Economic Risk and Decision Analysis
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By
Dr. Thitisak Boonpramote

Department of Mining and Petroleum Engineering, Chulalongkorn University

Conditional Probability

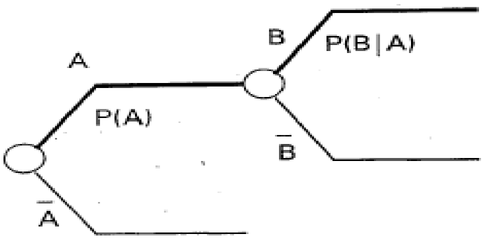
CONDITIONAL PROBABILITY

The probability that an event will occur given that other events have already occurred.

For example, the probability of event B given that event A has occurred is written

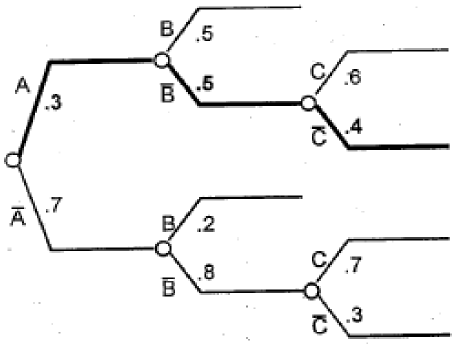
$P(B|A)$.

Read "the probability of B given A."



The Bayes' Rule is used often as a method to update probabilities. It provides a way to upgrade prior probability assessments based upon new information.

Joint Probability vs. Conditional Probability



The probabilities shown at nodes B and C are **conditional probabilities**, dependent on the outcomes of antecedent nodes.

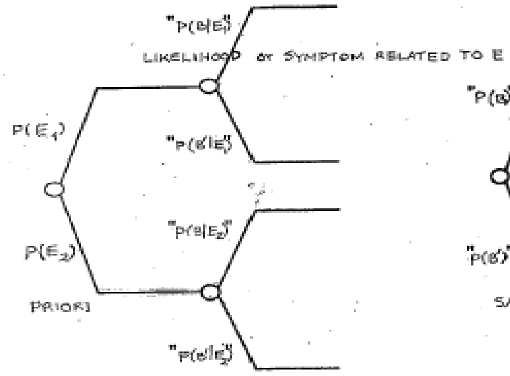
Each path through a decision or probability tree is a unique joint event.

For example, "the probability of A and not B and not C."

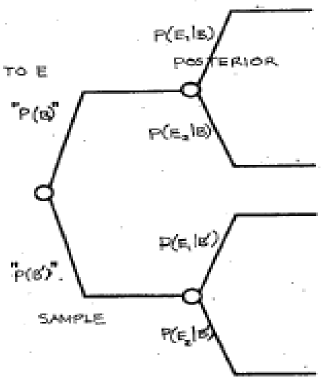
$$P(A\ \bar{B}\ \bar{C}) = P(A)\ P(\bar{B}|A)\ P(\bar{C}|A\ \bar{B}) = (.3)(.5)(.4)$$

Tree Inversion Method

Step 1. Judgments about the Quality of Information



Step 3. Inverted Tree

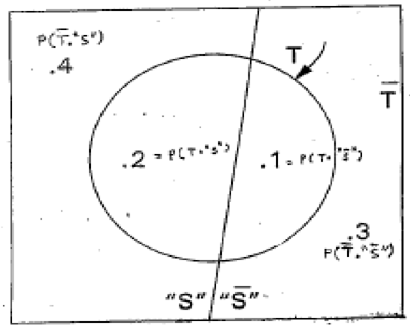


Step 2. Joint Probability Table

	" B "	" B′ "	Marginal for E
E_1	$P(E_1).P(B\|E_1)$	$P(E_1).P(B'\|E_1)$	$\Sigma\ P(E_1 \& B) + P(E_1 \& B') = P(E_1)$
E_2	$P(E_2)P(B\|E_2)$	$P(E_2).P(B'\|E_2)$	$\Sigma\ P(E_2 \& B) + P(E_2 \& B') = P(E_2)$
Marginal for B	$\Sigma\ P(E_1 \& B) + P(E_2 \& B) = P(B)$	$\Sigma\ P(E_1 \& B') + P(E_2 \& B') = P(B')$	

Joint Probability using Venn Diagram

English statistician J. Venn (1834-88) developed the famous diagram form to illustrate the possible outcomes from an experiment.



T = trap exists P(T) = 0.3

"S" = seismic shows an anomaly P("S") = 0.6

- The outer border encompasses the entire sample space.
- The areas should be drawn approximately proportional to probabilities.
- An **event** is a subset of the sample space. Remember that the probabilities of all partitions (events) totals 1.

Joint Probability Table

The Venn diagram provides a visualization of the possible events. Below is a **joint probability table** that provides the same data but in a form which is more difficult to imagine:

	T	T̄	
"S"	.2	.4	.6
"S̄"	.1	.3	.4
	.3	.7	1.0

The probabilities in the bottom and right margins are called, accordingly, **marginal probabilities**. The probabilities inside the table are **joint probabilities**.

Nomenclature: "·" means "and" (INTERSECTION) "+" means "or" (UNION)

$P(\bar{T}\cdot\text{"S"}) = .4$

$P(\text{"S"}) = P(\text{"S"}\cdot T + \text{"S"}\cdot\bar{T}) = .2 + .4 = .6$

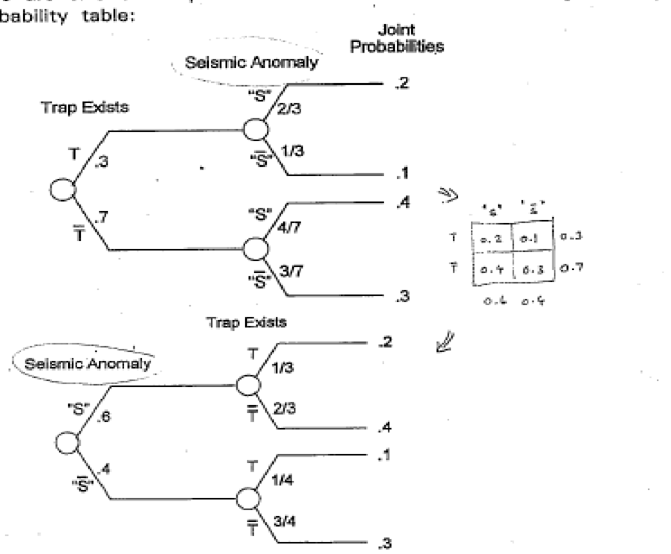
$P(T\,|\,\text{"S"}) = \dfrac{.2}{.2 + .4} = \dfrac{1}{3}$

What is $P(\bar{T}\cdot\text{"}\bar{S}\text{"})$? = 0.3

What is $P(\overline{T\cdot\text{"S"}})$? = 1-0.2 = 0.8

Probability Tree

Here are two tree representations of the same Venn diagram and probability table:



Baysian Analysis Example 1: Gravity Survey

This example shows how new information can be used to update prior probability assessments. Let's assume for a given prospect, that the geologist has assessed a .10 probability of testwell success (P_S) with the current knowledge. We want to evaluate whether or not to run a gravity survey. For simplicity, let's characterize the results of the survey as being either:

"A" = Anomaly seen	Symptom	D = Discovery if drilled	State of nature
"$\bar{A}$" = no anomaly seen		$\bar{D}$ = no discovery if drilled	

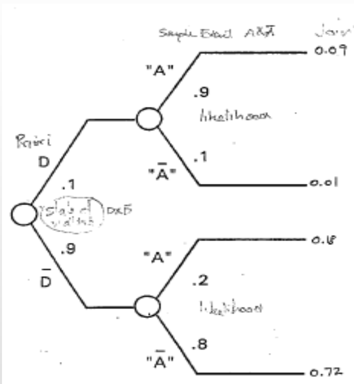
Our geophysicist judges:

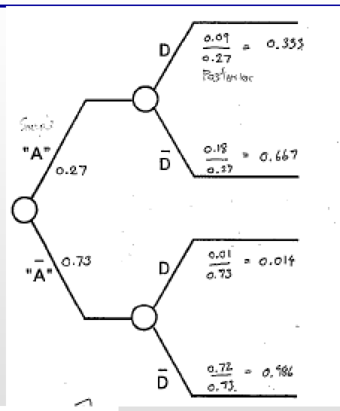
- There is a .90 chance of seeing an anomaly if a reservoir exists.
- There is a .20 chance of seeing a false anomaly if a reservoir does not exist.

Likelihood

In the figure, below, the left probability tree illustrates our current assessments. However, in order for the gravimetric data to be useful in our decision making, we need to "invert" the tree into the form on the right. The tree on the right represents part of a decision model to evaluate whether to invest in the additional information. There will be Drill decision nodes between the chance events. Bayes' theorem allows us to revise the original P_S judgment based upon the new information.

Gravity Survey: Solution





	D	D̄	
"A"	0.09	0.18	0.27
"Ā"	0.01	0.72	0.73
Marginal for	0.1	0.9	1

Revising Chance of Success

Suppose you have an exploration concession in which 15 anomalies have been found. The anomalies are similar in size and other characteristics. Your company's geologists and geophysicists have conceptualized three geologic scenarios and assigned probabilities:

e_1 = 2 productive reservoirs, 13 dry $P(e_1)$ = .58
e_2 = 4 productive reservoirs, 11 dry $P(e_2)$ = .27
e_3 = 6 productive reservoirs, 9 dry $P(e_3)$ = .15

You drill one of the prospects at random and it is dry. Revise the probabilities for the three geologic scenarios.

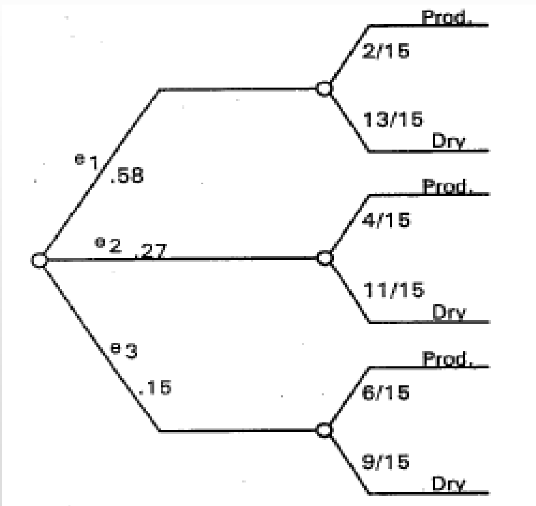
Develop:

1. A probability tree reflecting the judgments as given.
   What was the Probability of Success (P_s) for the first testwell?
2. A probability table for the two events:
   Well 1 outcome x Geologic Scenario
3. The inverted probability tree

Bonus question:

4. Expand the analysis to calculate P_s for testwell #2.

Revising Chance of Success: Probability Tree

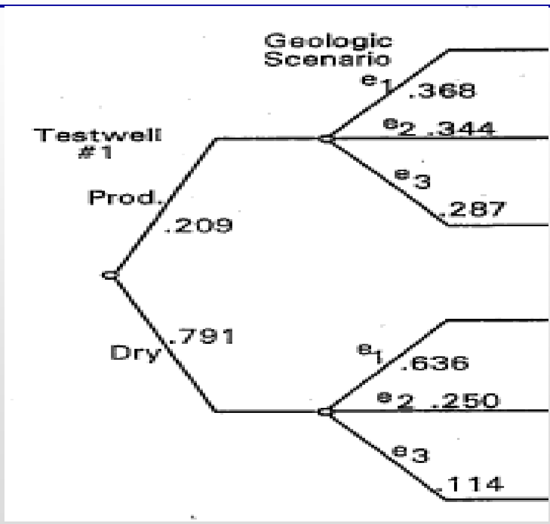


$P_s(\text{testwell } 1) = .58(2/15) + .27(4/15) + .15(6/15) = .209$

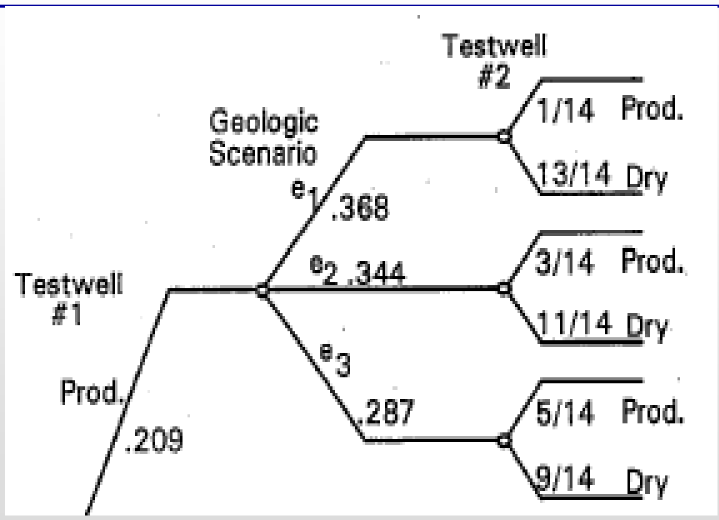
Revising Chance of Success: Joint Probability Table

	Productive	Dry	
e_1	.077	.503	.58
e_2	.072	.198	.27
e_3	.060	.090	.15
	.209	.791	

Revising Chance of Success: Flip the Tree

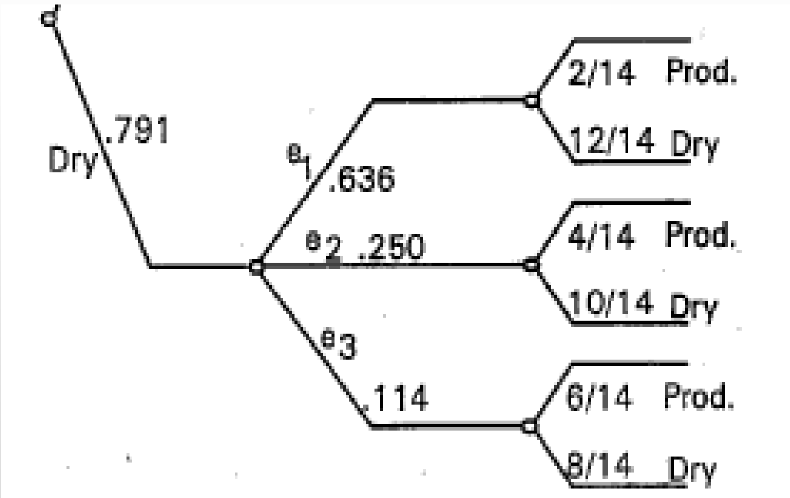


Revising Chance of Success from 2nd Well



P(Well #2 success | Well #1 Success) = .368(1/14) + .344(3/14) + .287(5/14) = .203

Revising Chance of Success from 2nd Well



P(Well #2 success | Well #1 Failure) =
.636(2/14) + .250(4/14) + .114(6/14) = .211

Decision Tree from Probability Tree

Site 1

If Anticlinal Structure Exists (p = 0.60)

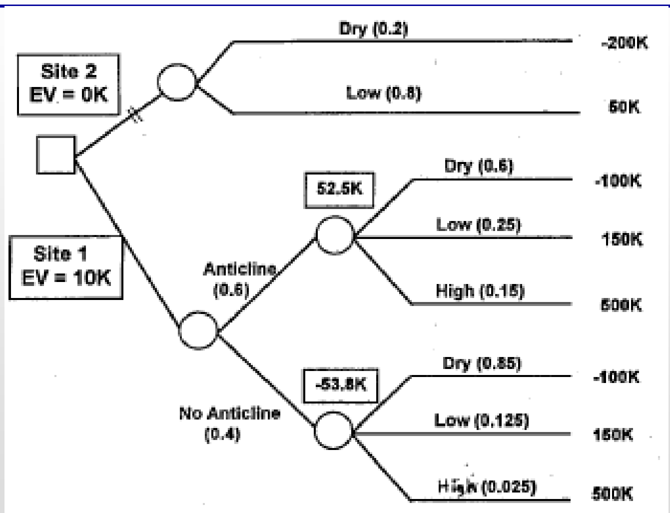
Outcome	P(Outcome|Anticline)	Payoff
Dry	0.600	-100K
Low	0.250	150K
High	0.150	500K

If No Anticlinal Structure Exists (p = 0.40)

Outcome	P(Outcome|No Anticline)	Payoff
Dry	0.850	-100K
Low	0.125	150K
High	0.025	500K

We want to compare the drilling location above to Site 2 which has a 20% chance of being a dry hole (payoff of -200K) and an 80% chance of being a "low" producer with a payoff of 50K.

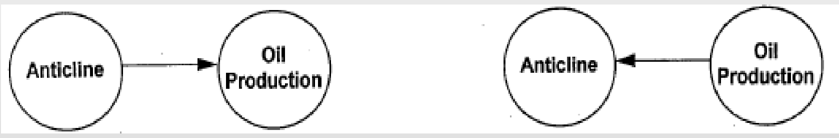
Decision Tree: Solution



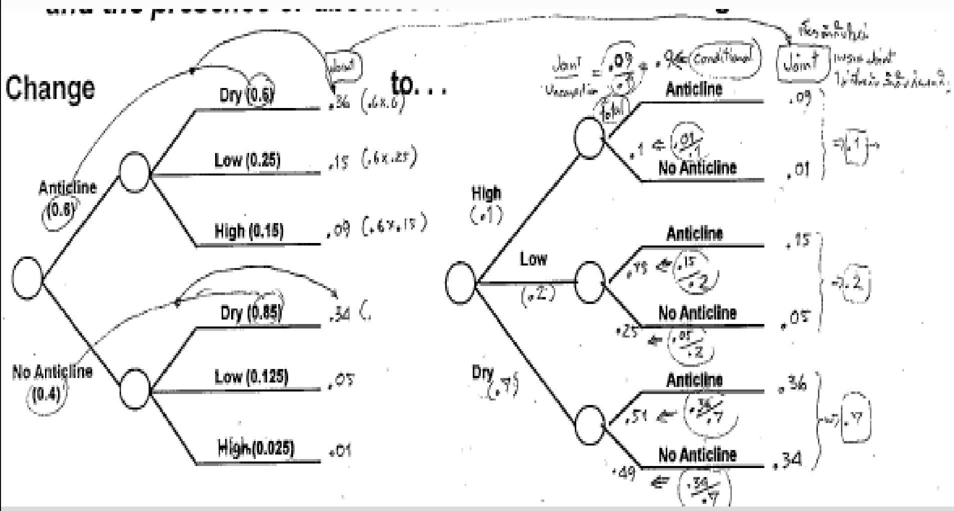
Probability Tree

Now suppose that the company selects Site 1, and the well is a high producer. In light of this evidence, does it seem more likely that a anticlinal structure exists? Can we figure out *P(Anticline|High)*?

Finding P(Anticline|High) is a matter of "flipping the tree" so that the chance node for the amount of oil is now on the left and the presence or absence of anticline is on the right.



Flip the Tree: Solution



Bayes Theorem as a short cut

$$P(Anticline|\,High) = \frac{P(High|\,Anticline)P(Anticline)}{P(High)}$$

$$= \frac{P(High|\,Anticline)P(Anticline)}{P(High|\,Anticline)P(Anticline) + P(High|\,No\ Anticline)P(No\ Anticline)}$$

$$= \frac{(0.15)\,(0.60)}{(0.15)\,(0.60)\ +\ (0.025)\,(0.40)}$$

$$=\ 0.90$$

Probabilities that have the same conditions must add to 1, and so P(No Anticline | High) must be equal to 0.10.

New information about well (high, low, or dry) allows us to update a priori probabilities.