Economic Risk and Decision Analysis for Oil and Gas Industry
CE81.9008

School of Engineering and Technology
Asian Institute of Technology

January Semester

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Review of Expected Operations

Expected Value of Random Variable

Expected value or mean

- **Discrete:**

$$E\{X\} = \sum_{i=1}^{n} x_i P(x_i)$$

where

E{*X*} = **expectation operator**, read "**expectation of**"

***P*(*x*$_i$)** = *P*(*X*=*x*$_i$), **unconditional probability** associated with variable *x*

E*(*x*)** often referred to as "**mean of *X"

- **Continuous:**

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx = \mu_X$$

Variance

Variance: the sum of squared deviations about the population mean.

Var(X)= E{[X-E(X)]²}= E(X2)-[E(X)]²

- **Discrete:**

$$s^2\{X\} = \sum_{i=1}^{n} \left(x_i - E\{X\}\right)^2 P(x_i)$$

where

S*²{*X*} = variance of *X

$$s\{X\} = \sqrt{s^2\{X\}}$$

- **Continuous:**

$$Var(X) = \int_{-\infty}^{\infty} [x - E(X)]^2 f(x) dx = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu_X^2$$

Multiplication of a random variable

Multiplication of a random variable by a constant

- **Expected value**

$$E(cX) = cE(X)$$

- **Variance**

$$Var(cX) = c^2 Var(X)$$

Multiplication of a random variable

- **Multiplication of two independent random variables**
  - **Expected value**

$$E(XY) = E(X)E(Y)$$

  - **Variance**

$$Var(XY)$$
$$= Var(X)[E(Y)]^2 + Var(Y)[E(X)]^2 + Var(X)Var(Y)$$

Sums of a random variable

Addition of two independent random variables

- **Expected value**

$$E\{X + Y\} = E\{X\} + E\{Y\}$$

- **Variance**

$$s^2\{X + Y\} = s^2\{X\} + s^2\{Y\}$$

Sums of a random variable

- **Linear combination of two or more independent variables.**
  - **Expected value**

$$E(c_1X + c_2Y) = c_1E(X) + c_2E(Y)$$

  - **Variance**

$$Var(c_1X + c_2Y) = c_1^2Var(X) + c_2^2Var(Y)$$

Example Calculation of Expected Value

Expected results from drilling prospect

- 30% chance of 20 MSTB
- 50% chance of 60 MSTB
- 20% chance of 95 MSTB

What are mean, variance, and standard deviation of expected reserves?

Example Calculation of Expected Value

Probability, p_i	Reserves, X_i, MSTB	$E\{X\} = p_i X_i$	$(X_i - E\{X\})^2$	Variance $p_i(Xi - E\{X\})$
0.30	20	6.0	1,225.0	367.5
0.50	60	30.0	25.0	12.5
0.20	95	19.0	1,600.0	320.0
1.00		55.0		700.0

Example Calculation of Expected Value

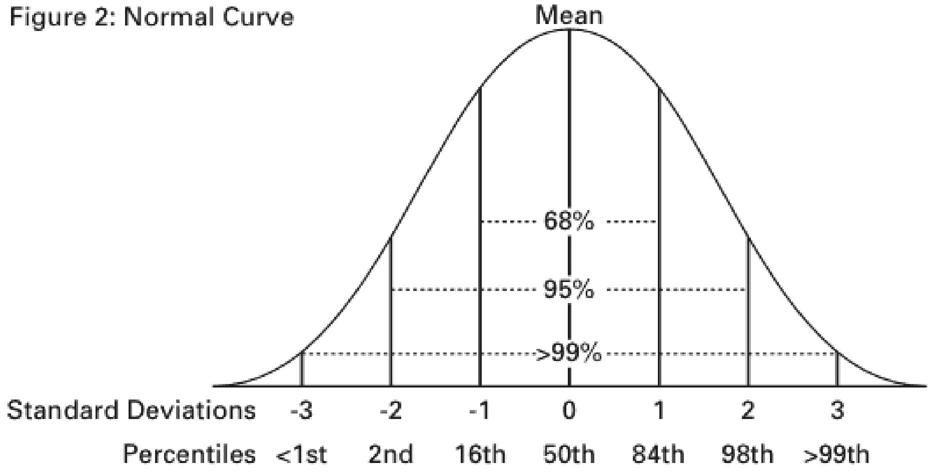
- Mean, or expected value, of reserves 55.0 MSTB
- Variance 700.0 $MSTB^2$
- Standard deviation 26.5 MSTB ($\sqrt{700}$)

<u>Interpretation</u>

- Over **large number of similar trials**, expect to recover 55 MSTB with **68% confidence** result will lie between 28.5 (55-26.5) and 81.5 (55+26.5) MSTB

The Standard Normal Curve

Figure 2: Normal Curve



Expected Value Concept

Expected Monetary Value

- When random variable in expected value is monetary value, calculated expected value called ***expected monetary value***, EMV
- **EMV** is weighted average of **possible monetary values (usually NPV’s),** weighted by respective **probabilities**
- Monetary values can be undiscounted or undiscounted
  - EMV of NPV’s called ***expected present value profit***

Structural Elements in EMV Calculations

- **Outcome state probabilities**, $P(S_j)$: probabilities assigned to outcome states
- **Criterion**: Basis decision makers use for most appropriate course of action from among the alternatives

- Two ways to display structural elements
  - Payoff table (tabular)
  - Decision tree (graphical)

EMV Example

If you spend **$500,000 drilling a wildcat oil well,** geologists estimate:

•the probability of a dry hole is 0.6

•with a probability of 0.3 that the well will be a producer can be sold immediately for **$2,000,000** and

•a probability of 0.1 that the well will produce at a rate that will generate a **$1,000,000** immediately sale value.

What is the project expected value?

EMV Example (Con'd)

Let P= probability of success, 1-P = probability of failure

Expected Value = Expected profits-expected Costs

= (P)(Income-Cost)-(1-P)(Cost)

= 0.3(2,000-500)+0.1(1,000-500)-0.6(500)

= $200

What does expected value of $200,000 means?

-50% chance that you'll get $200,000 on your investment

-the most probable outcome of selecting an alternative

-other interpretation

Interpretation of Expected Value

- What EV is not:
  - Not the **most probable outcome** of selecting an alternative
  - Not the number which we **expect** to **equal** or **exceed** 50% of the time
- Expected value is **average value *per decision*** realized when the alternative is repeated over *many trials*
- If the expected value concept is used on **rare occasions**, it becomes the same as a **one-time bet in the casino**.
- The concept of expected value represents a **play-the-average strategy**. It requires consistently applied to all project evaluations **over a long period**.

Interpretation of Expected Value

The **$200,000 expected value** is a statistical **long-term average profit or loss** that will be realized over **many repeated investments** of this type. It **<u>never guarantee this value over any individual try</u>**.

• It means if we drilled a large number of well **say 100 wells** of the type described, we expect statistics to begin to work out, we would **expect about 60 dry holes** out of 100 wells with about **30 wells producing a $2,000,000 income** and about **10 wells producing a $1,000,000 income**.

• This make **total income of $70,000,000** from 100 wells drilled **costing a total of $50,000,000** leaving **total profit of $20,000,000** after the costs, or **profit per well of $200,000**, which is the **expected value** result of the example.

Example 1: Drill vs. Farm out

Outcomes likely for **drilling prospect**

- **Dry hole** probability 65%, loss $250,000
- **Successful well** probability 35%, NPV of future net revenues $500,000
- Can **farm out** prospect, remove exposure to drilling expenditure, retain **overriding royalty interest**, NPV $50,000

Determine whether to **drill** or **farm out**

Example 1: Payoff Table Application

Outcome		Drill		Farm Out	
State	Probabililty	NPV, M$	EMV, M$	NPV, M$	EMV, M$
Dry hole	0.65	-250	-162.5	0	0
Producer	0.35	+500	+175	50	17.5
	1.00		12.5		**17.5**

Example 1: Payoff Table Application

- Since EMV of farm-out ($17.5M)>EMV of drilling, we should farm out
- Result highly **sensitive to probability of producer**
  - If increased from 35 to 36%, drillling better option
  - **Sensitivity analysis** useful if unsure about probabilities
- **Variance** of drill option much greater than variance of farm-out option (drilling much more risky)

Example 1: Sensitivity Analysis on Probabilities

- Probabilities used in EMV analysis usually most uncertain parameters
- We need to determine **influence of changes in probabilities** on apparent optimal decision to improve our decision making
- Consider example with two acts (drill or farm out) and two events (dry hole or producer)

Example 1: Sensitivity Analysis on Probabilities

Outcome State	Probabililty	Drill		Farm Out	
		NPV, M$	EMV, M$	NPV, M$	EMV, M$
Dry hole	**0.65**	-250	-162.5	0	0
Producer	**0.35**	+500	+175	50	17.5
	1.00		12.5		17.5

Example 1: Sensitivity Analysis on Probabilities

Let p = probability of dry hole
$(1-p)$ = probability of producer

Then

EV{drill} $= p(-250) + (1-p)(500)$
$= -750p + 500$

EV{farmout} $= p(0) + (1-p)(50)$
$= -50p + 50$

Example Sensitivity Analysis on Probabilities

- Decision maker **indifferent** if EV of two alternatives equal
- Probability at point of indifference given by

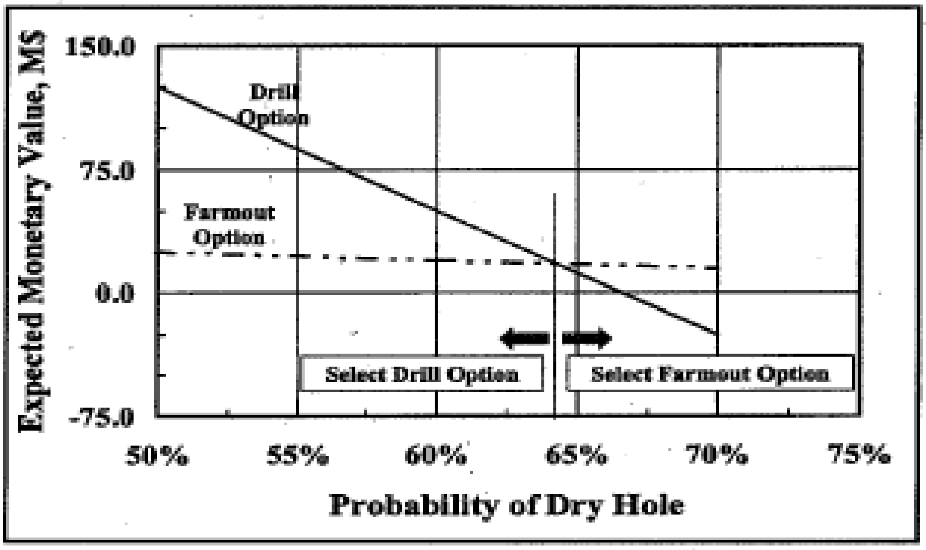
  EMV{Drill} = EMV {Farmout}

  $-750p + 50 = -50p + 50$

  $p = 0.6429$ or 64.29%

  - **Farm-out optimal for p>0.6429**, but results *highly sensitive* to change in probability
  - We must do our best to **ensure probability correct**

Example 1: Sensitivity Analysis on Probabilities



Example 2: Acreage acquisition

- A company with 100 acres leased wants to drill a well on 160-acre prospect area
- We can join unit by leasing remaining 60 acres in unit
  - Evaluation assumes we acquire acreage
- Gross well cost (with equipment) = $110M
- Gross dry hole cost = $80M

Example 2: Payoff Table Application

- We have identified 3 options and determined NPV's for several outcomes
  - **Participate in drilling with 37.5% non-operating WI** (60/160x100 = 37.5%)
  - **Farm out acreage and retain 1/8-th of 7/8-th's royalty interest** on 60 acres
  - Be **carried with back-in privilege** (37.5% WI) after investing parties have recovered 150% of investment

Example 2: Payoff Table Application

Outcomes	Probability	Net Present Value, M$		
		Drill with 37.5% WI	**Farm out Retain ORI**	**37.5% Back-in**
Dry hole	0.25	-30	0	0
20 MSTB	0.30	4.357	8.733	0.750
35 MSTB	0.25	45.448	14.646	34.142
50 MSTB	0.15	87.411	20.693	73.712
65 MSTB	0.05	125.863	26.401	111.141

Example 2: Payoff Table Application

Answer questions

- Should we lease adjacent land (mineral rights)?
  - If so, what maximum amount should we pay?
- If we lease adjacent land, which option will be most valuable to us?

Example 2: Payoff Table Application

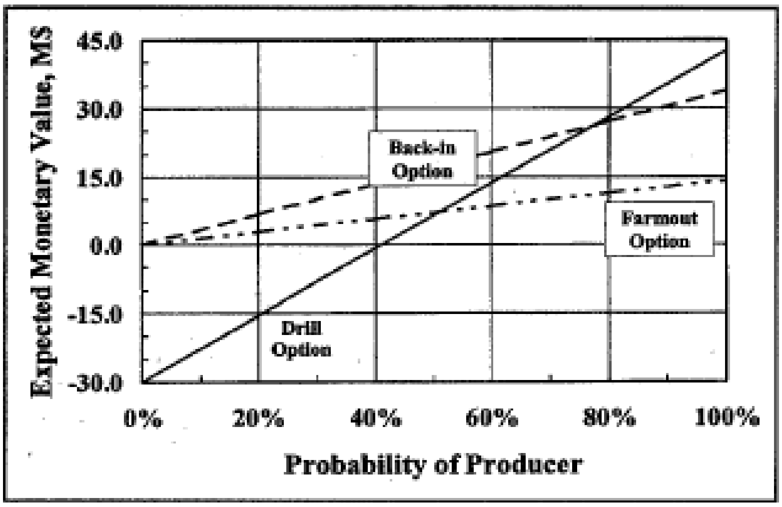
Out-come State	Proba-bility	Drill with 37.5%		Farm out with ORI		Back in with 37.5%	
		NPV	EMV	NPV	EMV	NPV	EMV
Dry hole	0.25	-30.000	-7.500	0	0	0	0
20 MSTB	0.30	4.357	1.307	8.733	2.620	0.750	0.225
35 MSTB	0.25	45.448	11.362	14.646	3.662	34.142	8.536
50 MSTB	0.15	87.411	13.112	20.693	3.104	73.712	11.057
65 MSTB	0.05	125.863	6.293	26.401	1.320	111.141	5.557
EMV, M$			24.574		10.706		**25.375**
Standard deviation, M$			45.622		7.809		32.869

Example 2: Payoff Table Application

- **Back-in** has largest EMV and is best option
- Maximum value of additional acreage is $25,375/60 = $423 per acre
  - If acreage acquired for exactly $25,375, rate of return will be 10% (discount rate used to determine NPV's)
  - Rate of return increases as we pay less to lease land

Example 2: Sensitivity Analysis on Probabilities

- For the example with **three alternatives** (drill, farm out, back-in), graphical method easier to implement



Example 2: Expected Opportunity Loss

- Definition: **EOL** is difference between ***actual*** profit or loss and profit or loss that would have ***resulted if decision maker had had perfect information*** at time decision made
- Example: choose to drill well, turns out to be dry hole, lose $30M
  - Farm-out would have had zero loss
  - EOL = $30M – 0 = $30M

Expected Opportunity Loss

- **EOL *minimization*** rule can be used in place of EMV *maximization* rule as basis for decision making
- Result same with either rule
- EMV easier to work with in complex situations

Example 2 – Expected Opportunity Loss

Analyze data in following table using EOL criterion (for drill, farm-out, back-in alternatives)

Outcomes	Probability	Net Present Value, M$		
		Drill with 37.5% WI	**Farm out Retain ORI**	**37.5% Back-in**
Dry hole	0.25	-30	**0**	**0**
20 MSTB	0.30	4.357	**8.733**	0.750
35 MSTB	0.25	**45.448**	14.646	34.142
50 MSTB	0.15	**87.411**	20.693	73.712
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Example 2 – Expected Opportunity Loss

Construct opportunity loss table

- Identify maximum value entry in each row in previous table
  - Subtract each entry in same row from maximum value
- Compute expected values by multiplying probabilities of outcomes by conditional opportunity losses
- Results in following table

Example 2 – Expected Opportunity Loss

Outcome State	Proba-bility	Drill with 37.5%		Farm out with ORI		Back-in with 37.5%	
		OL, M$	EOL, M$	OL, M$	EOL, M$	OL, M$	EOL, M$
Dry hole	0.25	30	**7.5**	0	**0**	0	**0**
20 MSTB	0.30	4.376	**1.312**	0	**0**	7.983	**2.395**
35 MSTB	0.25	0	**0**	30.802	**7.701**	11.301	**2.825**
50 MSTB	0.15	0	**0**	66.718	**10.008**	13.699	**2.055**
65 MSTB	0.05	0	**0**	99.462	**4.973**	14.722	**0.736**
	1.00		**8.812**		**22.682**		**8.011**

Example – Expected Opportunity Loss

- Best choice is **back-in alternative**, which has **minimum EOL**
  - Same decision as with maximum EMV criterion

Summary of Decision Criteria

- Choose alternative with **largest EMV** when profit is payoff variable and alternatives are mutually exclusive
- Choose alternative with **smallest EOL** when cost is payoff variable and alternatives are mutually exclusive