = P(A and B)/ P(A)

Example: 50% of people entering a store ask for help. (marginal prob.)

15% of people entering a store both buy and ask for help. (joint prob)

If someone asks for help, how likely is it that they will buy? (conditional)

Computing Conditional Probability

- The probability of event A given that event B has occurred:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

E.g.

$P(\text{Red Card given that it is an Ace})$

$$= \frac{2 \text{ Red Aces}}{4 \text{ Aces}} = \frac{1}{2}$$

Conditional Probability Using Contingency Table

Type	Color		Total
	Red	Black	
Ace	2	2	4
Non-Ace	24	24	48
Total	26	26	52

Revised Sample Space

$$P(\text{Ace} \mid \text{Red}) = \frac{P(\text{Ace and Red})}{P(\text{Red})} = \frac{2/52}{26/52} = \frac{2}{26}$$

Conditional Probability and Multiplication

- **Conditional probability:**

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

- **Multiplication rule:**

$$\begin{aligned} P(A \text{ and } B) &= P(A \mid B)\ P(B) \\ &= P(B \mid A)\ P(A) \end{aligned}$$

Conditional Probability and Statistical Independence

- Events A and B are **independent** if

$$\begin{aligned} & P(A \mid B) = P(A) \\ \text{or } & P(B \mid A) = P(B) \\ \text{or } & P(A \text{ and } B) = P(A)P(B) \end{aligned}$$

- Events A and B are independent when the probability of one event, A, is **not affected** by another event, B

Simple Probability Rules

Rule 4: If the ways in which one event can occur are a subset of those in which another event can occur, then the **probability of the subset event *cannot* be higher** than the probability of the one for which it is a subset.

Example : Suppose you are 18 and speculating about your future. You decide the probability you will eventually **get married and have children** is 75%.
By Rule 4, probability that you will eventually **get married is at least** 75%.

Summary of Probability Rules

Notation
Denote "events" or "outcomes" with capital letters A, B, C, and so on.
If A is one outcome, all other possible outcomes are part of "A complement" = A^C.

$P(A)$ is the probability that the event or outcome A occurs. For any event A,
$0 \le P(A) \le 1$.

Rule 1

$$P(A) + P(A^C) = 1$$

A useful formula that results from this is

$$P(A^C) = 1 - P(A)$$

Rule 2
If events A and B are *mutually exclusive*, then

$$P(A \text{ or } B) = P(A) + P(B)$$

Rule 3
If events A and B are *independent*, then

$$P(A \text{ and } B) = P(A)\,P(B)$$

Rule 4
If the ways in which an event B can occur are a subset of those for event A, then

$$P(B) \le P(A)$$

Probability Exercises

Reservoir Example

Information: 50 wells drilled in area with blanket sands

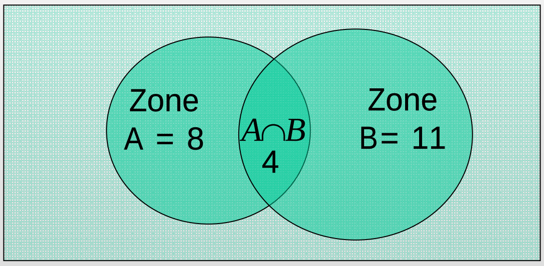
- **Zone A** has **8 productive wells**
- **Zone B** has **11 productive wells**
- **4 productive wells intersect both zones**

- Determine productive wells in
  - **Zone A but not B**
  - **Zone B but not A**
  - **Either A or B**
- Determine number of wells **discovered**
- Determine number of **dry holes**

Reservoir Example (cont.)

Information: 50 wells drilled in area with blanket sands

$n(S) = 50 \quad n(A) = 8 \quad n(B) = 11 \quad n(A \cap B) = 4$



Reservoir Example (cont.)

$n(S) = 50 \quad n(A) = 8 \quad n(B) = 11 \quad n(A \cap B) = 4$

- **Determine productive wells in:**
  - **Zone A but not B**

$$n(A \cap \overline{B}) = n(A) - n(A \cap B) = 8 - 4 = 4$$

  - **Zone B but not A**

$$n(\overline{A} \cap B) = n(B) - n(A \cap B) = 11 - 4 = 7$$

  - **Either A or B**

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 8 + 11 - 4 = 15$$

Reservoir Example (cont.)

$$n(S)=50 \quad n(A)=8 \quad n(B)=11 \quad n(A\cap B)=4$$

- Determine number of wells **discovered**

$$n(A\cap \overline{B})+n(\overline{A}\cap B)+n(A\cap B)=4+7+4=15$$

- Determine number of **dry holes**

$$n(S)-n(A\cup B)=50-15=35$$

Probability Table

- Alternative to Venn diagram for solving problems like previous examples

		Zone B		
		Yes	No	Total
Zone A	Yes	4		8
	No			
Total		11		50

Pivot Column
(100% probability)

Probability Table

- Alternative to Venn diagram for solving problems like previous examples

		Zone B		Total
		Yes	No	
Zone A	Yes	4	4	8
	No	7	35	42
Total		11	39	50

Marginal Probabilities   Pivot Column

Calculate Probability of Contamination

Information: 100 random samples of crude oil

- 45 contain sulfur
- 40 contain mercaptans
- 35 contain **both**

- Calculate probability that sample contains
  - Either sulfur or mercaptans
  - Sulfur only
  - Neither sulfur nor mercaptans

Calculate Probability of Contamination (cont.)

Information: 100 random samples of crude oil

$n(C) = 100 \quad n(S) = 45 \quad n(M) = 40 \quad n(S \cap M) = 35$

- **Calculate probability that sample contains:**
  - Either sulfur or mercaptans

$$(S \cup M) = \frac{[n(S) + n(M) - n(S \cap M)]}{100}$$

$$= \frac{45 + 40 - 35}{100} = 0.5 \text{ or } 50\%$$

Calculate Probability of Contamination (cont.)

$n(C) = 100 \quad n(S) = 45 \quad n(M) = 40 \quad n(S \cap M) = 35$

- **Calculate probability that sample contains:**
  - **Sulfur only**

$$p(S \cup \overline{M}) = \frac{n(S) - n(S \cap M)}{100}$$

$$= \frac{45 - 35}{100} = 0.1 \text{ or } 10\%$$

Calculate Probability of Contamination (cont.)

$n(C) = 100 \quad n(S) = 45 \quad n(M) = 40 \quad n(S \cap M) = 35$

- **Calculate probability that sample contains**
  - **Neither sulfur nor mercaptans**

$$p(\overline{S} \cap \overline{M}) = \frac{n(C) - n(S \cup M)}{100}$$

$$= \frac{100 - 50}{100} = 0.5 \text{ or } 50\%$$

Drilling Successive Discoveries

Information: Ten prospective leases acquired

- One well to be drilled in each lease
- All have equal chances of success
- Seismic shows 3 will be commercial

- Calculate probability of drilling **first two wells** as **successive discoveries**

Drilling Successive Discoveries

Let W_1 be Well 1 and W_2 be Well 2

$$P(W_1) = \frac{3}{10} \qquad P(W_2|W_1) = \frac{2}{9}$$

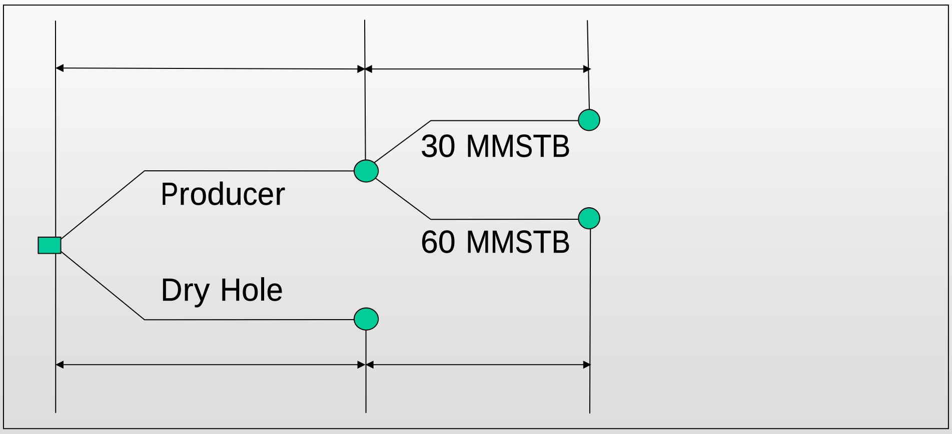
$$\therefore P(W_1 W_2) = P(W_1) \times P(W_2|W_1) = \frac{3}{10} \times \frac{2}{9}$$

$$= \frac{6}{90} = \frac{1}{15} = 0.0667 \text{ or } 6.67\%$$

Probability Trees

- Diagrams representing sequences of lines depicting probabilistic events
  - Branch to all possible sequences that can occur in any situation
- Pictorial presentation of **conditional probabilities**
- Show all possible branches for each event

Typical Probability Tree



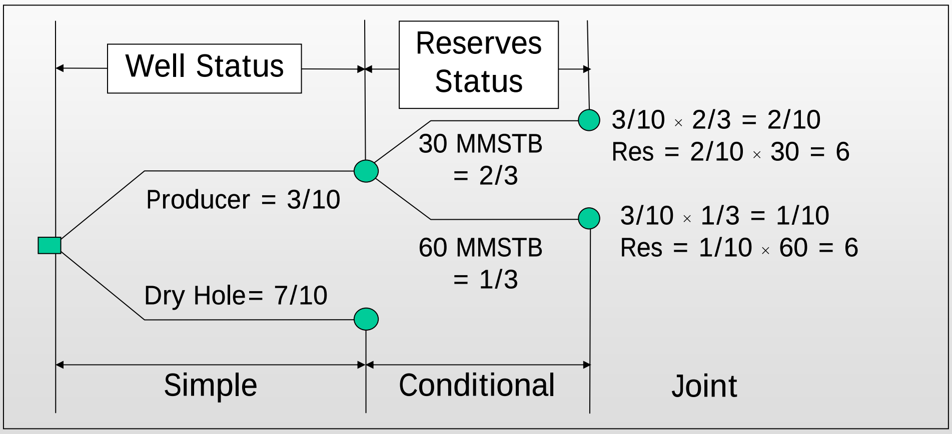
Probability Trees

- **Each branch** is labeled with **event** taking place at branch and corresponding **probability** of occurrence given the prior sequence of events required to reach that point in the tree
  - **Probabilities on first branch** of tree are **simple** or **unconditional probabilities**
  - **All probabilities after first branch** are **conditional probabilities** because they assume branches to the left model events that have already taken place

Probability Trees

- At **sources of branches**, called ***nodes***, **branches** must be **mutually exclusive and collectively exhaustive**
  - **Probabilities** at each node must **sum to one**
  - When probabilities from origin of tree to any terminal point are multiplied, **joint probability** of that particular sequence of events is obtained

Typical Probability Tree



Probability and Statistics

- **Statistics** deal with what we *observe* and how it compares to what might be *expected by chance*.

- A **set of probabilities** corresponding to **each possible value of some variable, X**, creates a **probability distribution**
  - Common examples include
    - Normal (Gaussian),
    - Poisson,
    - Exponential,
    - Binomial, etc