Economic Risk and Decision Analysis for Oil and Gas Industry

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Probability Distribution

Probability Distributions

- **Pattern of distribution of probabilities** over all possible events
- Can be used to determine **likelihood of occurrence** of all possible outcomes

Variable described by probability distribution is called random variable

Random Variable

Random variable

- Outcomes of an experiment expressed numerically
- e.g.: Toss double dies ;

  Count the number of times the number 12 appears (0, 1 or 2 times)

Classifications of Probability Distributions

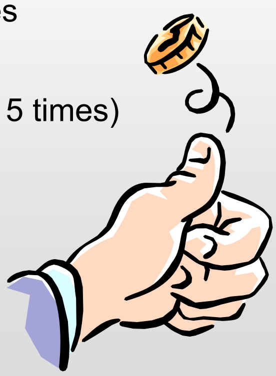
- **Discrete probability distribution** is associated with **random variable** that can take on only a ***finite*** number of values

- **Continuous probability distribution** is associated with **random variable** that can take on ***infinite*** number of values

Discrete Random Variable

Discrete random variable

- Obtained by counting (1, 2, 3, etc.)
- Usually a **finite number** of different values
- e.g.: Toss a coin five times;
  Count the number of tails (0, 1, 2, 3, 4, or 5 times)



Discrete Probability Distribution: Example

Event: Toss 2 Coins. Count # Tails.

Probability Distribution

Values	Probability
0	1/4 = .25
1	2/4 = .50
2	1/4 = .25

Discrete Probability Distribution

- List of all possible [X_j , $p(X_j)$] pairs
  - X_j = **value of random variable**
  - $P(X_j)$ = **probability associated** with value
- **Mutually exclusive** (nothing in common)
- **Collectively exhaustive** (nothing left out)

$$0 \le P(X_j) \le 1 \qquad \sum P(X_j) = 1$$

Summary Measures

Expected value (the mean):

Weighted average of value by the probability distribution:

$$\mu = E(X) = \sum_j X_j P(X_j)$$

Summary Measures

continued

Example of expected value (the mean):

Toss two coins, count the number of tails, compute expected value

$$\mu = \sum_j X_j P(X_j)$$

$$= (0)(2.5) + (1)(.5) + (2)(.25) = 1$$

Summary Measures

(continued)

Variance

- Weight average squared deviation about the mean

$$\sigma^2 = E\left[\left(X-\mu\right)^2\right] = \sum\left(X_j-\mu\right)^2 P\left(X_j\right)$$

Summary Measures

(continued)

Example of variance:

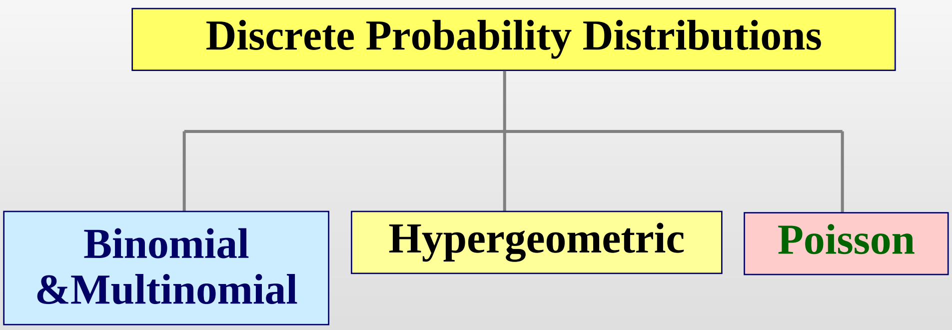
Toss two coins, count number of tails, compute variance

$$\sigma^2 = \sum\left(X_j-\mu\right)^2 P\left(X_j\right)$$

$$= \left(0-1\right)^2\left(.25\right)+\left(1-1\right)^2\left(.5\right)+\left(2-1\right)^2\left(.25\right) = .5$$

Discrete Probability Distribution

Important Discrete Probability Distributions



Binomial Probability Distribution

- Describe the probability of a given **number of outcomes (x)** in a specified **number of trials (n)**.
- **“n” Identical trials**
  - e.g.: 15 tosses of a coin; 10 light bulbs taken from a warehouse
- **Two mutually exclusive outcomes** on each trial
  - e.g.: Heads or tails in each toss of a coin; defective or not defective light bulb
- Trials are **independent**
  - The outcome of one trial does not affect the outcome of the other

Binomial Probability Distribution

(continued)

- **Constant probability** for each trial
  - e.g.: Probability of getting a tail is the same each time a coin is tossed
- Two **sampling methods**
  - Infinite population **without replacement**
  - Finite population **with replacement**
- **Example application**: early exploratory efforts in a newly recognized geological area

Binomial distribution

Probability of exactly **x successes** in **n trials**

Probability of success

Probability of failure, $(1 - p)$

$$P(x) = C_x^n \times p^x q^{n-x}$$

Parameters of distribution

$$C_x^n = \frac{n!}{x!(n-x)!}$$

Binomial Coefficient is a mathematical notation for "the **combination** of ***n*** things taken ***x*** at a time"

Number of trials considered

Number of successes, X = 0, 1, 2, ..., n

Binomial distribution

$$P(X) = \frac{n!}{X!(n-X)!} p^X (1-p)^{n-X}$$

$P(X)$: probability of X successes given n and p

X: number of "successes" in sample $(X = 0, 1, \cdots, n)$

p: the probability of each "success"

n: sample size

Tails in 2 Tosses of Coin

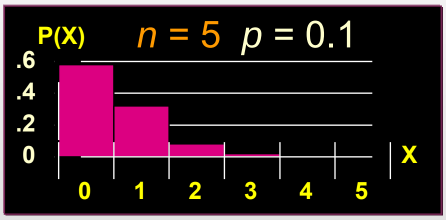
X	P(X)
0	1/4 = .25
1	2/4 = .50
2	1/4 = .25

Binomial Distribution Characteristics

- **Mean**
  - $\mu = E(X) = np$
  - e.g.: $\mu = np = 5(.1) = .5$
- **Variance and standard deviation**
  - $\sigma^2 = np(1-p)$

    $\sigma = \sqrt{np(1-p)}$
  - e.g.:

    $\sigma = \sqrt{np(1-p)} = \sqrt{5(.1)(1-.1)} = .6708$



Drilling Exploratory Wells

- Plan to drill **six exploratory wells ➔ n = 6**
- **Probability of success is 15%**
- Calculate probabilities
  - **Exactly two discoveries ➔ x = 2**
  - **Less than three successful wells ➔ x < 3**
  - **More than three successful wells➔ x > 3**

Drilling Exploratory Wells

- Plan to drill **six exploratory wells (n = 6)**, p = 15%
- Probability of **exactly two discoveries**

$$P(2) = C_x^n \times p^x q^{n-x} = \frac{6!}{2! \times (6-2)!}(0.15^2)(0.85)^{6-2}$$

$$= \frac{6\times5\times4\times3\times2\times1}{(2\times1)(4\times3\times2\times1)}(0.0225)(0.522)$$

$$= \frac{720}{2\times24}(0.0225)(0.522) = 0.1762 \text{ or } 17.62\%$$

Drilling Exploratory Wells

- Plan to drill six exploratory wells, p = 15%
- Probability of **less than three successful wells (x<3➔ x=0,1,2)**

$$P(2) = 0.1762$$

$$P(0) = \frac{6!}{(0!)(6!)}(0.15^0)(0.85)^{6-0} = 0.3771$$

$$P(1) = \frac{6!}{(1!)(6!)}(0.15^1)(0.85)^{6-1} = 0.3993$$

$$P(<3) = P(0) + P(1) + P(2) = 0.9526 \text{ or } 95.26\%$$

Drilling Exploratory Wells

- Plan to drill six exploratory wells, p = 15%
- Probability of **more than three successful wells** **(x > 3 ➔ x = 4, 5, 6)**

$$P(x>3)=P(4)+P(5)+P(6)$$
$$=0.0055+0.0004+0$$
$$=0.0059 \text{ or } 0.59\%$$

Negative Binomial Distribution

- **Number of failures (x) that will occur before any number of success (r) is attained**, assuming probability of success is constant for each trial.

Negative Binomial Distribution

Required number of successes

Probability of success on each trial

Probability of exactly *x* failures

$$P(x) = \binom{r+x-1}{x} p^r q^x$$

Number of failures before achieving r^{th} success

Probability of failure on each trial, 1 – *p*

Negative Binomial Distribution

$$P(x) = \binom{r+x-1}{x} p^r q^x$$

$$\binom{r+x-1}{x} = \frac{(r+x-1)!}{(r-1)!\,x!}$$

Probability of Drilling Success

- **Required number of successes *r* = 2**
- Probability of success is $p = 0.15$
- Calculate probability
  - **Zero failures**
  - **Exactly one failure**
  - **Drilling more than one dry hole**

Probability of Drilling Success

- Required number of successes ***r* = 2**
- Probability of success is $p = 0.15$
- Calculate probability of **zero failures (x=0)**

$$P(0) = \binom{2+0-1}{0} \times (0.15)^2 \times (0.85)^0$$

$$= \binom{1}{0} \times (0.15)^2 \times (0.85)^0$$

$$= 1 \times 0.0225 \times 1 = 0.0225 \text{ or } 2.25\%$$

Probability of Drilling Success

- Required number of successes ***r* = 2**
- Probability of success is *p* = 0.15
- Calculate probability of **exactly one failure (x=1)**

$$P(1) = \binom{2+1-1}{1} \times (0.15)^2 \times (0.85)^0$$
$$= \binom{2}{1} \times (0.15)^2 \times (0.85)^0$$
$$= 2 \times 0.0225 \times 0.85 = 0.0383 \text{ or } 3.83\%$$

Probability of Drilling Success

- Required number of successes *r* = 2
- Probability of success is *p* = 0.15
- Calculate probability of **more than one dry hole (x=0, 1)**

$$P(0) + P(1) = 0.0225 + 0.0383$$
$$= 0.0608 \text{ or } 6.08\%$$

$$100 - 6.08 = 93.92$$

Multinomial Probability Distribution

- Assumptions for applicability
  - Random variable can assume only one of **several possible values** in any single trial
  - Probability of each outcome remains constant from one trial to the next
  - In sequence of trials, outcome of any trial has no effect on outcome of any other trial (trials are independent events)
  - Number of trials is discrete

Multinomial Probability Distribution

- Example application: early exploratory efforts in a newly recognized geological area with **multiple possible outcomes**
  - Dry hole?
  - 10 million STB reserve?
  - 15 million STB reserve?

Multinomial Probability Distribution

Probability of this sample

Size of sample, $k_1 + k_2 + \ldots + k_m$

$$P(S) = \frac{N!}{k_1!k_2!\ldots k_m!} P_1^{k_1} P_2^{k_2} \ldots P_m^{k_m}$$

Probability of drawing types 1, 2, ..., *m* from population

Total number of outcomes of types 1, 2, ..., *m* in this sample

Total number of outcomes in sample

Drilling Three New Wells

- Possible outcomes of exploratory well
  - Dry hole (k_1) Probability = 0.5
  - Discovery with 12 million STB reserves (k_2) P = 0.35
  - Discovery with 18 million STB reserves (k_3) P = 0.15
- Calculate **probabilities of discovering various total reserves** with **three additional wells**

$m = 3$ $N = 3$ $P_1 = 0.5$ $P_2 = 0.35$ $P_3 = 0.15$

Drilling Three New Wells

- Possible outcomes of exploratory well

k_1	k_2	k_3	**Prob $P(S)$**	**Res, MMSTB**	**Prob of Res > than**	**Expected Res, MMSTB**
3	0	0	0.125	--	1.000	---
2	1	0	0.263	**12**	**0.875**	**3.150**
2	0	1	0.113	18	0.613	2.025
1	2	0	0.184	24	0.500	4.410
1	1	1	0.158	30	0.316	4.725
…	…	…	…	…	…	…
0	0	3	0.003	54	0.003	0.182
			1.000			**20.700**

Drilling Three New Wells

- Possible outcomes of exploratory well
  - **One possibility: k_1 = 2, k_2 = 1, k_3 = 0**

$$P(S) = \frac{3!}{2!1!0!}(0.5)^2 \times (0.35)^1 \times (0.15)^0$$

$$= \frac{3 \times 2 \times 1}{2 \times 1 \times 1 \times 1} = 0.25 \times 0.35 \times 1 = 0.263$$

Corresponding reserves = (2)(0) + (1)(12) + (0)(18) = **12 million STB**

Expected reserves = (12 × 0.263) = **3.15 million STB**

Hypergeometric Probability Distribution

- **"n" trials in a sample** taken from a finite **population of size N**
- Sample taken **without replacement**
- Trials are **dependent**
- Concerned with finding the **probability of "X" successes in the sample** where there are **"C" or "A" successes in the population**

Hypergeometric Probability Distribution

Number of successes in population is less than *N*

Number of items in population

Number of successes observed in sample

Number of total successes in population

$$P(x) = \frac{\binom{C}{x}\binom{N-C}{n-x}}{\binom{N}{n}}$$

Hypergeometric random variable

Discrete number; values = 0, 1, … , to smaller of *n* or *C*

Number of trials (size of sample)

Hypergeometric Distribution Function

$$P(X) = \frac{\binom{A}{X}\binom{N-A}{n-X}}{\binom{N}{n}}$$

e.g.: Three Light bulbs were selected from ten. Of the ten, four were defective. What is the probability that two of the three selected are defective?

$P(X)$: probability that X successes given $n, N,$ and A

n: sample size

N: population size

A: number of "successes" in population

X: number of "successes" in sample

$(X = 0, 1, 2, \cdots, n)$

$$P(2) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}} = .30$$

Hypergeometric Distribution Characteristics

- Mean
  - $$\mu = E(X) = n\frac{A}{N}$$
- Variance and Standard Deviation
  - $$\sigma^2 = \frac{nA(N-A)}{N^2}\frac{N-n}{N-1}$$
    $$\sigma = \sqrt{\frac{nA(N-A)}{N^2}}\sqrt{\frac{N-n}{N-1}}$$

Finite Population Correction Factor

Choosing Candy From a Box

- Box contains **20 candies**
  - **8 caramel**, 12 nut
- Person **selects 4 pieces**
- Calculate probability that **1 is caramel**

$N = 20$
$C = 8$
$n = 4$
$x = 1$

$$P(x) = \frac{\binom{8}{1}\binom{20-8}{4-1}}{\binom{20}{4}} = \frac{\binom{8}{1}\binom{12}{3}}{\binom{20}{4}} = \frac{\frac{8!}{1!(8-1)!} \times \frac{12!}{3!(12-3)!}}{\frac{20!}{4!(20-4)!}}$$

$$= \frac{8 \times 220}{4,845} = \frac{1,760}{4,845} = 0.3633 \text{ or } 36.33\%$$

Exploration Prospects

- Company has 10 exploration prospects
  - 4 expected to be productive
  - 3 wells will be drilled
- Calculate probability that **1 will be productive**

$N = 10$
$C = 4$
$n = 3$
$x = 1$

$$P(x) = \frac{\binom{4}{1}\binom{10-4}{3-1}}{\binom{10}{3}} = \frac{\binom{4}{1}\binom{6}{2}}{\binom{10}{3}} = \frac{\frac{4!}{1!(4-1)!} \times \frac{6!}{2!(6-2)!}}{\frac{10!}{3!(10-3)!}}$$

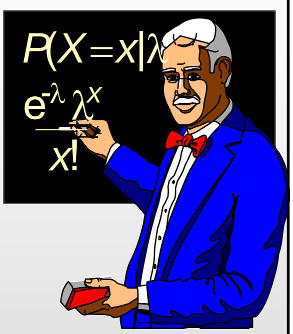
$$= \frac{4 \times 15}{120} = \frac{60}{120} = 0.5 \text{ or } 50\%$$

Poisson Probability Distribution

- Assumptions
  - Average number of events **over a unit of measure** (space or time) remains **constant** from one trial to the next
  - From one trial to the next, number of occurrences of the random variable must be **independent** of each other
  - Random variable must be **integer per unit of measurement**
  - Event must be **relatively rare** or **uncommon occurrence** with the unit of measurement so the probability of two or more occurrences in any time interval is negligible relative to the probability of one occurrence in the interval

Poisson Probability Distribution

- **Poisson process**
  - Discrete events in **an “interval”**
    - The probability of one success in an interval is stable
    - The probability of more than one success in this interval is 0
  - The probability of success is **independent** from interval to interval



  - e.g.: The number of customers arriving in 15 minutes
  - e.g.: The number of defects per case of light bulbs

Poisson Probability Distribution

- **Characteristics**
  - Discrete distribution, concerned with occurrences that can be described by **discrete random variable**
  - Applied in situations in which **probability of success, p, is very small** compared to probability of failure, q, and in which n is very large
  - Examples: frequency of pump breakdowns on given lease; well workover requirements

Poisson Probability Distribution

- **Characteristics**
  - Main parameter is **mean, λ,** from which other parameters can be obtained easily
  - Distribution is skewed to the left, and skewness increases as λ decreases

Poisson Probability Distribution

- Characteristics
  - **Binomial distribution** can be approximated with **Poisson distribution when *n* is large** (beyond binomial tables for *n*) and *p* small such that $np < 5$
  - **Binomial distribution** can be approximated by **normal distribution** if $n > 30$ and np and $n(1-p) > 5$
  - **Normal approximation** to **binomial distribution** can be extended to Poisson distribution when $\lambda > 10$

Poisson Probability Distribution

Average number of occurrences per interval of time or space

$$P(x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

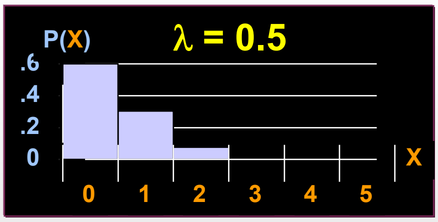
Probability of exactly *x* occurrences

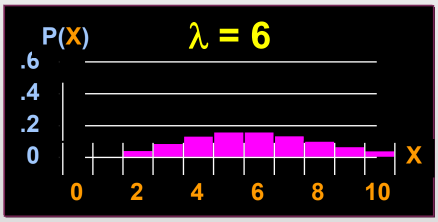
Number of occurrences per basic unit of measure

Poisson Distribution Characteristics

- Mean
  - $\mu = E(X) = \lambda$

    $= \sum_{i=1}^{N} X_i P(X_i)$
- Standard deviation and variance
  - $\sigma^2 = \lambda \qquad \sigma = \sqrt{\lambda}$





Poisson Probability Distribution Function

$$P(X) = \frac{e^{-\lambda}\lambda^X}{X!}$$

$P(X)$: probability of X "successes" given λ

X: number of "successes" per unit

λ: expected (average) number of "successes"

e: 2.71828 (base of natural logs)

e.g.: Find the probability of four customers arriving in three minutes when the mean is 3.6.

$$P(X) = \frac{e^{-3.6}3.6^4}{4!} = .1912$$

Pipeline Leaks

- Pipeline averages 3 leaks per year
- Calculate probability of exactly 4 leaks next year

$$P(x) = \frac{\lambda^x}{x!} e^{-\lambda} = \frac{3^4}{4!} e^{-3} = \frac{81 \times 0.0498}{24}$$

$$= 0.1681 \text{ or } 16.81\%$$

Well Workovers

- Lease averages 2 workovers per day
- Calculate number of days per month with no workovers

$$P(x) = \frac{\lambda^x}{x!} e^{-\lambda} = \frac{(2)^0}{0!} e^{-\lambda} = \frac{1 \times 0.1353}{1}$$

$$= 0.1353 \text{ or } 13.53\%$$

$$\therefore \text{ days per month} = 0.1353 \times \frac{365}{12} = 4.115 \text{ or } 4$$

Pipeline Leaks

- Pipeline averages 5 leaks per 1,000 miles
- Calculate probability of no leaks in first 100 miles

$$P(x) = \frac{\lambda^x}{x!} e^{-\lambda} = \frac{0.5^0}{0!} e^{-0.5} = \frac{1 \times 0.6065}{1}$$

$$= 0.6065 \text{ or } 60.65\%$$

Giant Oilfield Discoveries

- Company discovers 1 giant field every 2 years
- Show discovery distribution over 10 years

λ = 0.5 per year

Fields discovered during year, x	*P*(*x*)	*P*(*x*) × 10 years	**No. of years**	**Fields discovered**
0	0.6065	6.065	**6**	**0**
1	0.3033	3.033	**3**	**3**
2	0.0758	0.758	**1**	**2**
3	0.0126	0.126	**0**	**0**
4	0.0016	0.016	**0**	**0**
5	0.0002	0.002	**0**	**0**
	1.0000	10.000	**10**	**5**

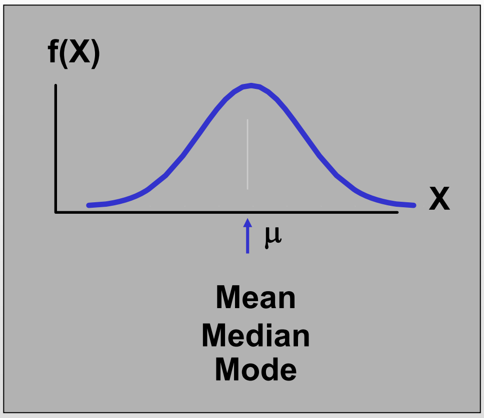
Continuous Probability Distribution

Continuous Probability Distributions

- **Continuous random variable**
  - Values from **interval of numbers**
  - Absence of gaps
- **Continuous probability distribution**
  - Distribution of continuous random variable
- Most important continuous probability distribution
  - The normal distribution
- **Cumulative distribution function**, ***F*** **(*x*)** or **CDF**, represents probability that outcome of *X* in a random trial will be **less than or equal to** any specified value of *x*

Normal Distribution

- "Bell shaped"
- Symmetrical
- Mean, median and mode are equal
- Interquartile range equals 1.33 σ
- Random variable has infinite range



The Mathematical Model

$$f(X) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(X-\mu)^2}$$

$f(X)$: density of random variable X

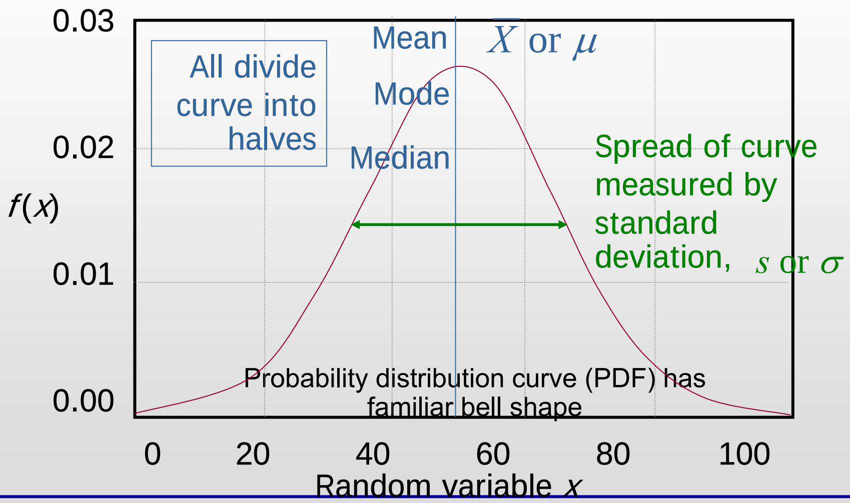
$\pi = 3.14159; \qquad e = 2.71828$

μ : population mean

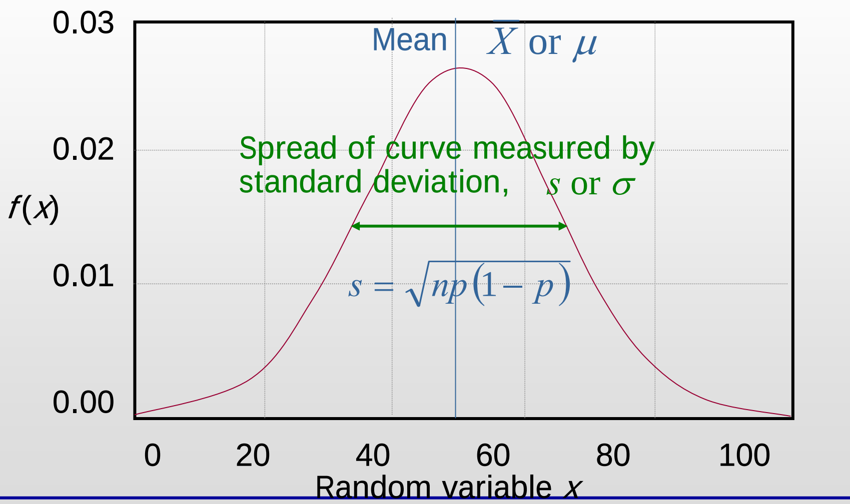
σ : population standard deviation

X : value of random variable $(-\infty < X < \infty)$

Normal (Gaussian) Probability Distribution

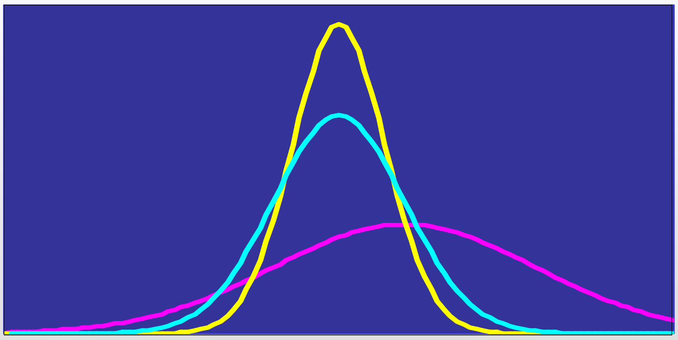


Normal (Gaussian) Probability Distribution



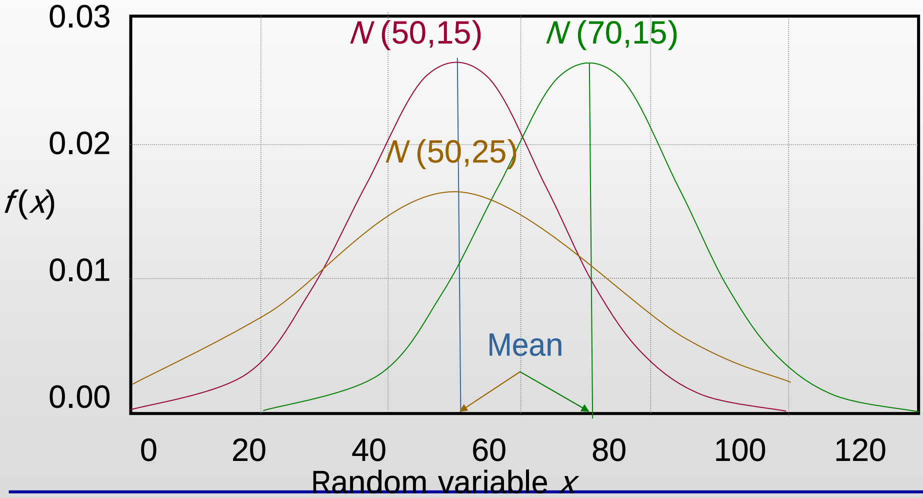
Many Normal Distributions

There are an infinite number of normal distributions



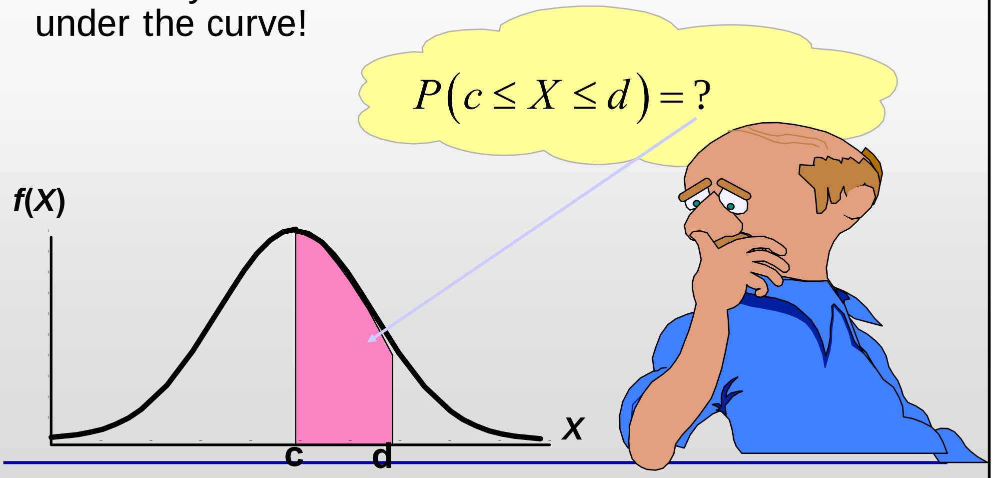
By varying the parameters σ and μ, we obtain different normal distributions

Normal (Gaussian) Probability Distribution

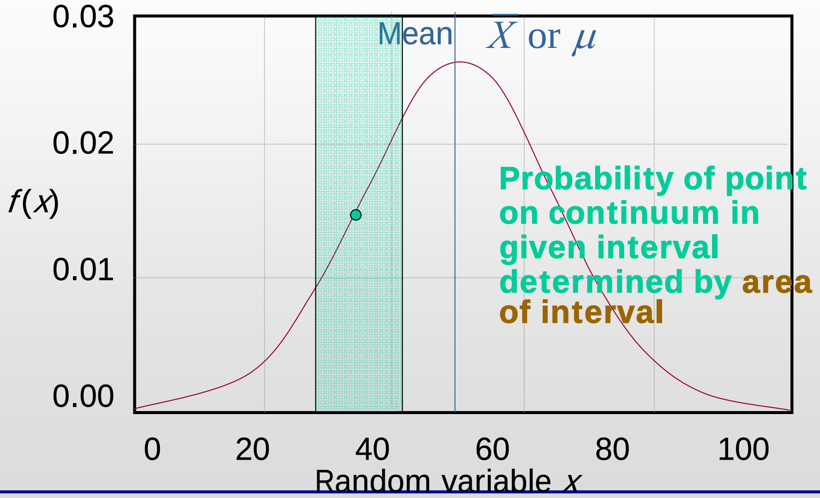


Finding Probabilities

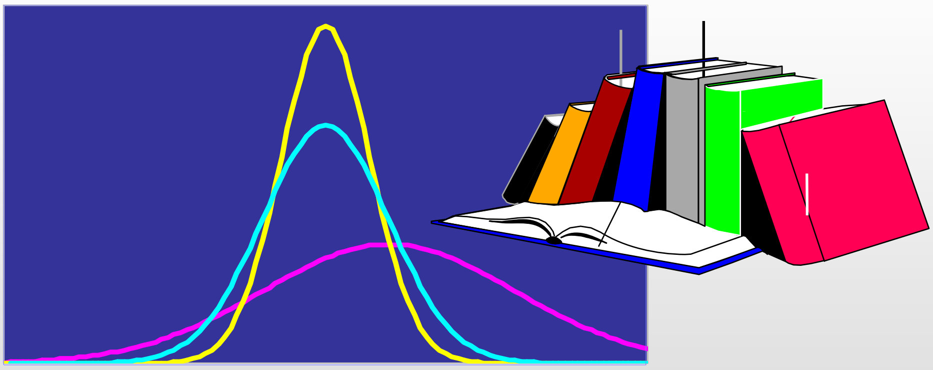
Probability is the area under the curve!



Normal (Gaussian) Probability Distribution



Which Table to Use?



An infinite number of normal distributions means an infinite number of tables to look up!

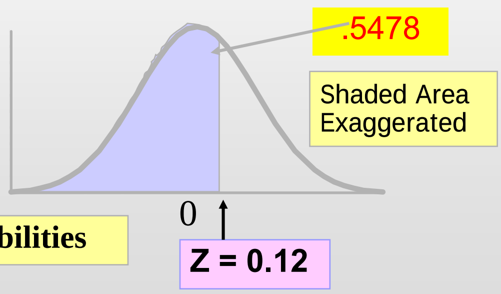
Solution: The Cumulative Standardized Normal Distribution

Cumulative Standardized Normal Distribution Table (Portion)

$\mu_Z = 0 \qquad \sigma_Z = 1$

Z	.00	.01	**.02**
0.0	.5000	.5040	.5080
0.1	.5398	.5438	.5478
0.2	.5793	.5832	.5871
0.3	.6179	.6217	.6255

Probabilities



Only One Table is Needed

Standard Normal Probability Distribution

- **Dimensionless variable *Z*** useful for analysis, and used in normal probability table

$$Z = \frac{X - \overline{X}}{s} \text{ or } \frac{X - \mu}{\sigma}$$

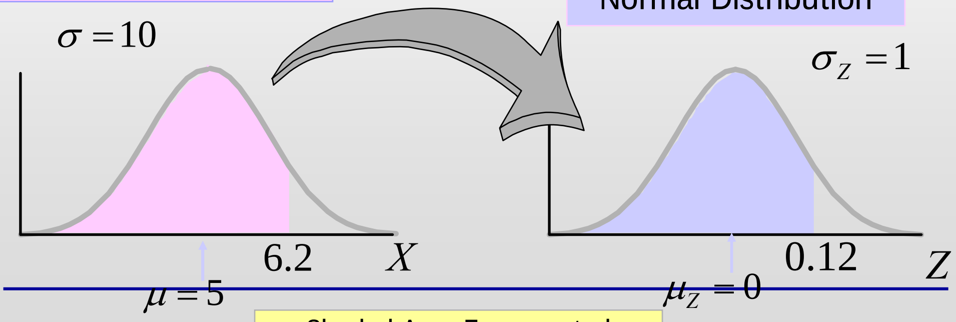
- Table gives area under normal curve between *Z* =0 and *Z* =*Z´*
- Area under curve is probability

Standardizing Example

$$Z = \frac{X - \mu}{\sigma} = \frac{6.2 - 5}{10} = 0.12$$

Normal Distribution

Standardized Normal Distribution

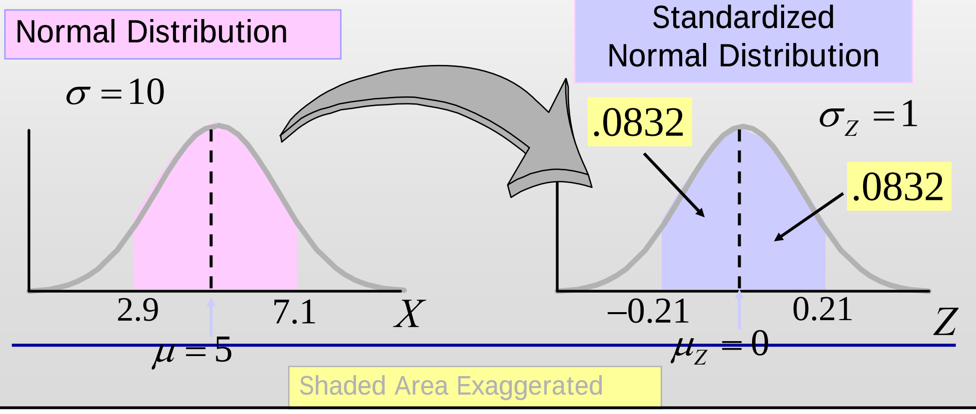


Shaded Area Exaggerated

Example:

$$P(2.9 \le X \le 7.1) = .1664$$

$$Z = \frac{X - \mu}{\sigma} = \frac{2.9 - 5}{10} = -.21 \qquad Z = \frac{X - \mu}{\sigma} = \frac{7.1 - 5}{10} = .21$$



Example:

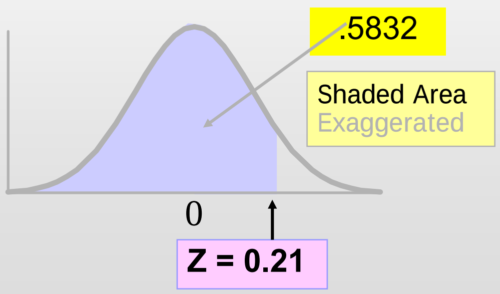
$$P(2.9 \le X \le 7.1) = .1664$$

(continued)

Cumulative Standardized Normal Distribution Table (Portion)

Z	.00	.01	.02
0.0	.5000	.5040	.5080
0.1	.5398	.5438	.5478
0.2	.5793	.5832	.5871
0.3	.6179	.6217	.6255

$\mu_Z = 0 \quad \sigma_Z = 1$



Example:

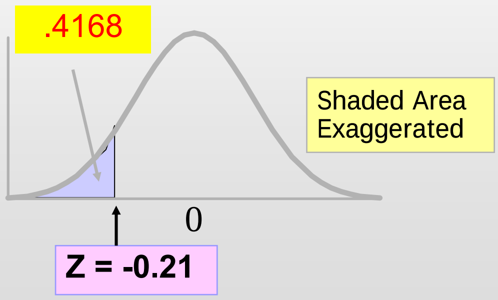
$$P(2.9 \le X \le 7.1) = .1664$$

(continued)

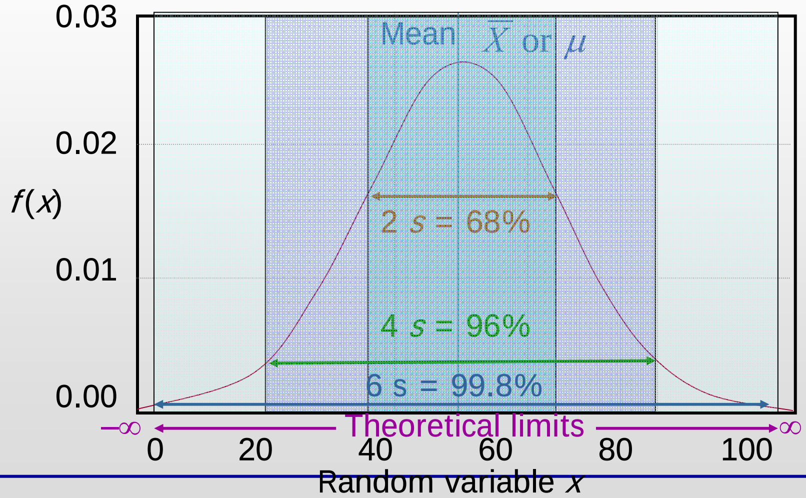
Cumulative Standardized Normal Distribution Table (Portion)

Z	.00	.01	.02
-03	.3821	.3783	.3745
-02	.4207	.4168	.4129
-0.1	.4602	.4562	.4522
0.0	.5000	.4960	.4920

$\mu_Z = 0 \quad \sigma_Z = 1$

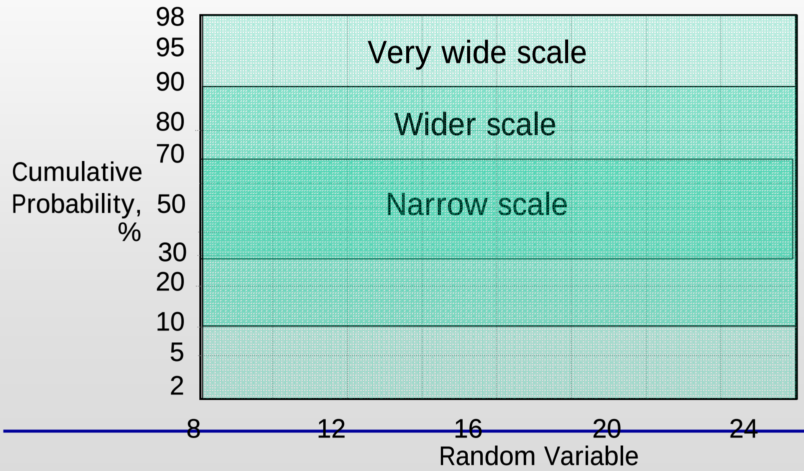


Normal (Gaussian) Probability Distribution



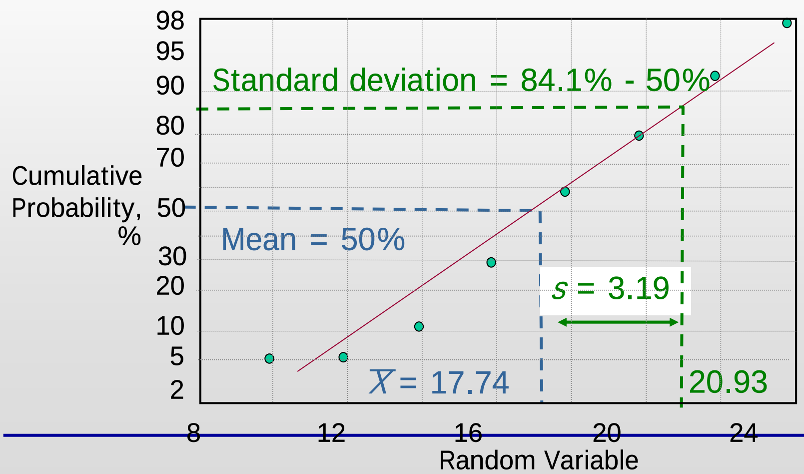
Normal (Gaussian) Probability Distribution

- Straight line on normal probability scale



Normal (Gaussian) Probability Distribution

- Straight line on normal probability scale



Assessing Normality

- Not all continuous random variables are normally distributed
- It is important to evaluate how well the data set seems to be adequately **approximated by a normal distribution**

Assessing Normality

(continued)

- **Construct charts**
  - For small- or moderate-sized data sets, do stem-and-leaf display and box-and-whisker plot look symmetric?
  - For large data sets, does the histogram or polygon appear bell-shaped?
- **Compute descriptive summary measures**
  - Do the mean, median and mode have similar values?
  - Is the interquartile range approximately 1.33 σ?
  - Is the range approximately 6 σ?

Assessing Normality

(continued)

- **Observe the distribution of the data set**
  - Do approximately 2/3 of the observations lie between mean $\pm$ 1 standard deviation?
  - Do approximately 4/5 of the observations lie between mean $\pm$ 1.28 standard deviations?
  - Do approximately 19/20 of the observations lie between mean $\pm$ 2 standard deviations?
- **Evaluate normal probability plot**
  - Do the points lie on or close to a straight line with positive slope?

Assessing Normality

(continued)

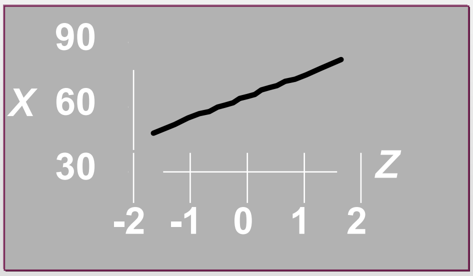
- **Normal probability plot**
  - Arrange data into ordered array
  - Find corresponding standardized normal quantile values
  - Plot the pairs of points with observed data values on the vertical axis and the standardized normal quantile values on the horizontal axis
  - Evaluate the plot for evidence of linearity

Assessing Normality

(continued)

Normal Probability Plot

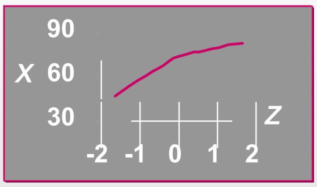
for Normal Distribution



Look for Straight Line!

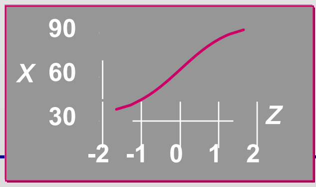
Normal Probability Plot

Left-Skewed

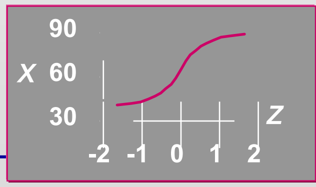


Right-Skewed

Rectangular



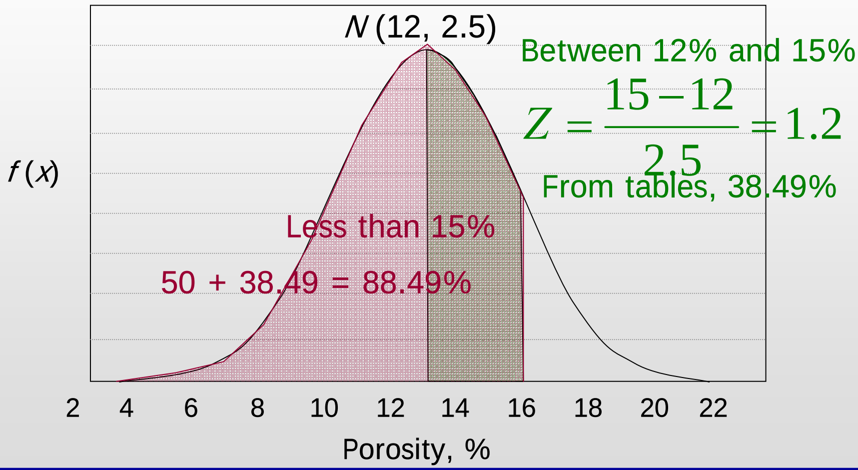
U-Shaped



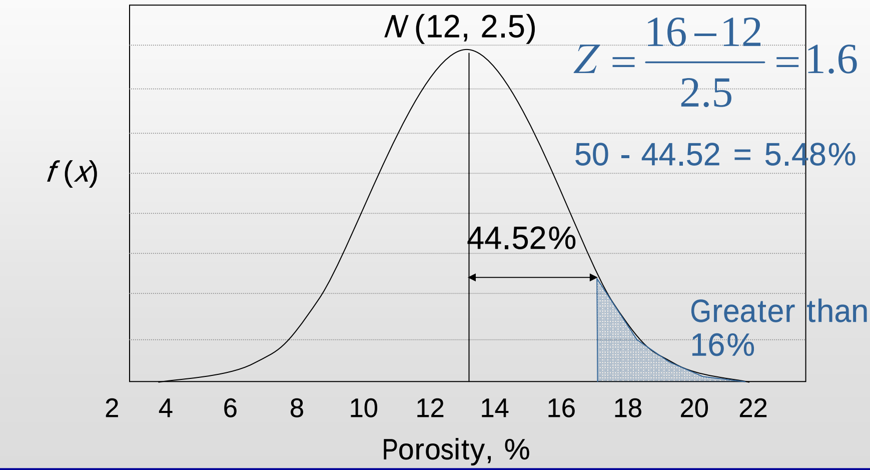
Porosity of a Formation

- Mean porosity = 12%, *s* = 2.5%
- Calculate probability of porosity
  - Between 12% and 15%
  - Less than 15%
  - Greater than 16%
  - Between 10% and 16%

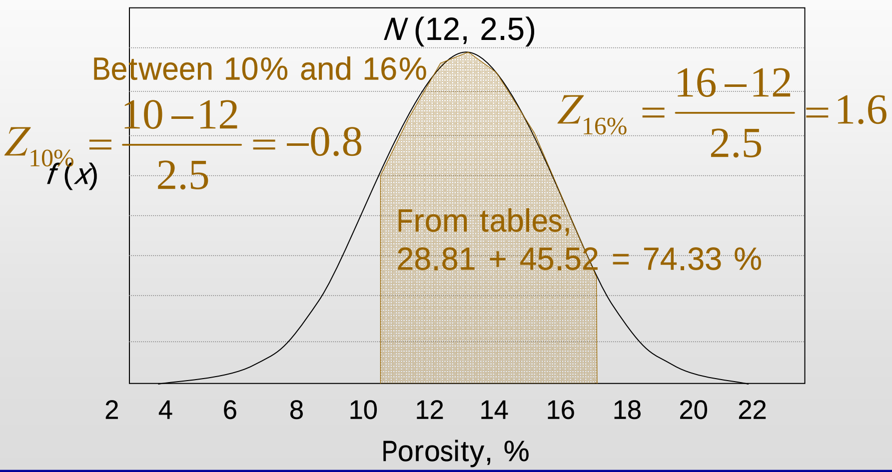
Porosity of a Formation



Porosity of a Formation



Porosity of a Formation



Chance of Discovering Oil

- Chance of discovery in prospect is 15%
- Funds available to drill 10 wells
- Calculate chance at least one discovery

$$\text{mean} = Np = 10 \times 0.15 = 1.5$$

$$s = \sqrt{10 \times 0.15 \times 0.85} = 1.275$$

$$Z = \frac{1 - 1.5}{1.275} - 0.3922 \cong -0.39$$

From tables, at Z = -0.39, CDF = 15.17%

Chance of Discovering Oil

- Chance of discovery in prospect is 15%
- Funds available to drill 10 wells
- Calculate chance at least one discovery

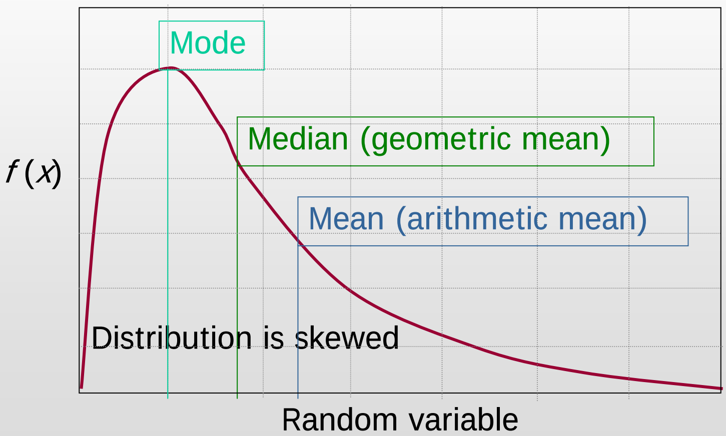
Probability of 1.5 to 10 successes is approximately 50%.

Probability of one or more successes is 15.17 + 50 = 65.17%

Probability of having between 1 and 1.5 successes

From tables, at Z = -0.39, CDF = 15.17%

Lognormal Probability Distribution

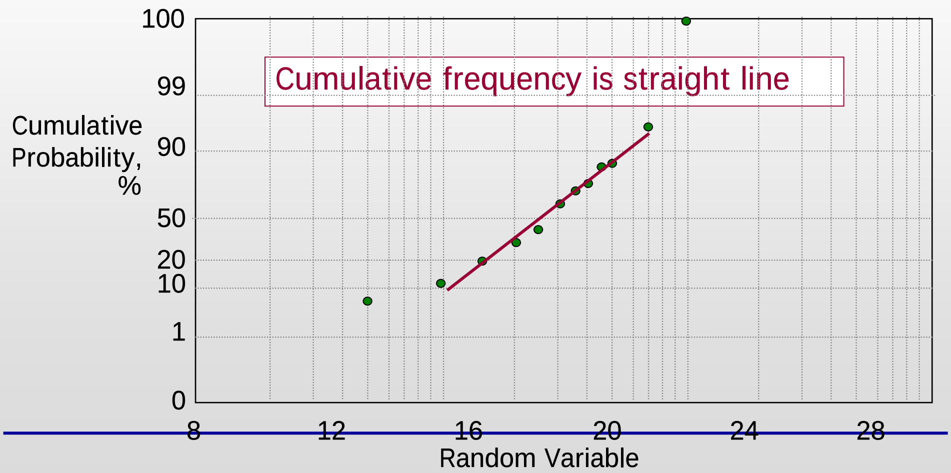


Lognormal Probability Distribution

- Distribution is transformed version of normal distribution
- When X is **lognormally distributed** with parameters s and $\overline{X}$, Y= ln(X) is normally distributed with these parameters

Lognormal Probability Distribution

- On log probability scale



Lognormal Probability Distribution

- Mean–

$$\overline{X} = \exp\left(\alpha + 0.5\beta^2\right)$$

- Mode–

$$\exp\left(\alpha - \beta^2\right)$$

Lognormal Probability Distribution

- Standard deviation

$$s = \sqrt{\exp\left(2\alpha + 2\beta^2\right) - \exp\left(2\alpha + \beta^2\right)}$$

Natural logarithm of variable read at 50th percentile

$$\beta = \frac{\ln\left(X_{2\%}\right) - \ln\left(X_{98\%}\right)}{z\text{, between two tails}}$$

$$s = \sqrt{\overline{X}^2\left(e^{\beta^2} - 1\right)}$$

Lognormal Probability Distribution

- Mode of *X*

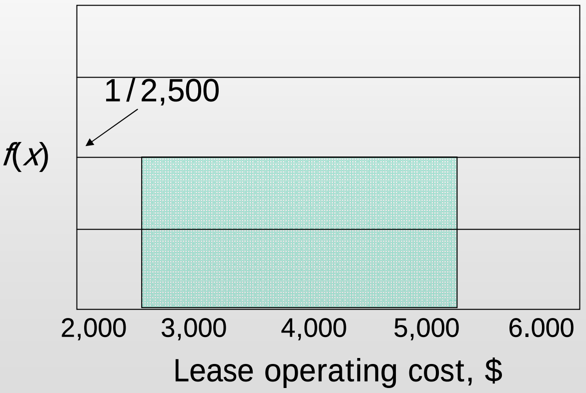
$$\frac{\overline{X}}{e^{1.5\beta^2}}$$

Uniform Probability Distribution

- Random variable that takes on **integer values** within **given interval (between a minimum and a maximum)** with **equal probabilities** is called **discrete random variable**
- Continuous analog is called **continuous uniform** or **rectangular probability distribution**

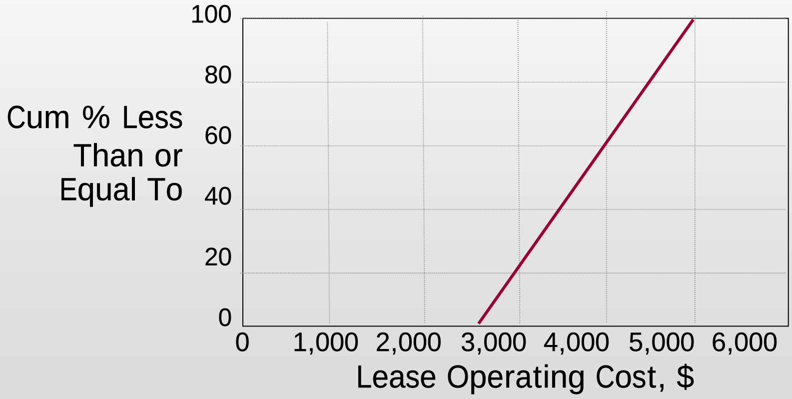
Uniform Probability Distribution

- Probability distribution function



Uniform Probability Distribution

- Cumulative distribution function



Uniform Probability Distribution

- Probability distribution function, PDF

$$f(x) = \frac{1}{x_{\max} - x_{\min}}$$

- Mean

$$\overline{X} = \frac{x_{\max} + x_{\min}}{2}$$

Uniform Probability Distribution

- Standard deviation

$$s = \sqrt{\frac{(x_{\max} - x_{\min})^2}{12}}$$

- Cumulative distribution function

$$P(X \le x) = \frac{x - x_{\min}}{x_{\max} - x_{\min}}$$

$$X = R_N (X_{\max} - X_{\min}) + X_{\min}$$

Cost of Operations

- Lease operating costs could vary between $2,500 and $5,000 per month
- All values within range are equally likely
- Calculate
  - Mean
  - Standard deviation
  - Continuous probability density function
  - Probability that operating costs will be $3,500 or less

Cost of Operations

- Mean

$$\overline{X} = \frac{x_{\max} + x_{\min}}{2} = \frac{5{,}000 + 2{,}500}{2} = \$3{,}750$$

- Standard deviation

$$s = \sqrt{\frac{(5{,}000 - 2{,}500)^2}{12}} = \sqrt{\frac{6{,}250{,}000}{12}}$$
$$= \sqrt{520{,}833.33} = \$721.69$$

Cost of Operations

- Density function

$$f(x) = \frac{1}{x_{\max} - x_{\min}} = \frac{1}{5{,}000 - 2{,}500} = \frac{1}{2{,}500}$$

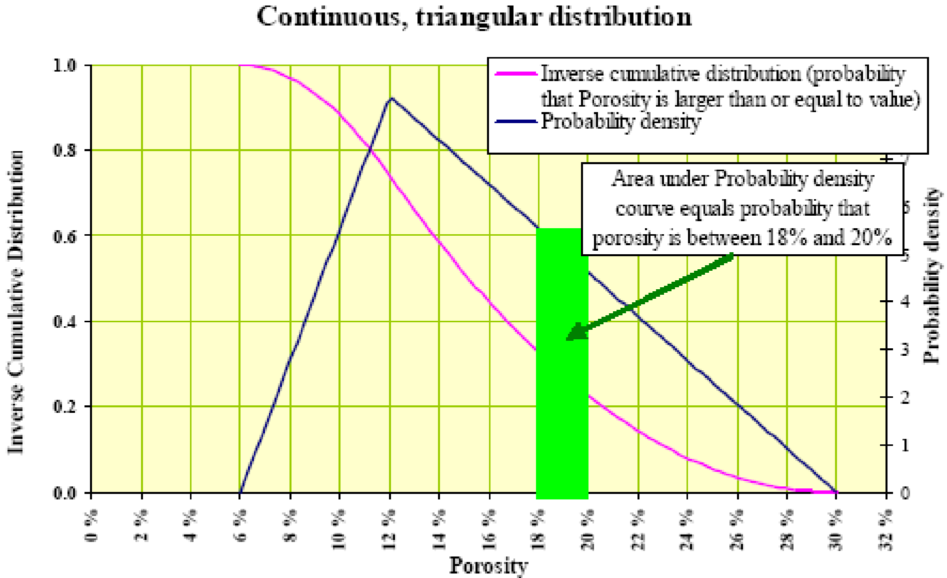
- Probability of cost \$3,500 or less

$$P(X \le x) = \frac{x - x_{\min}}{x_{\max} - x_{\min}} = \frac{3{,}500 - 2{,}500}{5{,}000 - 2{,}500}$$
$$= 0.40 \text{ or } 40\%$$

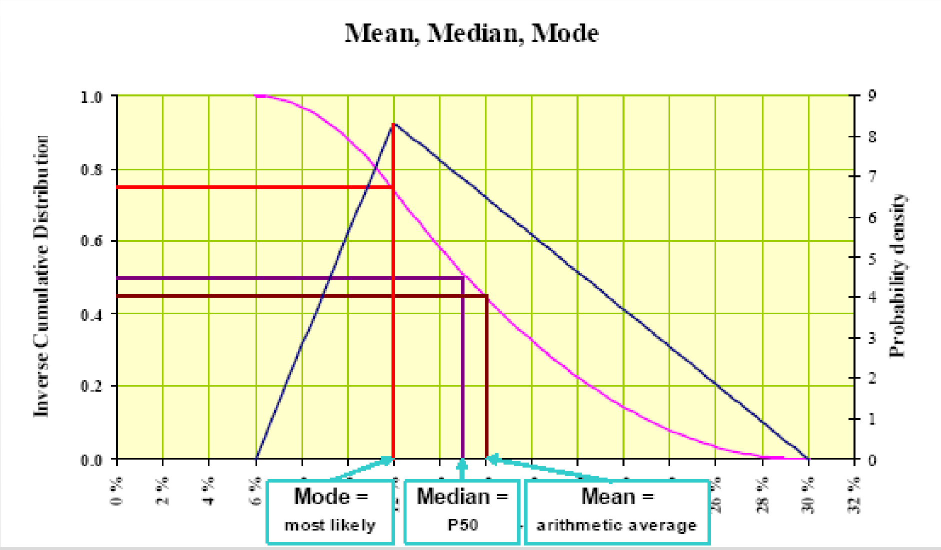
Triangular Probability Distribution

- Useful when we have upper limit, lower limit, and most likely values of random variable specified

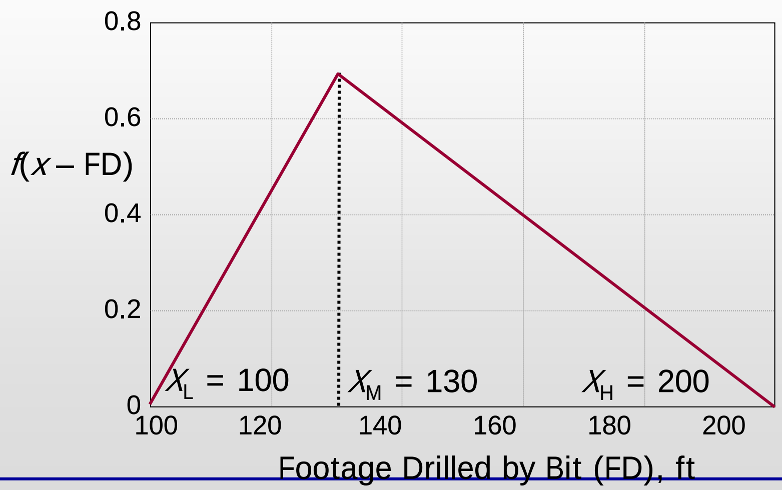
Triangular Probability Distribution



Triangular Probability Distribution



Triangular Probability Distribution



Triangular Probability Distribution

- Mean

Minimum value of random variable; Maximum value of random variable; Most likely value of random variable

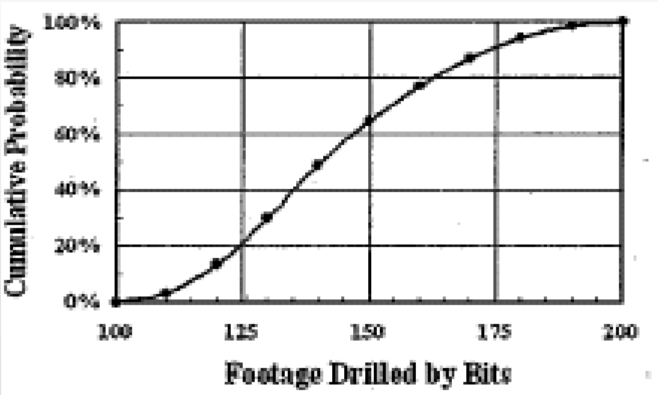
$$\overline{X} = \frac{X_L + X_H + X_M}{3}$$

- Standard deviation

$$s = \sqrt{\frac{(X_H - X_L)(X_H^2 - X_L X_H) - X_M X_H (X_H - X_M) - X_L X_M (X_M - X_L)}{18(X_H - X_L)}}$$

Triangular Probability Distribution

- Cumulative distribution function



Triangular Probability Distribution

- When $X_L \leq X \leq X_M$

$$f(x) = \left(\frac{X - X_L}{X_M - X_L}\right)^2 \left(\frac{X_M - X_L}{X_H - X_L}\right)$$

- When $X_M \leq X \leq X_H$

$$f(x) = 1 - \left(\frac{X_H - X}{X_H - X_M}\right)^2 \left(\frac{X_H - X_M}{X_H - X_L}\right)$$

Footage Drilled by Bit

- Most likely footage drilled is 130 ft
  - Minimum 100 ft, maximum 200 ft
- Footage follows triangular distribution
- Develop cumulative distribution function for drilling data

$X_L = 100$
$X_M = 130$
$X_H = 200$

Footage Drilled by Bit

- For X = 110 ft (less than mode of 130)

$$f(x)=\left(\frac{X-X_L}{X_M-X_L}\right)^2\left(\frac{X_M-X_L}{X_H-X_L}\right)=\left(\frac{110-100}{130-100}\right)^2\left(\frac{130-100}{200-100}\right)$$

$$=(0.3333)^2(0.3)=0.333 \text{ or } 3.33\%$$

Footage Drilled by Bit

- For X = 160 ft (more than mode of 130)

$$f(x)=1-\left(\frac{X_H-X}{X_H-X_M}\right)^2\left(\frac{X_H-X_M}{X_H-X_L}\right)$$

$$=1-\left(\frac{200-160}{200-130}\right)^2\left(\frac{200-130}{200-100}\right)$$

$$=1-(0.5714)^2(0.7)=1-0.2285$$

$$=0.7715 \text{ or } 77.15\%$$

Excel Built-In Functions

- Description is in Mian Vol II.
  - BINOMDIST – binomial distribution
  - NEGBINOMDIST – negative binomial distribution
  - HYPGEOMDIST – hypergeometric distribution
  - POISSON – Poisson distribution
  - NORMDIST – Normal distribution
  - LOGNORMDIST – Lognormal distribution

Probability Application

Problems in Audits of Probabilistic Estimates

- **Probabilistic cdf** may be result from several complex algorithms, each with pdf's for several variables
- Each **pdf** must be reasonable
- Possibilities of **correlations** between variables must be checked
- Validity of **algorithms** must be checked

Scenario Approach to Determine Range of Uncertainty

Requires three separate scenarios or estimates

- Estimate based on **minimum (most conservative) value** of parameters – considered "proved" or 1P
- Estimate based on **most likely values** of parameters, considered "proved plus probable" or 2P
- Estimate based on **maximum (most aggressive) value** of parameters – considered "proved plus probable plus possible" or 3P

Scenario method has real problems!

Problems with Scenario Approach

- Computation of 1P estimate assumes all minimum values of parameters occur in **same outcome** – extremely unlikely, resulting in unrealistically low reserve estimate with very small probability of occurrence
- 3P estimate unrealistically high, small probability of occurrence
- **Degree of uncertainty** in estimates neither quantified nor related to one another

Treatment of Uncertainties

- Uncertainties was classified in two broad categories:
  - **Category I:** related to geologic and engineering data in **drilled** areas or fault blocks and measurement accuracy and interpretation of data
  - **Category II:** related to geologic scenario in **undrilled** areas or fault blocks

Examples of Uncertainties in Data

- Gross rock volume in drilled areas
- Net-to-gross pay ratios and their spatial variation
- In-situ rock and fluid properties
- Location of fluid contacts
- Spatial distribution of permeability
- Nature and degree of principal heterogeneity
- Degree of reservoir compartmentalization
- Drainage areas of individual wells
- Recovery efficiencies of oil, gas, condensate

Examples of Geologic Uncertainties

- Amount of oil or gas in place
- Amount of commercially productive reservoir rock
- Areal extent of commercial accumulation

Treatment of Uncertainties

- **Data uncertainties** usually amendable to statistical analysis
  - Some uncertainties attributable to interpretation procedures rather than data – e.g., pressure transient test analysis, log interpretation
- **Geological uncertainties** more difficult to quantify, analyze using statistical methods

Typical pdf's for Parameters in Volumetric Reserve Estimates

- **Porosity** – typically **Gaussian**, correlated with initial water saturation
- **Interstitial water saturation** – typically **negative skew**, correlated with porosity
- **Net pay** – typically **log normal**, correlated with porosity and interstitial water saturation
- **Permeability** – typically **log normal**, correlated with porosity

Typical pdf's for Parameters in Volumetric Reserve Estimates (Cont'd)

- **OOIP (STB/acre)** – typically **positive skew**, **possibly log normally distributed**
- **Recovery efficiency** – typically **log normal**, correlated with porosity, irreducible water saturation, permeability, net-gross pay ratio, net pay
- **Initial well potential** – typically **log normal**, correlated with porosity, permeability, net pay, type of stimulation

pdf's of Reserves

- Statistics shows that pdf of product of pdf's tends to approach **log normal** as number of multipliers increases
  - Theoretical conclusion: pdf's of reserves estimates should be log normal
- Interestingly, field observations also indicate that pdf's of reserves attributed to sets of fields in comparable geologic settings tend to be **log normal**