Economic Risk and Decision Analysis for Oil and Gas Industry

CE81.9008

School of Engineering and Technology
Asian Institute of Technology

January Semester

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EMV Limitations

Limitations of Expected Value

- EMV fails to provide guidance on limiting the **downside exposure**.
- The firm is not **risk neutral**.
- The firm **does not possess an unlimited pool** of exploration capital.

EV - Failure to Provide Guidance on Limiting Downside Exposure

- **Consider the following two drilling ventures:**

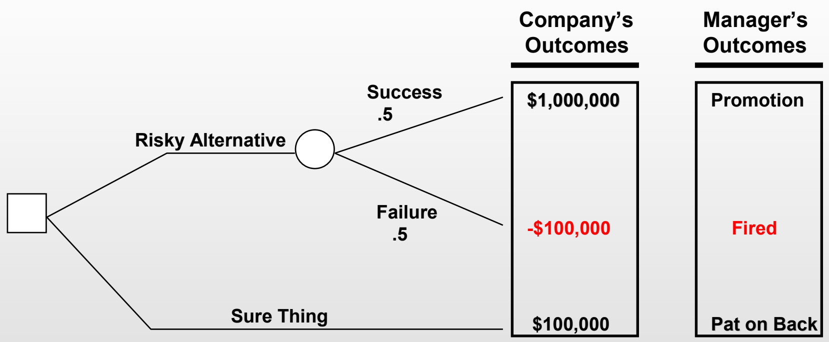
- Expected Value of A = Expected Value of B?

EMV does not consider the magnitude of money exposed to the chance of loss.

EMV - The Firm is not Risk Neutral

- The individuals who make decision for the firms may consider the **downside exposure** rather than the **expected value** on making their decision.
- This means they are not **risk neutral** but **risk averse**.

Incentives for Risky Decisions



- Middle management may be conservative (risk averse) because they use **their own outcomes** rather than the **company's outcomes** as the basis for decisions.

EMV - Unlimited Pool of Capital?

- Firm has an Exploration Budget of $20 Million and the Opportunity to Invest in the Following Four Drilling Prospects:

Prospect	***Outcome***	***Value***	***Probability***	***EMV***
1	Success	$40mm	0.20	
	Dry Hole	**-$5mm**	0.80	$4mm
2	Success	$15mm	0.10	
	Dry Hole	**-$5mm**	0.90	**-$3mm**
3	Success	$20mm	**0.50**	
	Dry Hole	**-$8mm**	0.50	$6mm
4	Success	**$80mm**	0.25	
	Dry Hole	**-$20mm**	0.75	$5mm

Unlimited Pool of Capital?

- EMV Decision Rule: Invest in Prospects 1, 3 & 4.

Total Required Capital:	$33 million
Total Exploration Budget:	$20 million
Deficiency	$13 million

How do we choose among (rank) Prospects with Positive Net Present Values?

Unsustainable Risk

- Person with $100M net worth might be able and willing to sustain $10M loss in quest for profit
- Person with $10M net worth probably can't and won't afford $10M loss
- Corporations need **formal policies** to deal with similar and other **risk management issues**

Example Project Alternatives

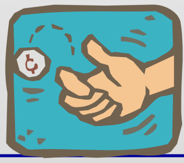
Project A		Project B	
Probability	**NPV (M$)**	**Probability**	**NPV (M$)**
0.80	80	0.80	30
0.20	**-40**	0.20	**-5**
EMV	**56**		23
Std. Dev.	48		**14**

Example Project Alternatives

- **Project A** has much greater NPV, EMV, would be choice if mutually exclusive using **NPV criterion**
- Project B has much smaller loss if project does not go well
- **Risk-averse decision maker** would **choose B**, so EMV not only criterion
- We need to **quantify attitude toward risk**

The St. Petersburg Paradox

- In the XVII century, **Nicolaus Bernoulli** formulated the following problem:
- Peter tosses a coin repeatedly until it comes up heads. Peter pays Paul:
  - One ducat if head comes up on the first toss
  - Two ducats if head comes up on the second toss
  - Four ducats if head comes up on the third toss
  - **Payment doubles with each toss** until head comes up



PS: One ducat ≈ $40

The St. Petersburg Paradox

Prob	Result	Payoff
0.50	H	1
0.50^2	TH	2
0.50^3	TTH	4
0.50^4	TTTH	8
0.50^5	TTTTH	16
………	………	……
0.50^n	(n-1)T H	2^{n-1}

- Note that Paul will earn some money from this game.
- What is the value of this game to Paul?
- What is the maximum he should pay to play it?
- What is the Expected Value of this game?

Expected Value of the Game

- The Expected Value of the game is:

$$E(x) = \sum_{n=1}^{\infty} 0{,}50^n \,.\, 2^{n-1} = \infty$$

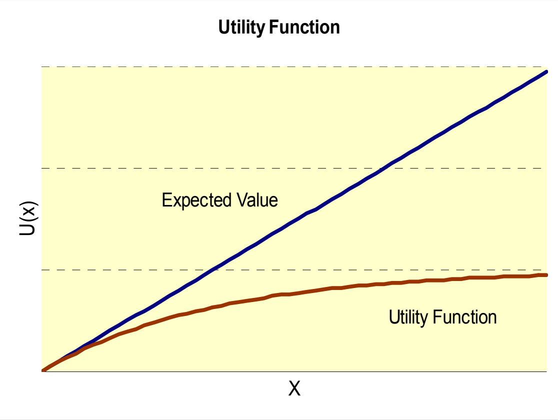
- According to the Expected Value criteria, the value of the game is infinite
- Paul should be willing to pay any price to be able to play it.
- Does this seem reasonable? Would you pay $1 million to play this game?

Bernoulli's Solution

- **Daniel Bernoulli**, a math professor at St. Petersburg and brother of Nicolaus, proposed a solution to this problem based on two observations:
- People are interested in **the utility of the payoffs**, not in their **monetary value**.
- The incremental utility derived from additional monetary return decreases as monetary return increases which follow the law of **diminishing marginal utility**.
    - A **$10,000 raise** has less utility for someone that is making $200,000 than if that same person is making $40,000.
    - The **utility of $1 billion** is not 1 billion times greater than the **utility of $1**.

Bernoulli's Solution

- **Expected Value** grows linearly with value
- **Utility functions** are non-linear
- Using a **Utility Function**, Bernoulli estimated the value of the game at 2.0 Ducats.



Risk Perception and Risk Taking Behavior

Theories on Risk Perception

- People perceive risks differently
- **But** there are some general principles we all use to evaluate risks
- By understanding these principles we can help people perceive risks more objectively

Theories on Risk Perception

There are 3 principles that govern the perception of risk

1. Feeling in control
2. Size of the possible harm
3. Familiarity with the risk

1. Feeling in Control

- Voluntary vs. involuntary risks
- **Involuntary risks** are situations where we believe we **have little control** over the situation **are perceived as having greater risk**
- **Voluntary risks** are situations we believe we **have some control** over are perceived as less risky
  - Example: Car Trip v. Plane Trip

2. The Size of the Possible Harm

- Risks that involve **greater possible harm** are **perceived as greater than** those involving less harm
- Even if the less harmful events are more likely
  - Example: Tornado v. Kitchen Fire

3. Familiarity with the Risk

- Risks that are **less familiar** are **perceived as being greater than** more familiar risks
  - Example: Nuclear plant accident vs. Food poisoning

Strategies for Improving Risk Perception

1. Examine the reality of “control”
2. Emphasize the likelihood of events
3. Educate to overcome familiarity

Theories on Risk Taking Behavior

- Risk Preference Theory
- Risk Homeostasis Theory

Risk Preference Theory

- People have a **natural propensity towards risk**
  - Risk Seeking
  - Risk Neutral
  - Risk Averse
- A personality trait
- People always **behave** according to their preference

Example: Let's Make a Deal

- **Let's Make a Deal**
  - Door A: 10% $1000 90% -$83.40
  - Door B: 50% $50 50% $0
  - Door C: 100% $25
- *Which door would you choose?*

Does Risk Preference Explain Real Behavior?

- Don’t you think at different times in your life you may make different choices?
- If this is a personality trait it should be fairly stable in spite of these kinds of changes
- If I changed the dollar amounts of the previous game would some of you change doors?

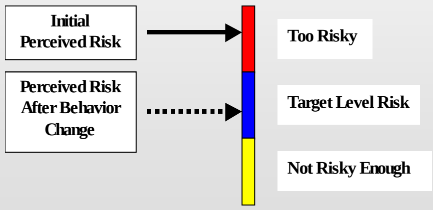
  Door D: 1%=$5000 or 99%= -$25

Approaches Under Risk Preference Theory

- If you still believe in risk preference theory then don’t hire risk takers,
- **or** at least don’t put them in positions where great harm would be the consequence of their behavior

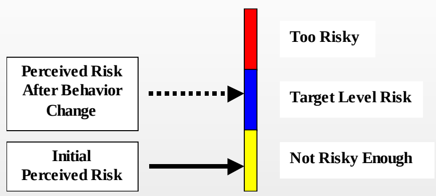
Risk Homeostasis Theory

- **Gerald Wilde** originally developed for studying driver behavior, but expanded to workplace behavior
- Each individual has a **target level of risk** which they are comfortable
- When we encounter situations where the **risk is perceived** to be **greater than the target level**, we adjust our behavior to **lower the risk**



Risk Homeostasis Theory (Continued)

- When we are in situations where the **risk is perceived** as **lower than our target level** we are change our behavior to **increase the risk**
- As long as there is some other benefit to increasing the risk
  - i.e. save time, save money, look good, etc.



Approaches Under Risk Homeostasis

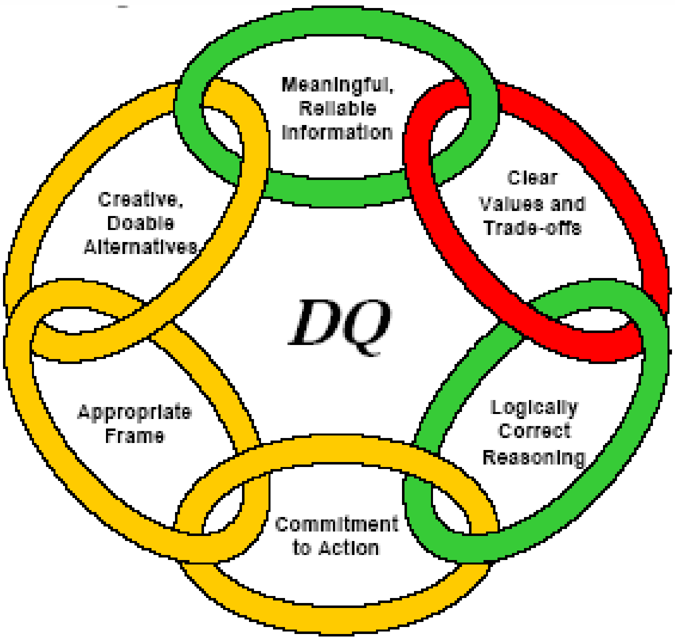
- Any engineering changes, administrative changes, or personal protective equipment will be **perceived as a decreased risk**
- This theory says that they will just adjust their behavior accordingly and the **actual risk will remain the same**
  - Example: Airbags and Speeding
- Approaches Under Risk Homeostasis:
  - Disguise the safety changes, so the user won't realize they are present
  - Educate people so they realize the true risk and consequences
    - Lower their target level of risk

Expected Utility Theory

Expected Utility Theory

- Dominant approach to the **theory of decision making** in both economics and finance.
- Establishes the process of modeling an **individual or firm's risk propensity**.
- Enables decision maker to **incorporate risk attitudes** into the decision process.
- Provides the firm a technique for determining the **appropriate level of diversification**.

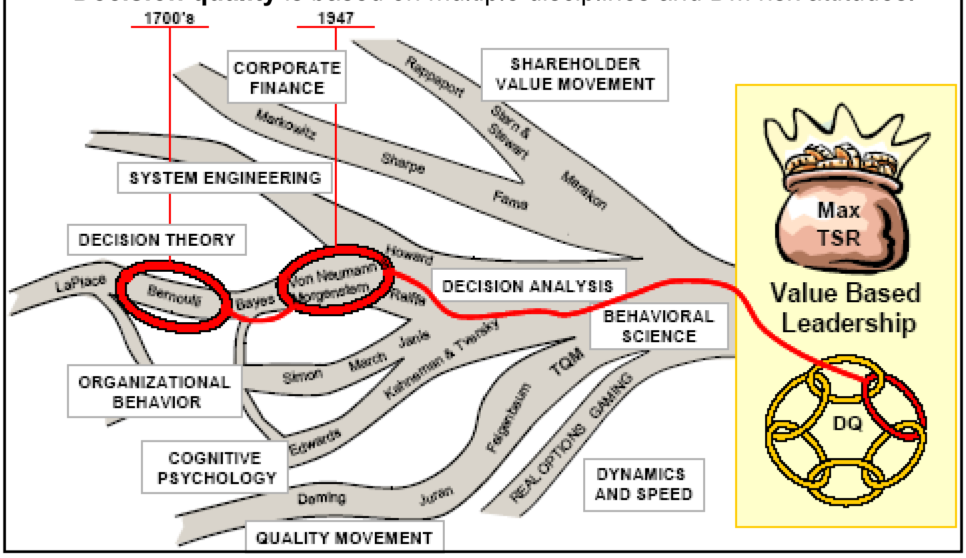
Decision Quality



Risk Tolerance
Risk Preference
Risk Aversion
Risk Attitude
Risk Premium
Risk Discounting

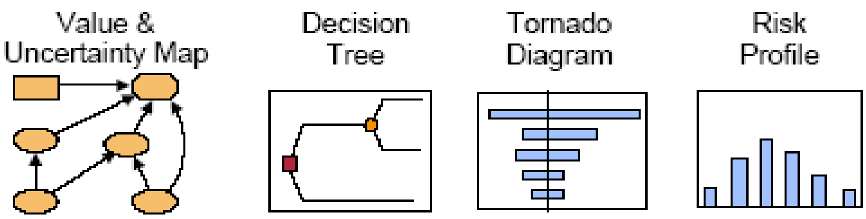
Improving Decision Quality

- **Decision quality** is based on multiple disciplines and DM risk attitudes.

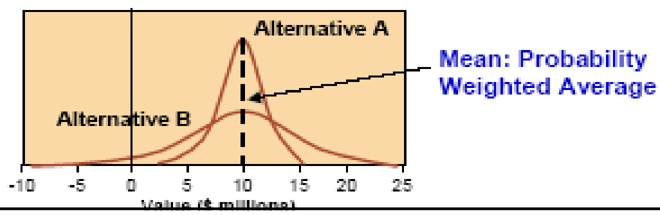


Embracing Uncertainty requires two parts:

1. Characterizing the value and risk of a specific course of action:

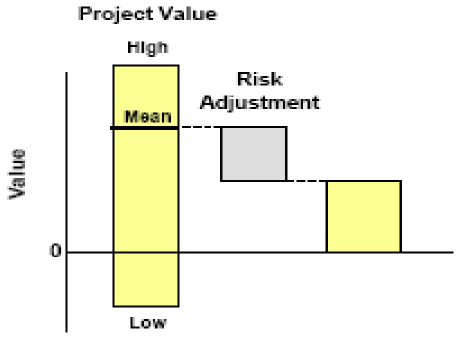


2. Applying the decision makers' risk preference to choose the best course of action



Risk aversion can be incorporated into the valuation procedure using a risk adjustment, which yields a project's certain equivalent.

Adjusting Project Value for Risk



The risk adjustment* depends on the corporation's risk tolerance and the risk of the project.

* Specifically, the risk adjustment is based on a utility function. A common and practically useful utility function is u(x) = -exp(-x/risk tolerance).

Attitudes Toward Risk

- Most researchers believe that if certain **basic behavioral assumptions** hold, people are **expected utility maximizers** - that is, they choose the alternative with the largest expected utility.
- An individual's **utility function** is a mathematical function that ***transforms*** **monetary values** - payoffs and costs - into **utility values**.
- Essentially an **individual's utility function** specifies the individual's preferences for various monetary payoffs and costs and, in doing so, it automatically encodes the individual's **attitudes toward risk**.

Expected Utility Concepts: Why insurance?

- *Insurance premium*---paid to insurer
- In return, insurer promises payment to individual if adverse event happens
- Examples: Health, car, property, farm crops,

Why do individuals value insurance?

- Individuals value because of *Diminishing marginal utility*
- They choose **2 years of smooth income** over 1 year of high consumption and 1 year of starving
- --because excessive consumption does not raise utility as much as starvation lowers it.
- They prefer to **smooth out consumption**

Why individuals value insurance?

- When outcomes are **uncertain**, individuals wish to smooth their consumption over possible *states of the world*
- *Examples:*
  - State1: get hit by a car
  - State2: not getting hit
- Goal is to **make choice today** that determines **consumption in future** for each of these states

Insurance, contd.

- Consumers smooth by using **some of today's income** to insure against **adverse outcome tomorrow**.
- Basic insurance theory suggests that individuals will demand full insurance to smooth their consumption across states of the world.
- Same consumption possible whether accident occurs or not

Expected Utility Model

$$EU = (1-p)\,U(C_0) + pU(C_1)$$

Where

- p stands for the probability of an adverse event
- C_0 and C_1 stand for consumption in the good and bad states of the world

Analyzing an individual's demand for insurance

- Assume, a **1% chance for and accident** with **$30,000 of damages**
- Sam can insure **some, none,** or **all** of these medical expenses
- Policy cost: **m cents per $1 of coverage**
  - A policy pays **$b for an accident**
  - His **premium is $mb**
- **Full insurance: m x $30,000**
  - State 0: $mb poorer
  - State 1: $b-$mb richer than if he doesn't buy insurance

Expected payoff

- Sam's desire to buy depends on price of insurance
- An **actuarially fair premium** sets the price charged equal to the expected payout
  - \$30,000 x .01 = \$300 (*act. fair prem.*)

Expected Utility Concepts

- Decision to buy insurance also affected by **risk preference**
- Assume a **utility function** U= √C.
  - C = 30,000
- Without insurance:
  .99√30,000+.01 √0 =171.5
- With actuarially fair insurance:
  .99√29,700 + .01√29,700 = 172.3
- Utility is higher with insurance
- Partial insurance is lower utility

Expected Utility Concepts

The expected utility model

If Sam …	And Sam is …	Consumption	Utility √C	Expected utility
Doesn't buy insurance	Not hit by a car (p=99%)	$30,000	173.2	0.99x173.2 + 0.01x0 = 171.5
	Hit by a car (p=1%)	0	0	
Buys full insurance (for $300)	Not hit by a car (p=99%)	$29,700	172.3	0.99x172.3 + 0.01x172.3 = **172.3**
	Hit by a car (p=1%)	$29,700	172.3	
Buys partial insurance (for $150)	Not hit by a car (p=99%)	$29,850	172.8	0.99x172.8 + 0.01x121.8 = 172.2
	Hit by a car (p=1%)	$14,850	121.8	

Role of risk aversion

- *Risk aversion*: extent to which an individual is willing to bear risk
  - **Risk avers individuals** have a **rapidly diminishing marginal utility** of consumption
  - Individuals with any degree of risk aversion will buy insurance priced fairly.
  - Even if insurance is expensive, if premium is actuarially fair, individuals will want to insure against adverse events.
- Implication: The efficient market outcome is full insurance and thus full consumption smoothing

Risk attitudes

Example 1:

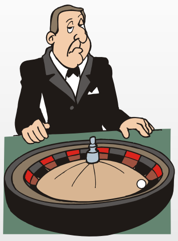
A: You get 10 000 baht for sure.

B: You participate in the following lottery:

20 000 baht p = 0.6

10 000 baht p = 0.3

- 5 000 baht p = 0.1

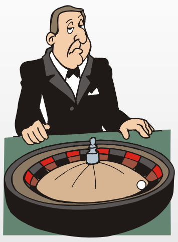


- The **expected monetary value (EMV)** in B is 14 500 baht
- But still many people choose A.
- People do not necessarily maximise expected monetary value.

Risk attitudes -- continued

Example 2:

A:	5 baht	p = 0.5
	- 5 baht	p = 0.5
B:	50,000 baht	p = 0.5
	- 50,000 baht	p = 0.5



- The expected monetary value is same in both cases.
- You **may participate** in a lottery similar to A when you play blackjack, but you **might not necessarily want to participate in B**. B entails more **risk**.

Risk attitudes -- continued

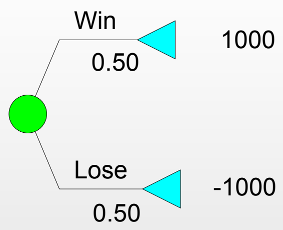
- One might be willing to pay a certain amount, for **not having to participate** in lottery B. Then one pays for *avoiding risk* (e.g. insurance or hedging).

Risk = An uncertain situation with possibility of loss.

- Now, the amount of risk does not depend only on the **probability** but the **amount of loss** as well.

Attitudes Toward Risk

- You are forced to play the following game
- It has as expected value of 0.
- Would you pay to avoid playing the game?



- Risk averse individuals will **pay to avoid taking the risk** of his game
- Most individuals are **risk averse**, which means intuitively that they are willing to **sacrifice some Expected Value** to **avoid risky gambles**.

Utility Curve and Axioms

Risk attitudes -- continued

The risk attitude of the DM:

- **Risk-averse: Risk Premium > 0 ⇔ concave utility function**
- **Risk-neutral: Risk Premium = 0 ⇔ linear utility function**
- **Risk-seeking: Risk Premium < 0 ⇔ convex utility function**

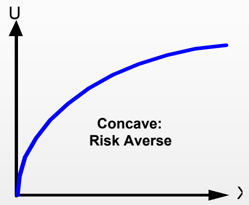
One ***avoids risk*** if one would rather take a **smaller** amount of money than the **EMV** than participating in the game.

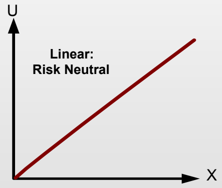
if one is willing to pay **exactly** the EMV for participating, one is ***risk-neutral***.

Otherwise, one is ***risk-seeking***

Risk Attitudes

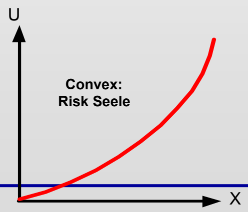
For a **risk averse** individual, utility functions are said to be **increasing and concave**. These individuals will pay less than the Expected Value of a gamble. (Ex: Purchasing insurance)



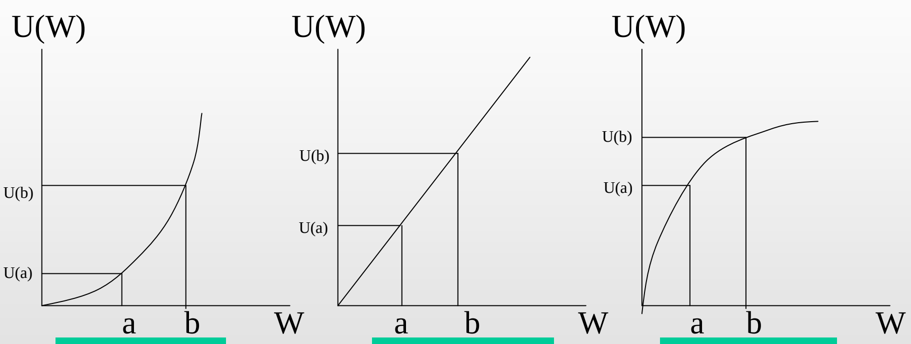


A **linear** utility function is simply the Expected Value and represents an individual that is **risk neutral**

A **risk seeker** will pay more than the EV of a gamble. This behavior is represented by a **convex** utility function. (Ex: Purchasing lottery tickets)

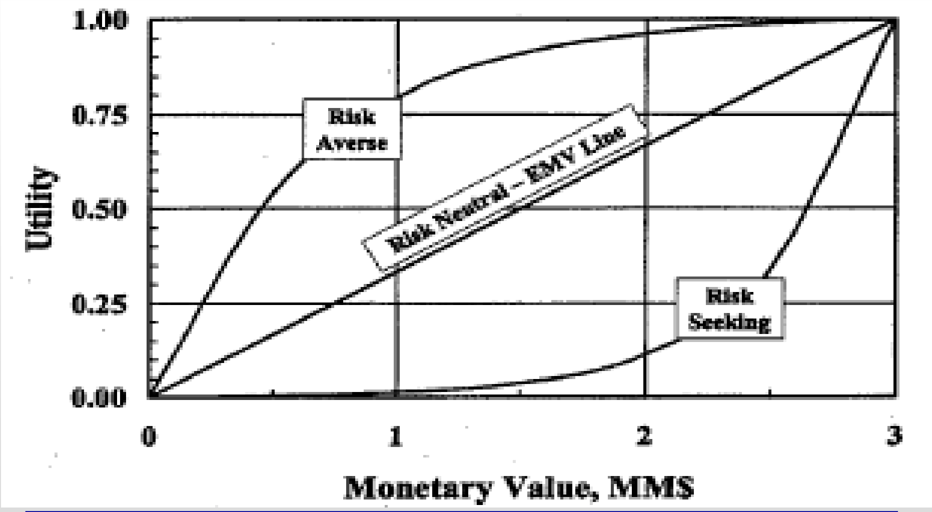


Theoretical Utility Curve



Risk-loving	Risk Neutral	Risk Averse
$U'(W) > 0$	$U'(W) > 0$	$U'(W) > 0$
$U''(W) > 0$	$U''(W) = 0$	$U''(W) < 0$

Theoretical Utility (Preference) Curve

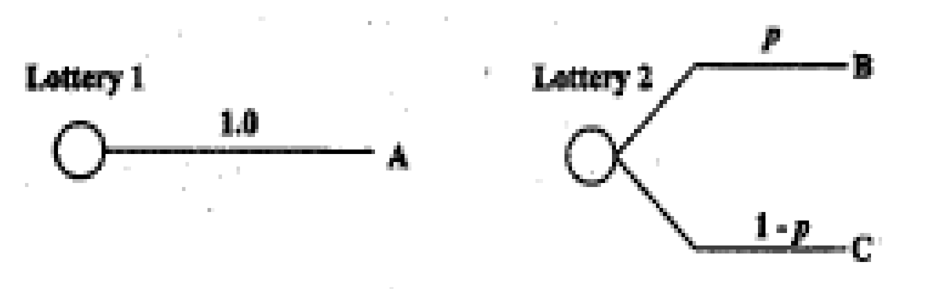


Axioms of Utility Theory

- Mathematical preference theory based on **certain assumptions (axioms)**
- If we accept these axioms as basis of **rational decision**, then our attitudes toward money can probably be described by **preference (utility) curve**
- **Transitivity**
  - If we prefer A to B and B to C, then we must prefer A to C
- **Complete Ordering**
  - We are able to order our preferences or indifference to any two alternatives
  - Given alternatives A and B, we either prefer A to B or B to A, or we are indifferent to choice of A or B

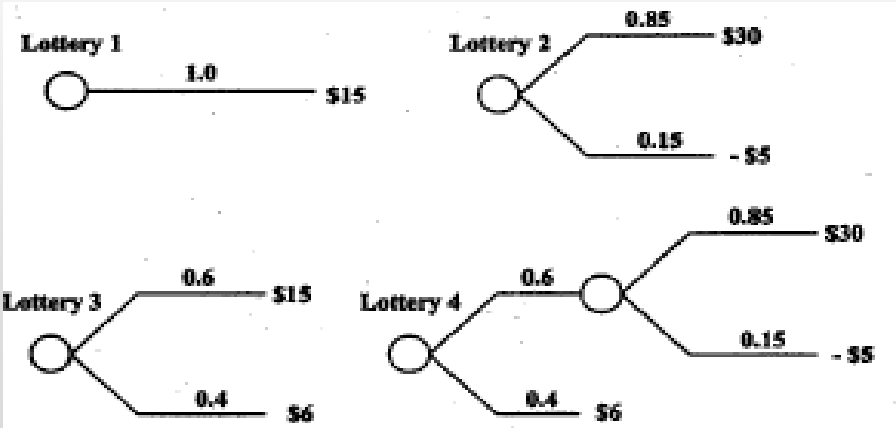
Axioms of Utility Theory

- **Continuity**
  - Given two "lotteries," 1 and 2, there is some probability *p* at which we are indifferent to the choice of lotteries 1 or 2
  - B preferable to A and A preferable to C



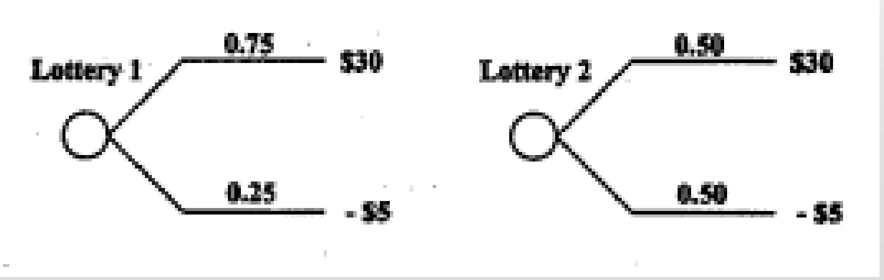
Axioms of Utility Theory

- **Substitution**
  - If we are indifferent to choice between lotteries 1 and 2, then if lottery 1 occurs in another lottery, we can substitute lottery 2 for it
  - We will be indifferent to choice of lotteries 3 or 4



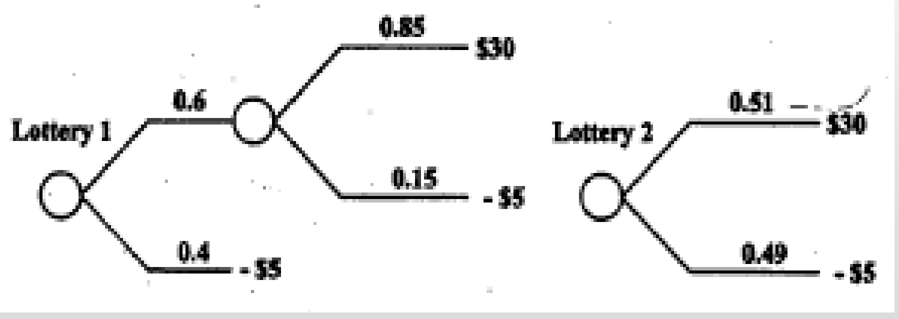
Axioms of Utility Theory

- **Unequal probability**
  - If we prefer reward B to reward C, then, if we are offered two lotteries with only outcomes B and C, we will prefer the lottery offering the highest probability of reward B
  - In example, we prefer lottery 1 to lottery 2



Axioms of Utility Theory

- **Compound Lottery**
  - Decision maker will be indifferent to compound lottery and simple lottery that have same outcomes with same probabilities



Axioms of Utility Theory

- **Invariance**
  - All that we need to determine decision maker's preference among uncertain events are **payoffs (consequences**) and **associated probabilities**
- **Finiteness**
  - No consequences are infinitely bad or infinitely good

Properties of Utility Curves

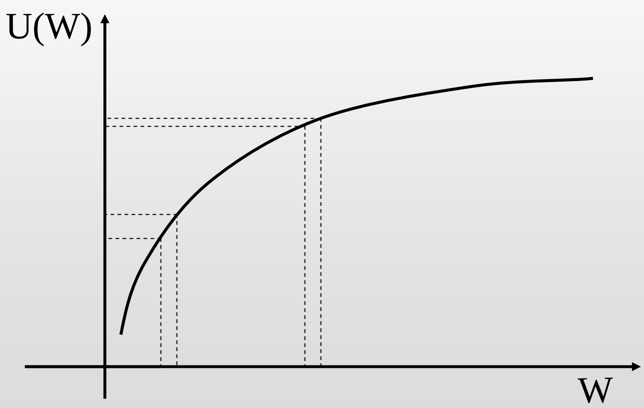
- Vertical scale **dimensionles**s, represents relative desirability of given amount of money – scale usually 0 to 1
- Horizontal axis represents **monetary values** (NPV, costs, incremental cash flows, etc.)

Properties of Utility Curves

- Curve increases monotonically (getting more always better)
- We can multiply preference values by probabilities of occurrence to arrive at expected preference value, EU, for decision alternative, similar to EMV calculation

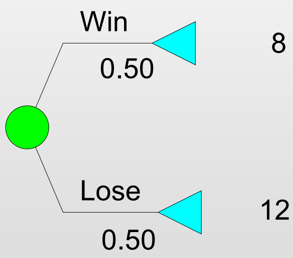
$$EU = \sum_{i=1}^{N} p_i U(x_i)$$

Risk-Averse Investors: Increasing and Concave Utility Function



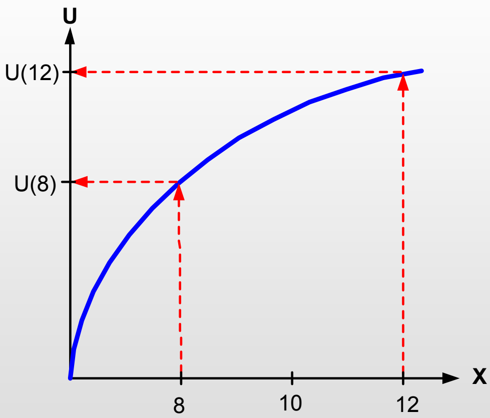
Using an Utility Curve

- Lottery X has payoffs of 8 and 12 with equal probabilities
- Expected Value of Lottery X is 10.



Using an Utility Curve

- Instead of using the **Expected Value of** X, we will determine the **Expected Value of U(X).**
- Using a utility function, we compute **utility of payoffs of X**.
- Utility of 8 is U(8)
- Utility of 12 is U(12)

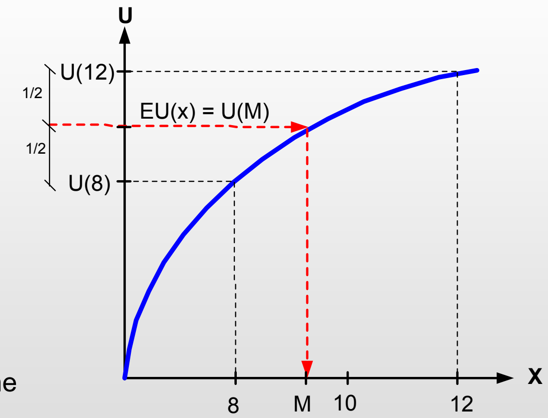


Using an Utility Curve

- The Expected Utility of the lottery is:

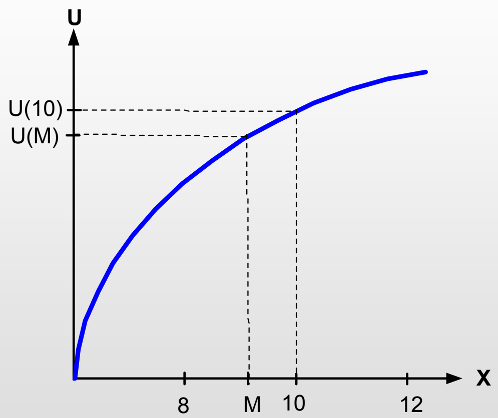
$$EU(x) = 0.50U(8) + 0.50U(12)$$

- We then determine the **value of M that corresponds to this utility of *EU(x)***
- Note that because of the concave shape of the utility function ➔M < 10



Using a Utility Curve

- U(10) is greater than U(M)
- **U(M)** is the **Expected Utility of the lottery X** = EU(x)
- This means that investor will **prefer to receive 10** than play the lottery.
- The lottery is not worth 10 to him, even though this is his expected value.



The Expected Utility of Risky Returns

Let $\mathbf{D_A}$ be the dollar return of asset A (a random variable), $\mathbf{D_A(i)}$ be the dollar return of asset A in state i (i = 1, … n) and

$\mathbf{p_i}$ be the probability of state i.

If you invest in A, your expected dollar wealth is

$$E[W_0+D_A] = [W_0+D_A(1)]\cdot p_1 + \ldots + [W_0+D_A(n)]\cdot p_n$$

Your **expected utility** is

$$E[U(W_0+D_A)] = U[W_0+D_A(1)]\cdot p_1 + \ldots + U[W_0+D_A(n)]\cdot p_n$$

Example I

Consider a simple prospect where all your **wealth** of **$100,000** is invested in a ***fair gamble***:

you will get **$150,000** with probability 0.5 or **$50,000** with probability 0.5.

(Note that this is called **a fair gamble** since **the expected profit is zero**)

$$E(\text{profit}) = (150{,}000-100{,}000)\cdot 0.5 + (50{,}000-100{,}000)\cdot 0.5 = 0$$

Example - continued

a. Calculate the **expected final wealth**. ($100,000)

b. Assuming that your **utility function** is **logarithmic (i.e. U(W) = ln(W)),** calculate your **utility of the final wealth** for each possible outcome. (11.9184, 10.8198)

c. Calculate the **expected utility of the final wealth** and compare it to the **utility of the initial wealth**. Will you enter the game? (f: 11.3691, i: 11.5129)

d. How much will you pay me for the right to enter this game? Or should I pay you? ($13,397.5)

Certainty Equivalent and Risk Premium

- **Certainty Equivalent (CE)** is a certain **amount of money**, that is **equally preferred** to a given simple game.

Example:

θ	$P(\theta)$	a_1	a_2	a_3
heads	0.5	550	700	400
tails	0.5	0	-100	100
CE		400	150	300

- **Risk Premium:** EMV decision maker is willing to give up (pay) to **avoid risky decision**

RP = EMV - CE

Certainty Equivalent and Risk Premium

The **certainty equivalent** (**CE**) determines the **maximum dollar price** an investor will pay for a **risky asset** with an uncertain dollar return D.

The expected utility of the investment in the risky asset is equal to that of the certainty equivalent. In our example:

$E[U(W_0+D)] = 0.5 \cdot U(\$50K) + 0.5 \cdot U(\$150K) = 11.3691$

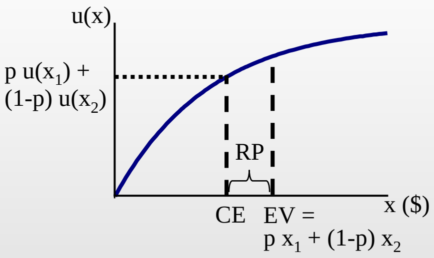
$11.3691 = U[CE]$

$CE = \$86{,}602.5$

That means that **you will not invest more than $86,602.5 in that game**, or that you will enter the game only if I will pay you a **risk premium**: $100,000 - $86,602.5 = **$13,397.5**

Certainty Equivalent and Risk Premium

The **Certainty Equivalent** and **Risk Premium (RP)** give enough information for analysing choices.



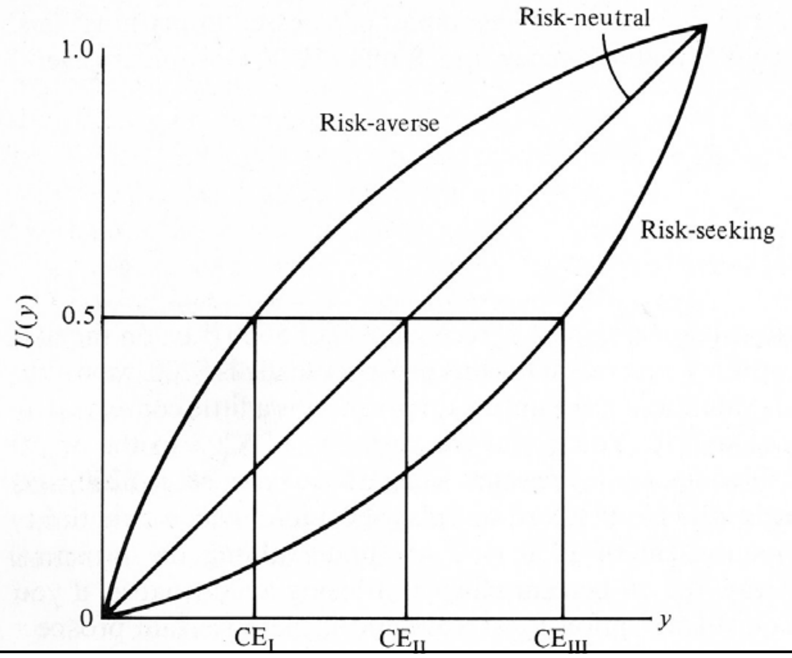
- In **multiple criteria utility models** utilities related to different objectives can be combined with for instance additive models.

Certainty Equivalent and Risk Premium

Relationship Between Risk Attitude and Certain Equivalent

Risk Adjustment	Relationship Between the Mean and the Certain Equivalent	Interpretation
Zero	Certain Equivalent = Mean	Risk Neutral
Positive	Certain Equivalent < Mean	Risk Averse
Negative	Certain Equivalent > Mean	Risk Preferring

Risk attitudes -- continued



Example: Certainty Equivalent and Risk Premium

- Lottery has two possible outcomes
  - Win $3,000 with probability of 0.5
  - Lose $200 with probability of 0.5
- EMV $1,400
- Owner of lottery ticket willing to sell it for $500, and no less
- Then CE = $500, RP = $900
- Owner willing to give up $900 in EMV to avoid risk of loss
- RP is **premium we will pay to avoid risk**

Step in Determining Risk Premium for Investment

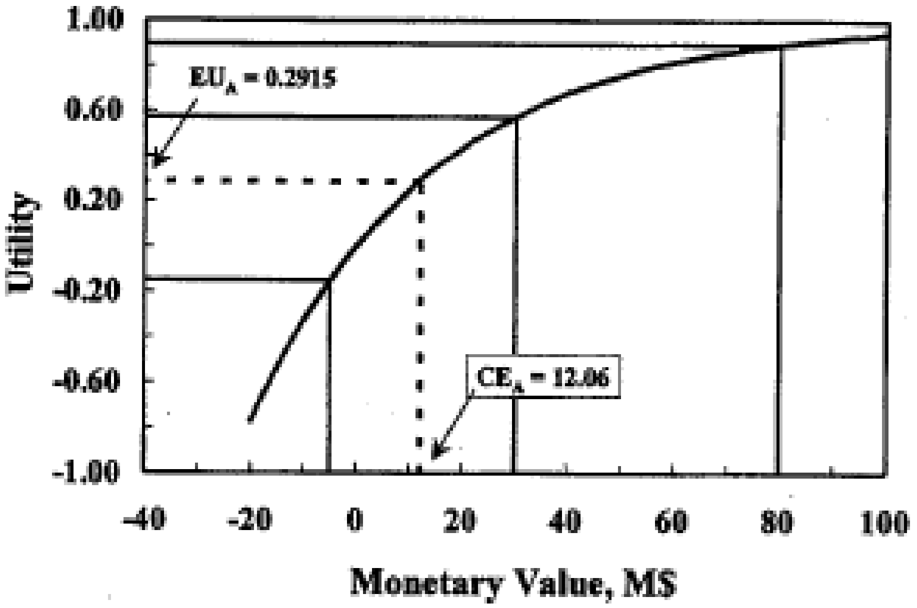
- Obtain **utility function** of decision maker (methodology discussed later)
- Find **EU** of investment
- Find **CE**, the certain amount with utility value equal to EU
- Calculate **EMV** of investment
- Subtract CE from EMV to **determine RP**

Example EU, CE and RP Calculation

For information in table, calculate EU, CE, and RP for each project, and select preferred project

Project A		Project B	
Probability	NPV (M$)	Probability	NPV (M$)
0.80	80	0.80	30
0.20	-40	0.20	-5
EMV	56		23
Std. Dev.	48		14

Example EU, CE and RP Calculation



Example EU, CE and RP Calculation

- Convert $ payoffs to utility values, using figure
  - U($80M) = 0.8983
  - U($30M) = 0.5756
  - U(-$5M) = -0.1536
  - U(-$40M)= -2.3157 (off scale, from fitting equation)

Example EU, CE and RP Calculation

- Calculate expected utility (EU) for each project

  EU_A = 0.8x0.8983+0.20x(-2.1357)

  = 0.2915

  EU_B = 0.8x0.5756+0.20x(-0.1536)

  = 0.4298

Example EU, CE and RP Calculation

Read CE from utility curve for each project

- For $EU_A = 0.2915$, $CE_A = \$12.06M$
- For $EU_B = 0.4298$, $CE_B = \$19.66M$

Since $EU_B > EU_A$, B is preferred project even though $EMV_A(\$56M) > EMV_B(\$23M)$

Example EU, CE and RP Calculation

$RP_A = EMV_A - CE_A = \$56M - \$12.06M$
$= \$43.94M$

$RP_B = EMV_B - CE_B = \$23M - \$19.66M$
$= \$3.34M$

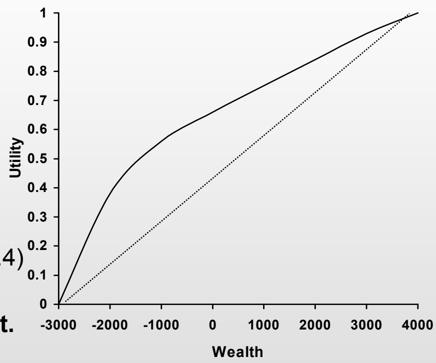
Consider the following utility function and gamble. . .

Win $4000 with probability of 0.40
Win $2000 with probability of 0.20
Win $0 with probability of 0.15
Lose $2000 with probability of 0.25

Step 1. Find the Expected Utility
EU = 0.4U($4000) + 0.2U($2000) + 0.15U($0) + 0.25U(-$2000
= 0.4(1) + .2(.81) + .15(.65) + .25(.4)
= 0.76

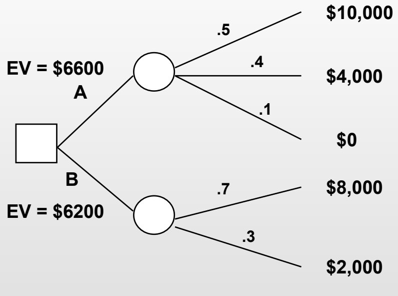
Step 2. Find the certainty equivalent.
Approximately $900

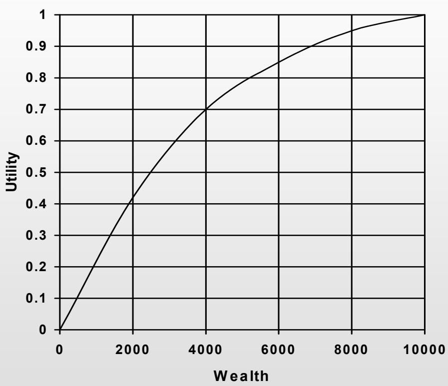
Step 3. Find the expected value.
EV = $1500



Step 4. Find the risk premium.
Risk Premium = EV - CEQ
$600 = $1500 - $900

Consider the following investment choices. . .





- **What is the dominant choice, given the decision maker's utility function?**

Decision Analysis and Finance

- There is general agreement that individual investors are **risk averse**.
- But should firms also behave as individuals when considering investment opportunities?
- DA approach:
  - Decision makers are **risk averse** and this will influence their decision
- Finance approach:
  - **Diversified shareholders** are risk neutral.
  - Managers ***should*** make the decision that is best for the shareholders and be risk neutral.

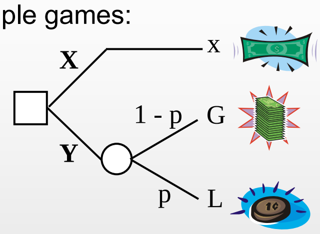
Risk attitudes implementation

- There are two problems in implementing utility maximization in a real decision analysis:
  - The first is obtaining an **individual's utility function**.
  - The second is using the resulting utility function to **find the best decision**.

Utility Function Assessment

Utility Function Assessment

Utility functions are assessed using simple games:



X: Certain return x

Y: Return G with probability p

Return L with probability 1 - p

Variables x, G, L, p

General idea: Vary the parameters of the simple games until the decision maker (DM) is ***indifferent*** between X and Y:

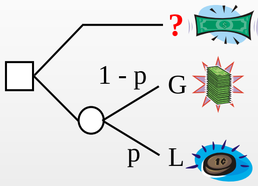
$$X \sim Y \Rightarrow u(x) = p\,u(G) + (1 - p)\,u(L)$$

Questions are asked until sufficiently many points for the utility function have been obtained.

Elicitation styles

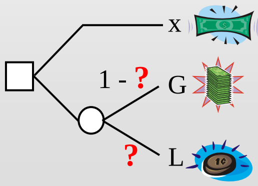
1. Certainty equivalence:

- The DM assesses x.



2. Probability equivalence:

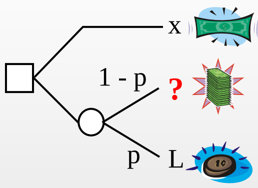
- The DM assesses p.



Elicitation styles -- continued

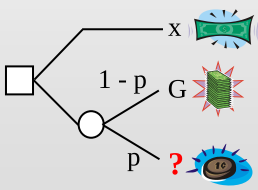
3. Gain equivalence:

- The DM assesses G.



4. Loss equivalence:

- The DM assesses L.



Constructing Utility Curve

- Good method is the first method to determine **certainty equivalences (CE)** and have decision maker think in terms of 50:50 gambles
- Illustrate with drill vs. farm out problem considered earlier
- We will determine five points on utility curve

Data for Drill vs. Farm Out Decision

Outcome		Drill		Farm Out	
State	Probability	NPV, M$	EMV, M$	NPV, M$	EMV, M$
Dry hole	0.65	-250	-162.5	0	0
Producer	0.35	+500	+175	50	17.5
	1.00		12.5		**17.5**

Constructing Utility Function

- **Point 1: identify worst possible outcome**

  Negative payoff of -$250M (dry hole)

  We assign utility of zero

  U(-$250M) = 0
- **Point 2: identify best possible outcome**

  $500M for producer

  We assign utility of one

  U(+$500M) = 1

Constructing Utility Function

- **Point 3: Assess midpoint, decision maker then plays lottery, called reference lottery**

  Win $500M with probability 0.5

  Lose $250M with probability 0.5

  EMV = 0.5x500M + 0.5x(-250M) = $125M
- What is **minimum amount, CE,** for which decision maker would be willing to sell opportunity to play game?
- Decision maker **chooses $50M**
- Means decision maker *indifferent* between sure $50M and risky gamble with EMV = $125M

Constructing Utility Function

- **Point 3**

  Utility of $50M must equal EU of gamble

  U($50M) = 0.5xU($500M)+0.5xU(-$250)

  = 0.5x1 + 0.5x0

  = 0.5

Constructing Utility Function

- **Point 4: Decision maker plays another lottery, between U($50M) from point 3 and U($500M) from point 2**

  Win $50M with probability 0.5

  Win $500M with probability 0.5

  EMV = 0.5(50+500)$M = $275M
- Decision maker would sell chance to play lottery for **$225M**
- U($225M) = 0.5xU ($50M) + 0.5xU(500M)

  = 0.75

Constructing Utility Function

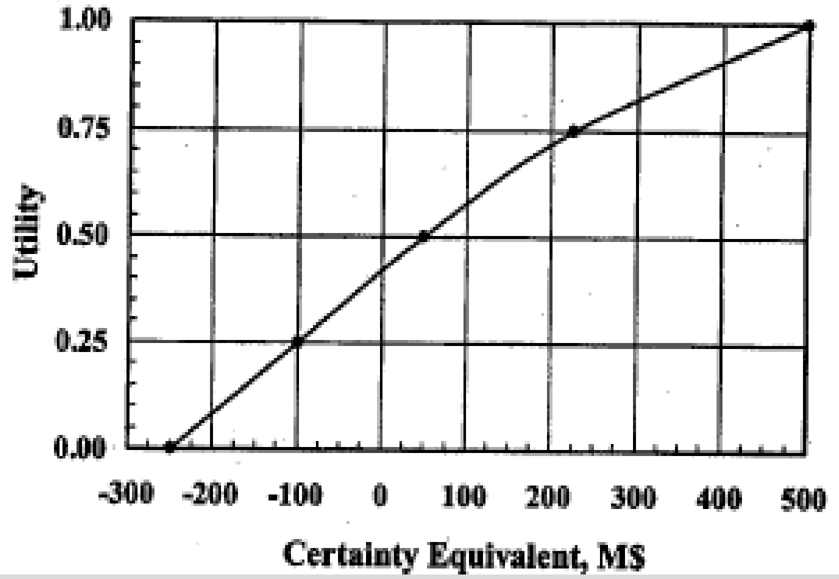
- **Point 5: Decision maker plays another lottery, between U($50M), p = 0.5 from point 3 and U(-$250M), p = 0.5 from point 1**, EMV = -$100M
- Decision maker **selects -$100M** as payment he would take (pay) make to avoid gamble
- U(-$100M) = 0.5x[U($50M)+U(-$250M)]
  = 0.5x(0.5+0)
  = 0.25

Constructing Utility Function

- We have found 5 points in interview with decision maker

Monetary, M$	Utility
-250	0.00
-100	0.25
50	0.50
225	0.75
500	1.00

Constructing Utility Function



Example Using Utility Function

- Use utility curve we constructed to determine our optimal choice in the drill vs. farm-out decision

Data for Drill vs. Farm Out Decision

Outcome		Drill		Farm Out	
State	Probability	NPV, M$	EMV, M$	NPV, M$	EMV, M$
Dry hole	0.65	-250	-162.5	0	0
Producer	0.35	+500	+175	50	17.5
	1.00		12.5		**17.5**

Analysis of Drill vs. Farm-Out Decision

Outcome		Drill		Farm Out	
State	Probability	NPV, M$	U(NPV)	NPV, M$	U(NPV)
Dry hole	0.65	-250	0	0	0.40
Producer	0.35	+500	1.0	50	0.50
	1.00		0.35		**0.435**

EU(Farm Out) = 0.435 > EU(Drill) = 0.35
So, we should farm out

Comments on Preference Assessment : Framing Effect

Framing 1

- Choose between A and B:
  - A: A 50% chance of gaining $1000.
  - B: A sure gain of $500.

Framing 2

- Choose between C and D:
  - C: A 50% chance of losing $1000.
  - D: A sure loss of $500.

Comments on Preference Assessment : Framing Effect

400 people had an accident. There are two alternative rescue plans.

Which one would you choose?

A: 200 are rescued for sure

B: 100 are rescued with probability 0.6

(400 are rescued with probability 0.4)

What about these?

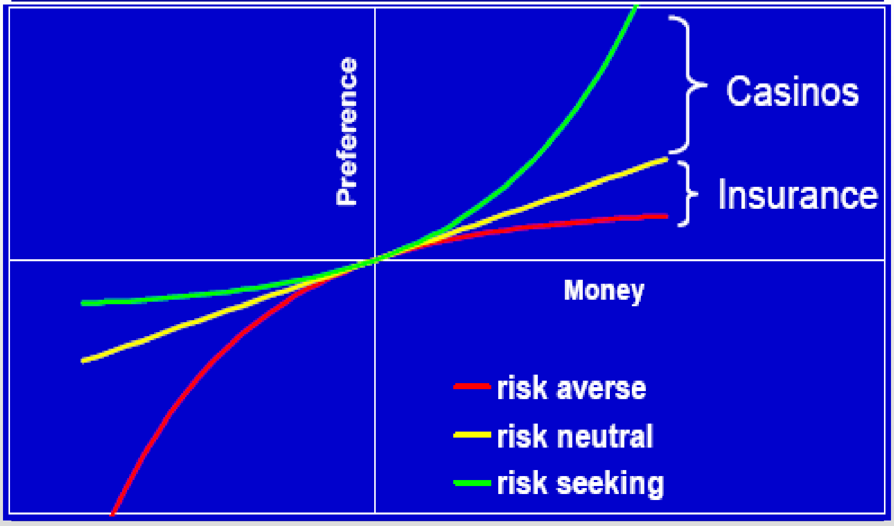
A: 200 are killed for sure

B: nobody are killed with probability 0.4

(300 are killed with probability 0.6)

Framing effect: Most people are **risk-averse about gains** and **risk-seeking about losses**.

Risk attitudes: Gain vs. Loss



Comments on Preference Assessment : Certainty Effect

The Certainty Effect 1: Russian Roulette

Gun A has 4 bullets.

Paying some amount will remove 1 bullet.

Gun B has 1 bullet.

How much would you pay to remove the bullet?

More, less, the same?

Comments on Preference Assessment : Certainty Effect

Probability of being shot in Case A:

Before payment: 4/6 = 0.67;

After payment: 3/6 = 0.50.

Risk reduction: 0.17.

Probability of being shot in Case B:

Before payment: 1/6 = 0.17;

After payment: 0/6 = 0.00.

Risk reduction = 0.17.

Comments on Preference Assessment : Certainty Effect

Standard Finance:

Utility of the payment is 0.17 in both cases, so prediction is payment would be the same.

Behavioral Finance or Utility Theory:

Predicts people would much rather *eliminate* risk than *reduce* it since in BF small probailities are overweighted.

Comments on Preference Assessment

- Choice among preference assessment procedures should be based on **ease of use** by the decision maker.
- Assess preferences on outcomes that **represent realistic ranges** of outcomes for the decision maker.
- Individuals are **not perfectly consistent** and since some risks are more meaningful than others, it makes sense to use the range that corresponds to the problem at hand.
- Check for **consistency of preferences**.

Mathematical Representation of Utility Functions

- **Interviews** time-consuming method to construct utility functions
- **Approximations** to decision maker's actual utility function often adequate with **standard utility function forms**