Economic Risk and Decision Analysis for Oil and Gas Industry
CE81.9008

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Option Concepts

What is an Option?

- An **opportunity** or a **contract** that gives you a **right**, but not an **obligation**…
  - Asymmetric returns
  - Will exercise only if advantageous
  - Acquired at some cost
- … **to take some action**…
  - Usually to buy or sell something (the underlying asset)
- … **now, or in the future**…
  - Usually during a limited time frame
  - Option expires after time limit
- … **for a pre-determined price.**
  - ***Price of action*** distinct from **option cost**

Terminology

- **Underlying Asset (S)**
  - The asset that will be received or given away if the option is exercised.
- **Financial Option**
  - An option where the underlying asset is a ***traded financial security*** or ***commodity***.
- **Real Option**
  - An option where the underlying asset is a ***real asset***.
- **Call Option**
  - The right to ***purchase*** the underlying asset.
- **Put Option**
  - The right to ***sell*** the underlying asset.

Terminology

- **Exercise Price (X)**
  - The pre-determined price at which the option holder can buy or sell the asset to the option seller. Also known as **Strike Price**
- **Expiration date (T)**
  - The date on which the option expires.
- **Premium**
  - The cost the holder paid for the option contract. This is the value of the option

Review of Financial Options

The Option Contract:

A contract in which the **writer (seller)** of the option grants the **holder (buyer)** the **right**, **but <u>not the obligation</u>**, to ***buy from*** or to ***sell to*** the writer an **underlying asset** at a fixed **strike (exercise) price** at or before an **exercise date.**

- **Call Option**
  - Grants its **holder** the **right to buy** the underlying asset
  - **Writer** (seller) of the call option makes a **commitment to sell** the underlying asset at the strike price if the option is exercised.

Review of Financial Options

- **Put Option**
  - Grants its holder the right to sell the underlying asset
  - Writer (seller) of the put option makes a commitment to buy the underlying asset at the strike price if the option is exercised

- Options may be:

  Exercised, traded in the market or allowed to expire

- **European vs. American Options**

  - European options can be exercised ***only at the exercise date***

  - American options can be exercised ***anytime*** between the date they are written and the exercise date

Options Terminology

“In the Money”

- An option whose exercise would generate a profit, given the current market price of the underlying asset.
  - Calls (puts) with exercise prices lower (higher) than the current price of the underlying

“Out of the Money”

- An option whose exercise would generate a loss.
  - Calls (puts) with exercise prices lower (lower) than the current price of the underlying

“At the Money”

- The exercise price is roughly equal to the current price of the underlying

Options Payoffs at Expiration

- **Payoff Graph or Diagram:** Graph of the **profit and loss positions** for each possible price of the underlying asset at the exercise date

A. Call Options

Value of the call option at expiration:

Payoff to call owner $= S_T - X$ if $S_T > X$

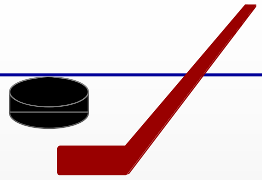
$= 0$ if $S_T < X$

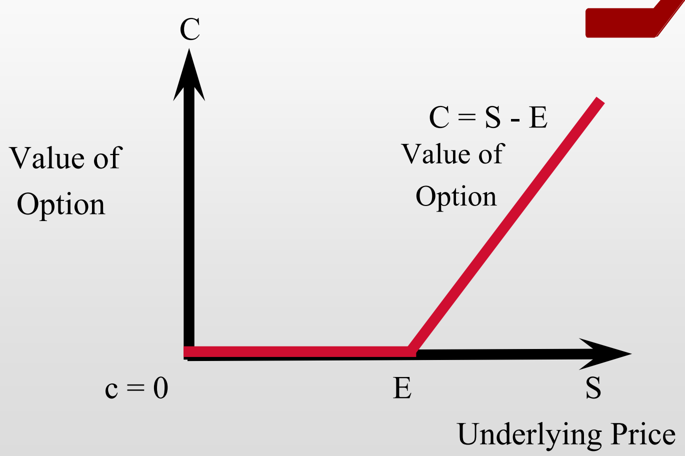
where:

S_T = value of the underlying at expiration

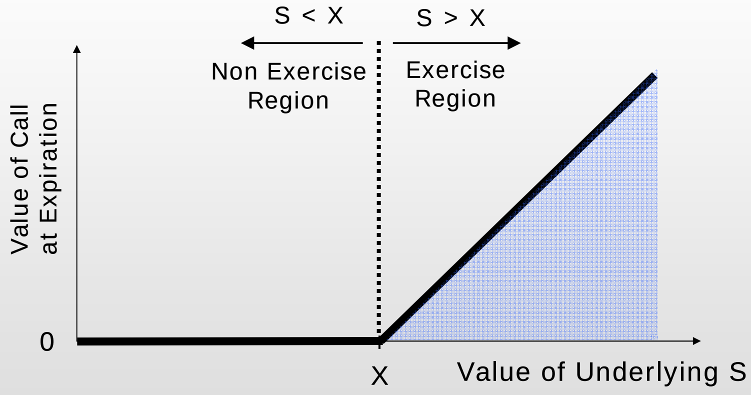
X = Strike price

Call Option: Value at Expiration





Call Option: Value at Expiration



- Value at Expiration is F = max (S - X , 0)

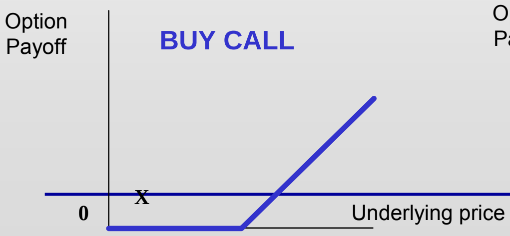
Call Option Value: example

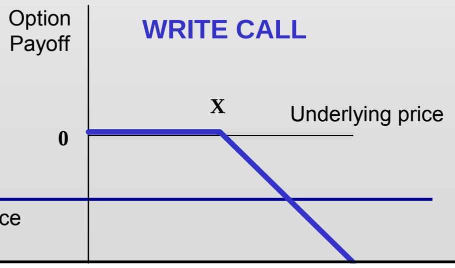
Call option on Stock: **X =$4.0**:

Stock Price	$3.2	**$4.0**	$4.2	$4.4	$4.6
Option Payoff	$0.0	$0.0	$0.2	$0.4	$0.6

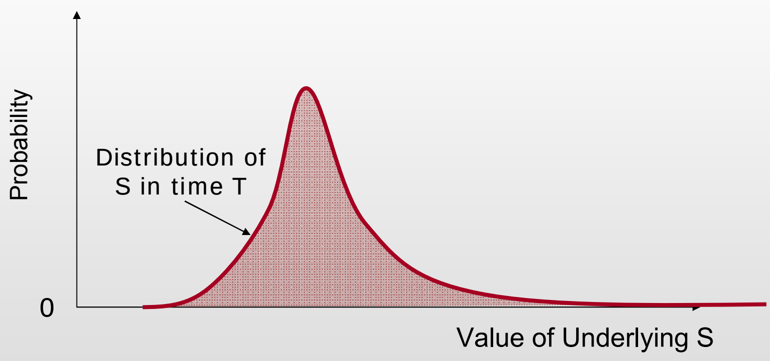
Payoff to call writer = $-(S_T - X)$ if $S_T > X$

Payoff to call writer = 0 if $S_T < X$

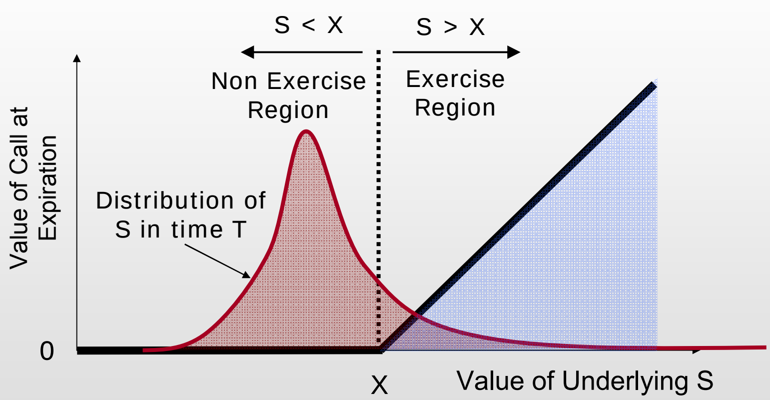




Call Payoff is Asymmetric

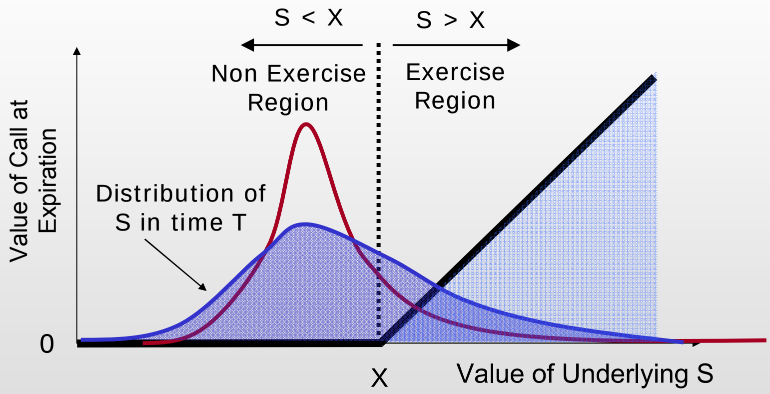


Call Payoff is Asymmetric



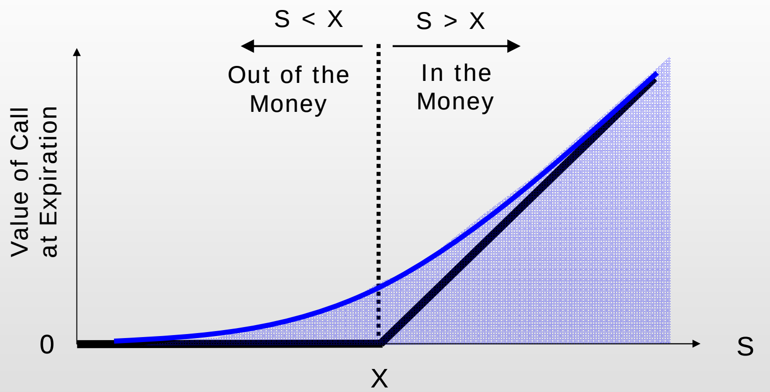
- **Option Value** can **never be *negative***

Expected Call Payoff increases with Volatility



- **Probability of S > X** increases with **Volatility of S**

Call Option: Value before Expiration



- ***Before expiration*** option may have value even if S < X.
- This is because of the ***uncertainty*** about the future value of S at expiration.

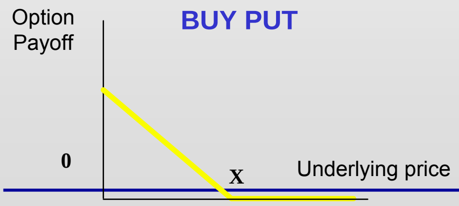
Options Payoffs at Expiration

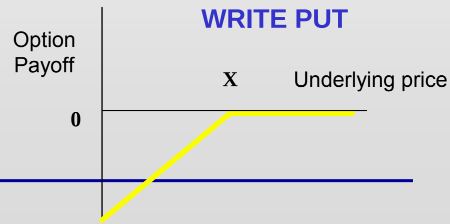
B. Put Options

Value of the put option at expiration:

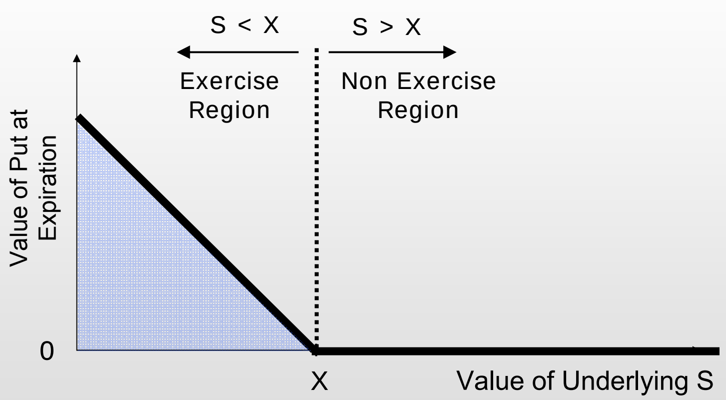
Payoff to put owner = 0 if $S_T > X$

Payoff to put owner = $X - S_T$ if $S_T < X$



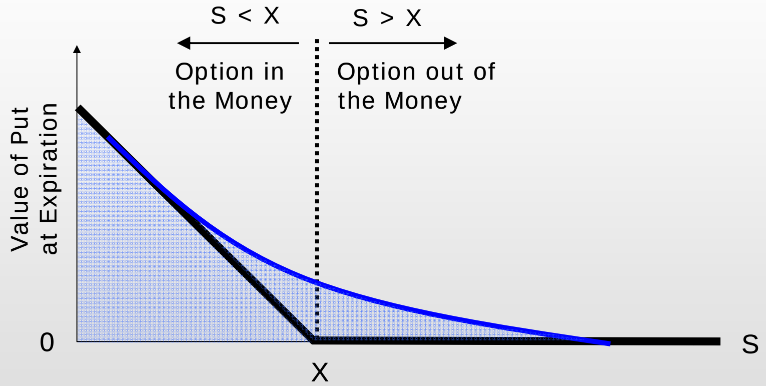


Put Option: Value at Expiration



- Value at Expiration is F = max (0, X - S)

Put Option: Value before Expiration



- ***Before expiration*** option may have value even if S > X.
- This is because of the ***uncertainty*** about the future value of S at expiration.

Put-Call Parity

Consider a **European call** and a **European put** with the same ***strike price X*** and the same ***expiration date T***

- **Buy the call** and **write the put**:
- What is the payoff from this portfolio at ?

  Examine the two possible scenarios:

	$S_T < X$	$S_T > X$
Payoff from call owner	0	$S_T - X$
Payoff from put written	$-(X - S_T)$	0
Total	$S_T - X$	$S_T - X$

- So this position always **nets a payoff of $S_T - X$**

Put-Call Parity

- What is the value of this option portfolio?
  - Find a **tracking portfolio** that ***replicates the future payoffs***. The value of the option portfolio must equal the price of the tracking portfolio if there is *no arbitrage*
- The **tracking portfolio** (levered equity):
  - Borrow $\frac{X}{(1+r_f)^T}$ today and repay X at maturity
  - Buy one share of stock
- The payoff of the tracking portfolio at time T is $\mathbf{S_T - X}$
  - The **tracking portfolio** perfectly replicates the payoff of the **option portfolio**
  - Hence the tracking portfolio and the option portfolio must have the same value at date 0

Value of the option portfolio at date 0

- Purchase the call option for price C_0
- Write (sell) the put option for price P_0
- Total cost of establishing the position is $C_0 - P_0$

- The value of the tracking portfolio at date 0:
  - Borrowed funds: $\frac{X}{(1+r_f)^T}$, Buy one share of stock: S_0
  - The value of the tracking portfolio is then $S_0 - \frac{X}{(1+r_f)^T}$
- The option portfolio and the tracking portfolio must have the same value:

Put-Call Parity: $$C_0 - P_0 = S_0 - \frac{X}{(1+r_f)^T}$$

Option Pricing

Behavior of Call Option Prices

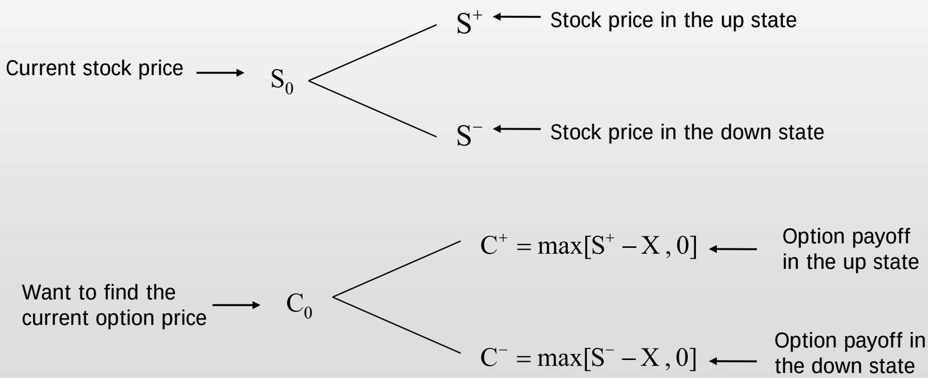
Key Variables			*Call Prices*
Stock price	=	*S* ↑	*C* ↑
Time	=	*T* ↑	*C* ↑
Exercise price	=	*E* ↑	*C* ↓
Variance	=	*Var* ↑	*C* ↑
Risk-free rate	=	*R* ↑	*C* ↑

Factors that affect Option Value

Factor	Effect on Call Option	Effect on Put Option
Increase in Price of Underlying (S)	Increases	Decreases
Increase in Exercise Price (X)	Decreases	Increases
Increase in Volatility (σ)	Increases	Increases
Increase in Time to Expiration (T)	Increases	Increases
Increase in Interest Rate (r)	Increases	Decreases
Increase in Dividends Paid (δ)	Decreases	Increases

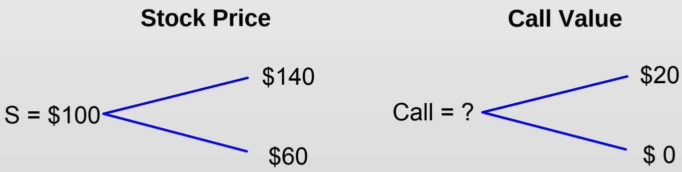
The Binomial Model

Consider **a stock** and **a European call on the stock** one period before expiration:



Valuing Options: Binomial Approximation

- Let **C** be the value of a **one year Call Option** on a stock.
- Let **X = $120** be the exercise price of a stock that is worth **S = $100** today.
- Assume that the stock price in one year will be either $140 or $60 and that the riskfree rate is 10%.
- In this case, the value of the Call at expiration will be either $20 or $0.

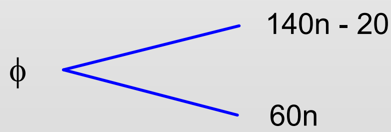


Valuing a Call Option: Binomial Approximation (discrete time)

- So we know what the value of the Call will be in one year, but what is the value of the Call today?
- To solve we use a **riskless portfolio**. This portfolio will have ***n*** **shares of the stock (S)** and **one short position on the Call option (C)**.

$$\phi = nS - C$$

- At the end of one year the **value of this portfolio** will be:



Valuing a Call Option: Binomial Approximation (discrete time)

- We make this portfolio riskless by choosing an appropriate value for *n*:

$$140\,n - 20 = 60\,n$$

$$n = 0.25$$

- With **n = 0.25** the value of the portfolio at the end of one year will be 15, regardless of the value of the stock.

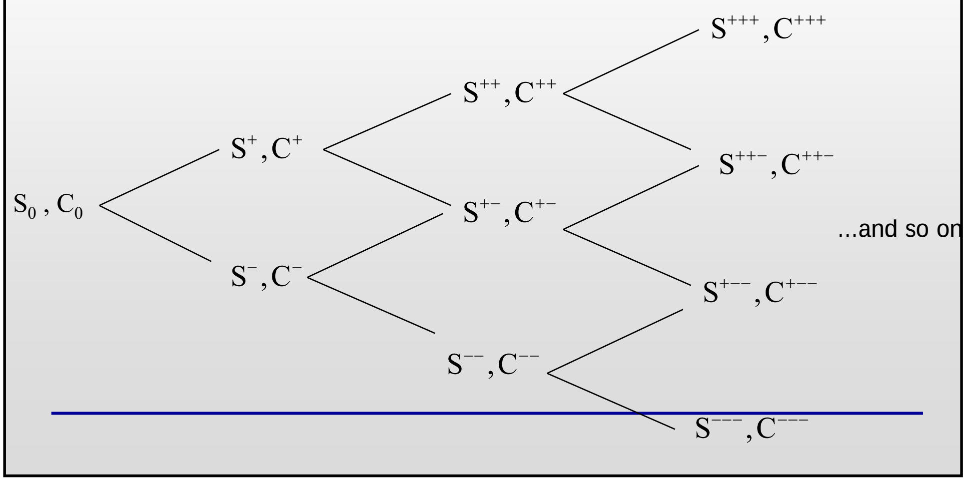
- If this portfolio is riskless, it should be discounted at the **risk free rate**. Its value today will then be 15/(1 + r)=**13.64**

- The value of the Call today will then be:

$$\phi = n\,S - C \rightarrow (0.25)100 - C = 13.64$$

C = $11.36

Generalization of the Binomial Model



Valuing a Call Option: Black and Scholes Formula

- Developed in 1973: $c = SN(d_1) - X\ e^{-rt}\ N(d_2)$

$$d_1 = \frac{\ln\frac{S}{X} + (r + \frac{\sigma^2}{2}) \cdot t}{\sigma\sqrt{t}} \qquad d_2 = d_1 - \sigma\sqrt{t}$$

Where **N(.)** is the **cumulative normal distribution function**

•**Black-Scholes formula** gives the value of **a European call option in the continuous-time case**

• Same as the option price given by the binomial tree approach, where the time intervals are taken to be very small

Elements of Black-Scholes Formula

- C_0 = Current call option value
- X= Exercise price
- S_0 = Current stock price
- e = 2.71828
- r_f = Annual risk-free rate
- T= Time to maturity (in years)
- ln=Natural logarithm
- N(d) = Probability that a random draw from a standard normal distribution will be less than d
- σ= Standard deviation of stock's annualized continuously compounded return

Valuing a Call Option: Black and Scholes Formula

$$c = SN(d_1) - X\ e^{-rt}\ N(d_2)$$

$$d_1 = \frac{\ln \frac{S}{X} + (r + \frac{\sigma^2}{2}) \cdot t}{\sigma \sqrt{t}} \qquad d_2 = d_1 - \sigma\sqrt{t}$$

- ***Ex:***

  S = $100
  X = $120
  σ = 35%
  r = 10%
  T = 1

  } C = 10.59

Assumptions of the Black and Scholes Model

1. Assumptions required for Black-Scholes formula
   - -European call option without dividends
   - -Both r_f and σ are constant
   - -Stock price changes are continuous
2. σ that makes observed call option price consistent with Black-Scholes formula is called *implied volatility*
3. Value of underlying asset grows exponentially (is lognormally distributed)

Financial vs. Real Options

Analogy between Financial and Real Options

Financial Options		Real Options
Call Option	⇒	Option to Invest
Stock Price	⇒	PV of Project
Exercise Price	⇒	PV of Investment
Time to Expiration	⇒	Time to Expiration
Risk Free Rate	⇒	Risk Free Rate
Volatility of Stock Price	⇒	Volatility of Project Value
Dividends	⇒	Project Cash Flows

Classes of Real Options

- **I. Call-like Options**
  - Allow holder to **capture benefits** from ***increase in project value***
  - Exercise involves investing money into project
  - Exercise when expectations of positive return increase
- **II. Put-like Options**
  - Allow holder to **insure against losses** from ***decrease in project value***
  - Exercise may involve costs
  - Exercise when expectations of positive return decrease
- **III. Compound Options**
  - Projects might contain multiple interacting options
  - Exercise rule based on overall profit maximization

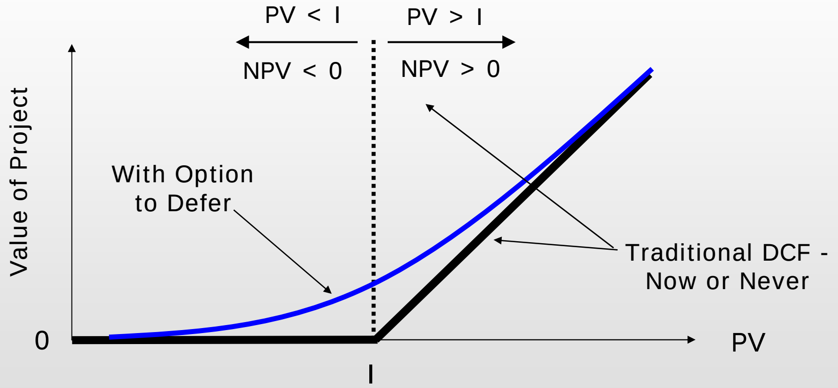
Firm Value depends on:

- **Value of Assets**
  - Current Productive Capacity
  - Expected Cash Flow Stream
  - Usually valued with **DCF method**
- **Option Value**
  - Option to grow/expand: Investment opportunities
  - Option to defer investment
  - Option to abandon operations
  - Option to suspend, resume, switch inputs or outputs
  - **Cannot be valued by DCF methods** – must use Option Pricing Methods

I. Call-like Options

- **Option to Defer Investment**
  - Situation where firm has an opportunity to invest in a project or an option to buy valuable land or resources.
  - Firm can wait a period of time to see if new information justifies constructing a building, a plant or developing a field.
  - Has greatest value in mineral resources, agricultural and real estate projects.
  - Leaving open opportunity to invest is similar to holding a call option. Investing kills the option.
  - If option to defer investment is greater than value of investing now investment should be postponed.
  - Optimal decision based on Max (Invest now, Defer)

Project with Option to Defer



- With Option to Defer, project may have value even if NPV < 0.

Call-like Options

- **Option to Expand an Existing Project**
  - Situation where firm has opportunity to increase output if market conditions turn out to be more favorable than expected.
  - This option allows the firm to capture of upside potential of project.
  - This option may sometimes be included in the original project at an additional cost. (Ex: building a four lane bridge in a two lane road).
  - Cost of expansion is the option exercise price
  - Leaving open opportunity to expand is like holding a call
  - Optimal decision based on Max (Do nothing, Expand Project)

Call-like Options

- **Option to Restart a Project**
  - In practice, not all projects have to operate continuously.
  - Some project may have the flexibility to restart after a temporary shutdown if new market conditions indicate that is the best decision.
  - This is similar to the option to defer investment
  - Cost of restarting is the option exercise price
  - Optimal decision based on Max (Remain Closed, Restart)

II. Put-like Options

- **Option to Abandon a Project**
  - Abandon means to eliminate all fixed costs in exchange for a salvage or abandon value.
  - May occur when market condition take a turn for the worse and expect value of project is less than salvage value.
  - Allows elimination of further exposure
  - Might entail shutdown costs
  - Optimal decision based on Max (Continue, Abandon)
- **Option to Contract Operation**
  - Decelerate or reduce exposure to potential losses
  - Might entail short term scale down costs
  - Optimal decision based on Max (Do nothing, Contract)

Put-like Options

- **Option to Temporarily Suspend Operation**
  - Common in mineral resources and consumer product.
  - If market conditions take a turn for the worse and shutdown and restart costs are low, firm may decide to suspend operations and wait for better market conditions.
  - Temporarily eliminates exposure
  - Might entail shut down costs
  - Optimal decision based on Max (Continue, Suspend)

III. Compounded or Nested Options

- **Combination of Options**
  - Occurs when multiple options exist simultaneously
  - May include invest, contract, expand and abandon options, for example
  - Complex valuation due to interaction among options as options may be interdependent
- **Switching Options**
  - Exits when firm has product or process flexibility that allows it to switch outputs or inputs in response to changes in demand, costs or supplies.
  - Flexible systems may contain an infinite number of options allowing continuous switching of modes of operation.
  - Switching cost is the option exercise price.

Example

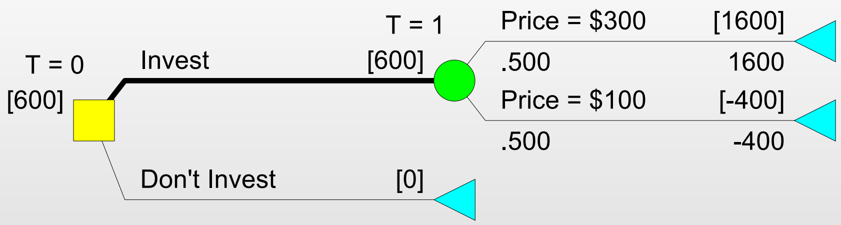
Project with Option to Defer[1]

- Firm is deciding on an **irreversible investment of I=$1,600** in a widget plant.
- Plant will produce one widget per year in perpetuity at no cost.
- **Initial price of P=$200** will have a one time increase of 50% to **$300** or reduction of 50% to **$100 in period 1** with probability q = 0.50 each.
- Project has only private risk, i.e., its risk can be completely diversified away.
- Discount rate is the **risk free rate r = 10%.**
- Project can be ***deferred*** until next year.

[1] From Dixit and Pindyck (1994)

Project without Option

- Immediate Investment is optimal and yields $600



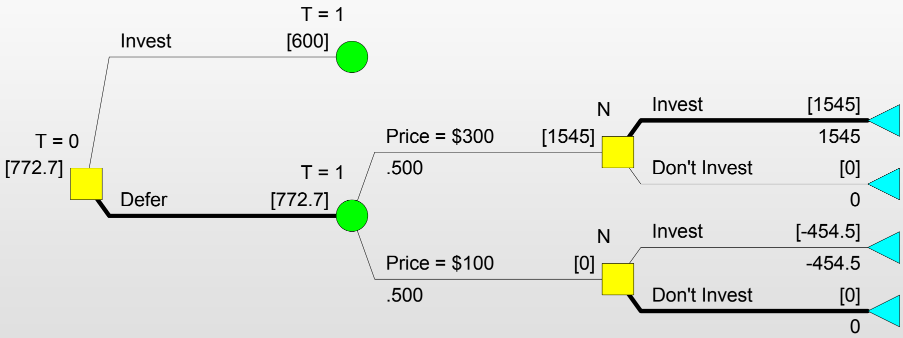
$$NPV = -1600 + \sum_{t=0}^{\infty} \frac{200}{(1.1)^t} = \$600$$

Option to Defer

- Investing immediately ***kills*** the **option to defer**
- This option has value because deferring investment for one year from T = 0 to T = 1 ***allows the price uncertainty to be resolved***.
- With this new information, a better investment decision can be arrived at.
- Deferring for more periods has no value as there is no uncertainty after T = 1 and no new information will be obtained.

Project with Option

- Deferring to T = 1 is optimal and yields $772.7



$$NPV = 0.5.\left[\frac{-1600}{(1.1)} + \sum_{t=1}^{\infty}\frac{300}{(1.1)^t}\right] = \$772.7$$

Time 0 Price Threshold

- We can also determine the ***price threshold*** which would trigger the investment in T = 0 rather than defer.
- This can be done by equating the NPV of investing now and deferring and yields P = $249 with a NPV of $1,143.
- For **P < $249** it is always **optimal to defer the investment**.
- Note that this contradicts the traditional NPV rule that you should invest immediately whenever NPV > 0.

Time 1 Price Threshold

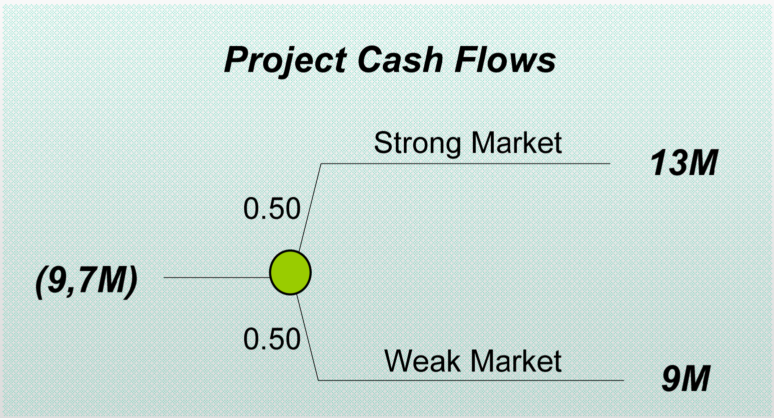
- The **minimum price level** that triggers investment in T = 1 is **P = $97**
- This is the price that yields a zero NPV considering the option to defer.
- At this price level, the traditional NPV of the project without option to defer is -$533.
- Note that this contradicts the traditional NPV rule that you should never invest if NPV < 0.

Example: An Investment Decision

- You have an opportunity to invest in a real estate project deal now or one year from now.
- Total construction costs are $9.7 million and the property will be ready for sale in one year.
- It is expected that if the economy improves the property can be sold for $13 million. If not, the price will be only $9 million.
- Assume that the chances of either state occurring are the same, that your cost of capital is 10% and that market conditions will stabilize after next year.

An Investment Decision

- Should you invest now?

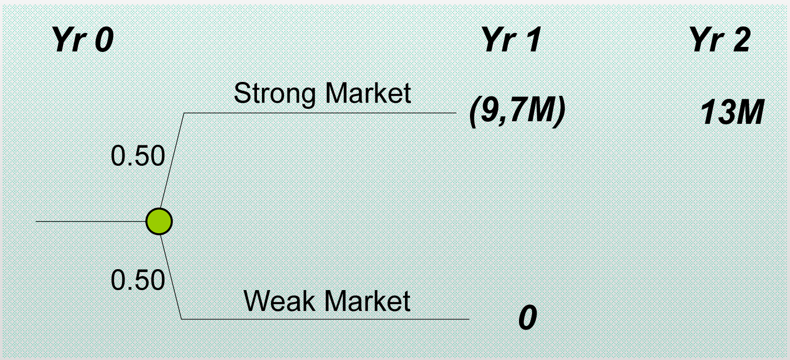


Option Thinking

- Now assume that you will **defer the investment** for one year.
- Think of this problem as an option to invest in the project.
- To **exercise the option** you must pay \$9.7 million
- The expected value of the project is \$10.0M, so your net gain is \$0.3M if you invest.
- On the other hand, by investing you ***kill*** the option.
- What if this option is worth more than \$0.3M?

Option Thinking

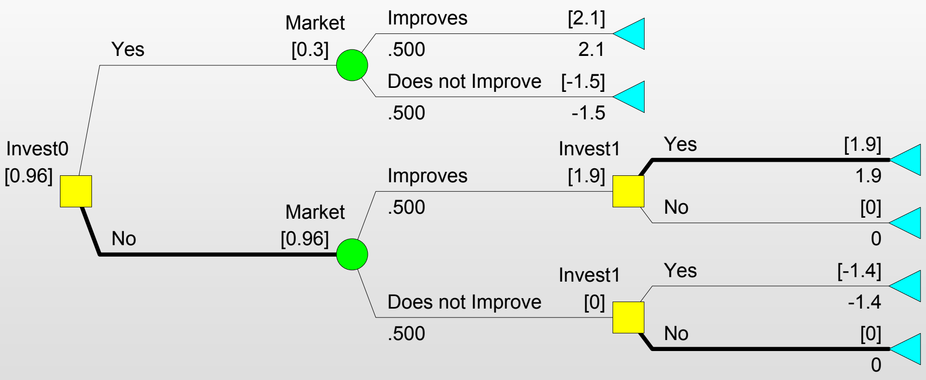
- By deferring the investment until Yr 1 you can eliminate the downside of the project by only **investing if market conditions improve.**



Option Thinking

- Deferring the investment increases the value of this project from **$0.30M** to **$0.96M**.
- The value of the option is **$0.66** (= 0.96 - 0.30)
- This happened because during the wait time **the market uncertainty was resolved** and this information allowed you to make a better decision.
- If you ignore the option to defer the investment, you will lead you to invest before it is optimal to do so.

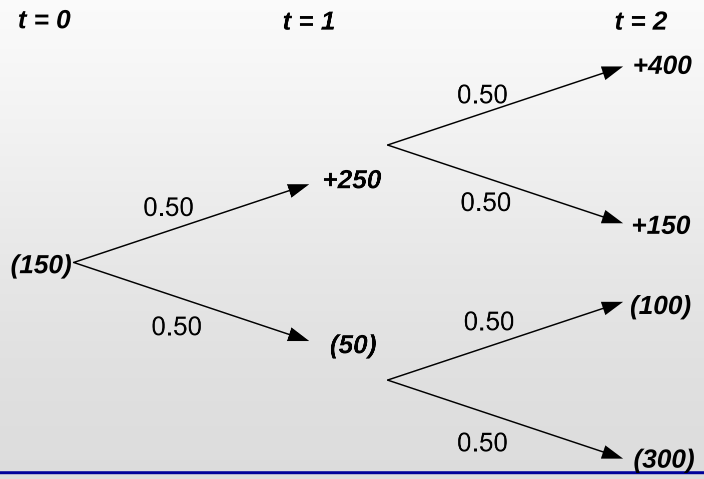
Decision Tree Modeling



Example: Option to Abandon

- Biodata Inc. is about to launch a new product that will have a market life of two years.
- Investment cost is $150 million.
- It is known that competitors are actively working to develop a similar product. Whether new competitors enter the market now, next year or not at all will affect the revenues Biodata expects to receive from its product.
- It is estimated that the probability of each scenario is 0.50 and that the cost of capital is 10% per year.

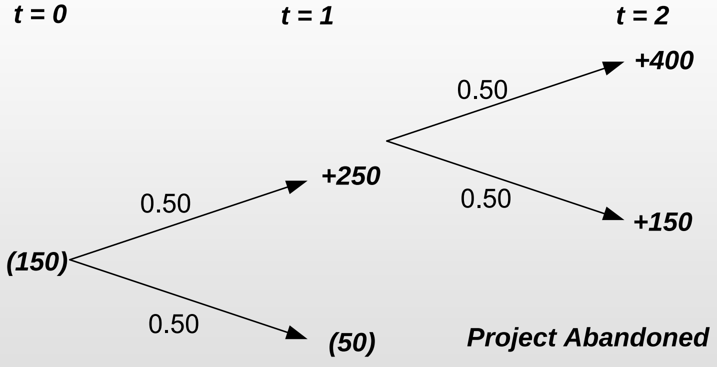
Biodata Inc: Cash Flow Uncertainty



Biodata Inc: NPV

- What is the project NPV?
  - NPV = -150+ 0.5(250 + (50))/1.1 +
    + 0.25(400+150)/1.12 +
    + 0.25((100)+(300))/1.12 = -28,10
- Suppose that the firm has the option to abandon production after the first year of operation.
- How does this affect the project NPV?

Biodata: Option to Abandon



Biodata Inc: Option to Abandon

- What is the project NPV now?
- What is the value of the option to abandon?
- Does this option affect the project risk? How?

Real Option Value

- Real Option Valuation is most useful when traditional NPV is small or close to zero.
- The degree of ***managerial flexibility*** and ***level of uncertainty of future cash flows*** also affect the value of the project real options.
- ***Greater flexibility to react*** to information and ***greater uncertainty*** allow for greater option value.

How Uncertainty affects Option Value

- **Scenario A:**
  - You decide to sell your house
  - Your real estate agent tells you that in the past year five houses similar to yours were sold for $100,000 each.
  - You receive an offer of $100,000
  - Is there value in waiting for a better offer?
  - Does the option to wait have any value in this case?

How Uncertainty affects Option Value

- **Scenario B:**
  - You decide to sell your house
  - Your real estate agent tells you that the market is currently very volatile and in the past year five houses similar to yours were sold for prices ranging from $70,000 to $130,000 each, which a mean price of $100,000.
  - You receive an offer of $100,000
  - Is there value in waiting for a better offer?
  - Does the option to wait have any value?

When is Flexibility Valuable?