Economic Risk and Decision Analysis for Oil and Gas Industry
CE81.9008

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Presented by
Dr. Thitisak Boonpramote

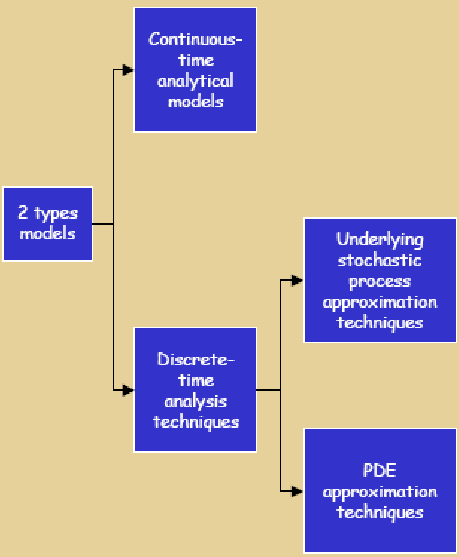
Department of Mining and Petroleum Engineering, Chulalongkorn University

Option Pricing Theory (OPT)

Real Options as an Analytic Tool

- There are several approaches used in practice to **value options**:
  - Black-Scholes formula (or other "standard" formulas)
  - Binomial Option Pricing Model
  - Risk-adjusted Probability
  - Monte Carlo Simulation
- All of these are based on the same underlying principles:
  - Map out evolution of some **underlying variable(s)** over time
  - Determine **cash flows** for each scenario
  - Risk-adjust the **probabilities** of obtaining different cash flows (or the expected future cash flows), rather than the discount rates
  - Discount back **risk-adjusted expected cash flows** at risk-free rate

Three types of OPT



- Limited in their practical use to more complex real-life situations (e.g., compound options)

- Monte Carlo simulation:
  Of limited use for situations with American-type option flexibility
- Lattice approaches:
  Capable of handling complex real option situations

- Are less intuitive (reserved for the quants)
- May be impossible to write down the set of partial differential equations

I) The Black-Scholes Model

The Black-Scholes Model of Option Pricing

- Probably the most famous tool associated with option pricing
- Black and Scholes developed a simple model that can be programmed in a spreadsheet to price options
- Based on the **model of stock price movements** incorporated into the lattice models
- Uses the solution to **partial differential equations (PDE)** to determine the basic result

The Model

$$C = SN(d_1) - Ee^{-rt}N(d_2)$$

$$d_1 = \frac{\ln(S/E) + (r + \frac{1}{2}\sigma^2)t}{\sqrt{\sigma^2 t}} \qquad d_2 = d_1 - \sqrt{\sigma^2 t}$$

where:

S = Current stock price
E = Exercise price of call
r = Continuous risk-free rate of return (annualized)
σ^2 = Variance (per year) of the continuous return on the stock
t = Time (in years) to expiration date

In addition, **N(d)** equals probability that a standardized, normally distributed, random variable will be **less than or equal to d**.

A Simple Example of Black-Scholes

- Consider the Big Oil Company (BOC). On 10/4/2008, **BOC April 49 call option** had a closing value of $4. The stock itself is selling at $50. On 10/4, the option had 199 days to expiration (maturity date = 4/21/2009). The annual risk-free interest rate is 7 percent.

  From this we can determine the following variables:

  1. Stock price, S, is $50.
  2. Exercise price, E, is $49.
  3. Risk-free rate, r, is 0.07.
  4. Time to maturity in years, t, = 199/365.

- Estimates of Variance may differ; but must obviously involve analysis of a series of past price movements for the stock. Let's assume variance of returns on BOC is estimated at 0.09/year.

Black-Scholes Example (cont'd)

Step 1: **Calculate d_1 *and* d_2**

$$d_1 = \frac{\ln(S/E) + (r + \frac{1}{2}\sigma^2)t}{\sqrt{\sigma^2 t}}$$

$$d_1 = \frac{\ln(50/49) + (0.07[\frac{1}{2}][0.09]).545}{\sqrt{0.09(.545)}}$$

$$d_1 = 0.3743$$

$$d_2 = d_1 - \sqrt{\sigma^2 t}$$

$$d_2 = 0.1528$$

Step 2: **Calculate N(d_1) *and N(d_2)***

From a table of the cumulative probabilities of the standard normal distribution, we know that:

N(d_1) = N(0.3743) = 0.6459

N(d_2) = N(0.1528) = 0.5607

Interpretation: N(d) is the cumulative probability of d. For example, N(d_1) tells us that there is a 64.59 percent probability that a drawing from the standardized normal distribution will be below 0.3743.

Black-Scholes Example (cont'd)

Step 3: **Calculate the call option value (C)**

$$C = SN(d_1) - Ee^{-rt}N(d_2)$$

$$C = \$50\text{x}N(d_1) - \$49e^{-.07(.545)}N(d_2)$$

$$C = \$32.295 - \$26.447$$

$$C = \$5.85$$

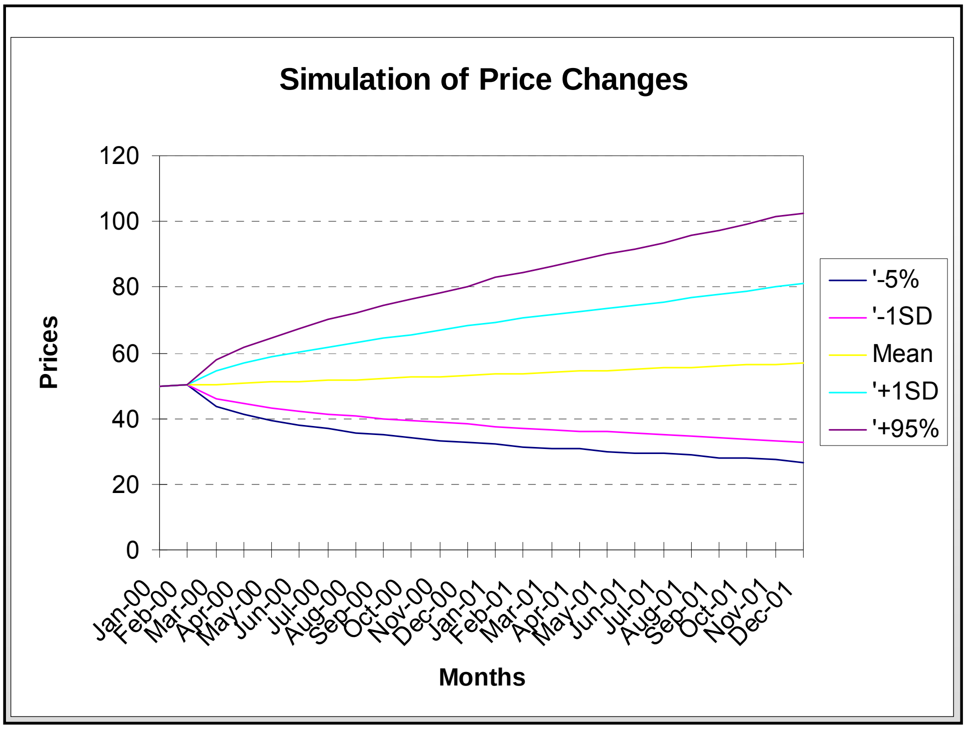
***Note**:* The estimated price of \$5.85 is greater than the actual price of \$4; this implies the call option is underpriced!

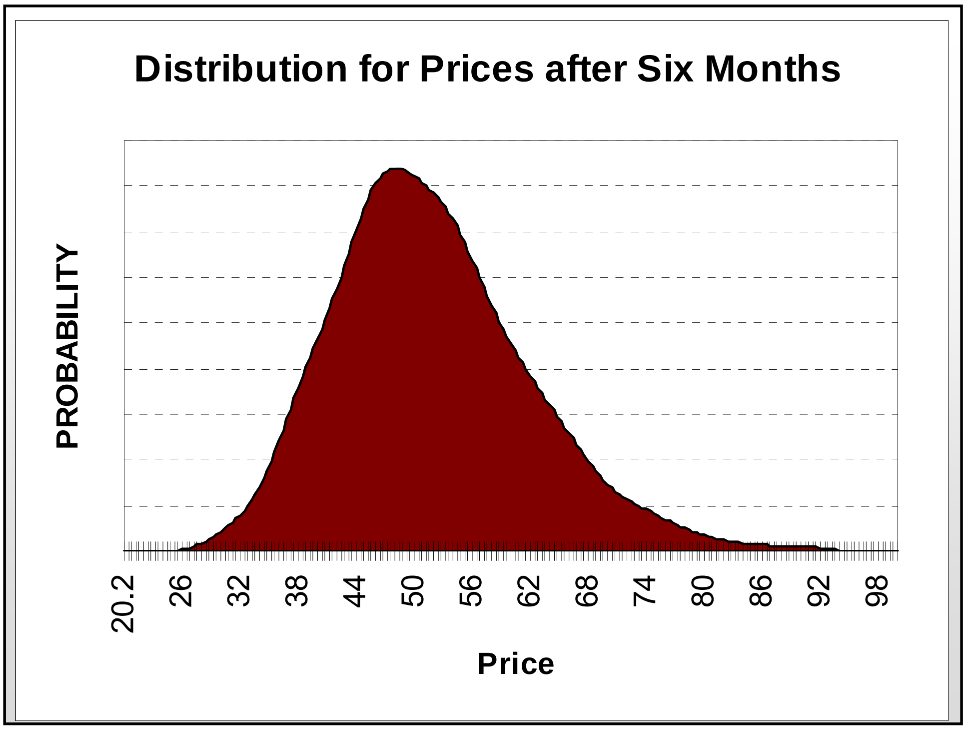
Spreadsheet Implementation

	A	B	C	D	E
1	**Call with dividends**				
2					
3					
4	**Input data**				
5	Stock price	$50			
6	Exercise price	$49			
7	Duration	0.545205			
8	Interest rate	7.00%			
9	dividend rate	0			
10	volatility	30.00%			
11					
12				Predicted	
13	Call price		=	$5.85	
14	put			$3.01	
15					
16					
17	Other quantities for option price				
18	d1	0.374249		N(d1)	0.64589
19	d2	0.152735		N(d2)	0.560696

Some intuition

- The Black Scholes model assumes that **prices change** according to a **geometric Brownian motion (GBM)** (the stochastic process will be discussed later)
- Basically, this means that prices appear to **move randomly over time**, but during any “slice” of time t, the probability distribution of the price is a lognormal distribution
- This price process can be simulated using Monte Carlo simulation with a simple @Risk model


Simulation of Price Changes
Prices
120
100
80
60
40
20
0
Jan-00
Feb-00
Mar-00
Apr-00
May-00
Jun-00
Jul-00
Aug-00
Sep-00
Oct-00
Nov-00
Dec-00
Jan-01
Feb-01
Mar-01
Apr-01
May-01
Jun-01
Jul-01
Aug-01
Sep-01
Oct-01
Nov-01
Dec-01
Months
'-5%
'-1SD
Mean
'+1SD
'+95%


Distribution for Prices after Six Months
PROBABILITY
20.2
26
32
38
44
50
56
62
68
74
80
86
92
98
Price

Intuition

- The Black Scholes value for a **European call option** is the **expected value of the outcomes** (or area) from the **lognormal price distribution** above the strike price
- This result is discounted at the risk free rate due to the arguments regarding replication, based on the assumption of a "complete market"
- Therefore, the Black Scholes valuation is often called **a *risk neutral* valuation**

OPT and the Black-Scholes Model

- Extension of the two-state model.
- B&S model allows us to value a call in the real world by:
  - Determining the duplicating combination at any moment;
  - Valuing the option based on the duplicating strategy.

Why is Black-Scholes so attractive?

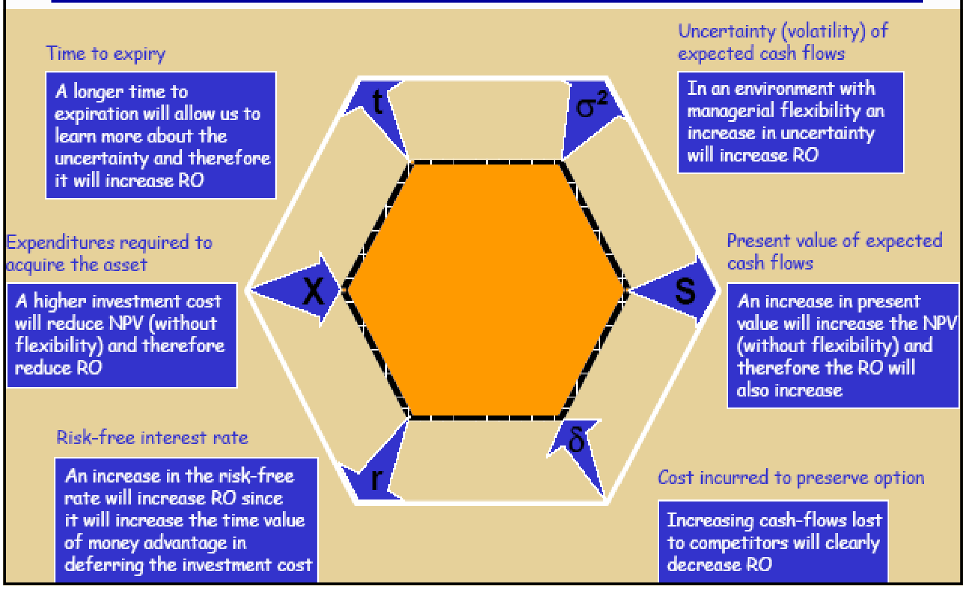
- Four of the five necessary parameters are observable.
- **Investor's risk aversion** does not affect value; Formula can be used by anyone, regardless of willingness to bear risk.
- It does not depend on the **expected return of the stock**.
- Investors with different assessments of the stock's expected return will nevertheless agree on the call price.
- As in the two-state example, the call depends on the stock price, and that price already balances investors' divergent views.

Black-Scholes Model Application to Real Options

Projects As Call Options

- **Opportunity to invest** in a corporate project bears an obvious similarity to a financial option.
- By establishing a mapping between **project characteristics** and the **determinants of call-option value**, a corporate project can be valued in the same way.

Option value determines by six levers

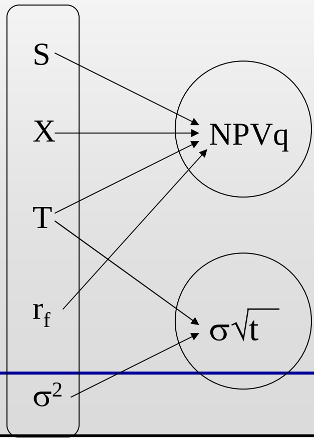


Metrics of the Black-Scholes Model

Converting the **five variables in the Black-Scholes model** to **two new metrics**. Combining five variables into two lets us **locate opportunities in two-dimensional space**.

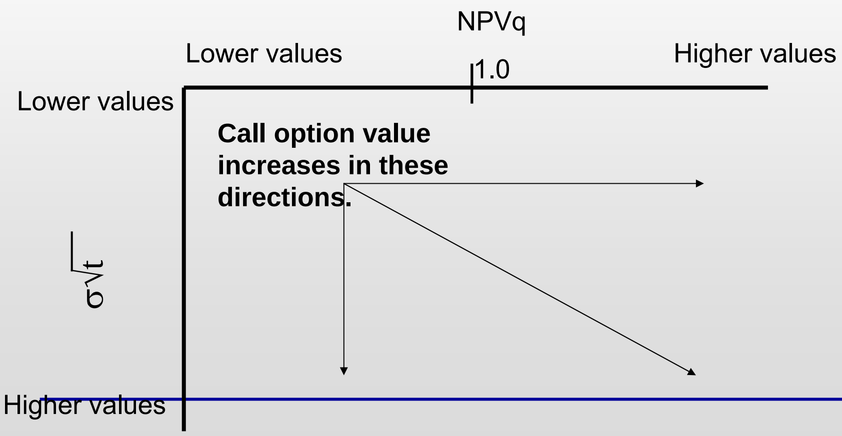
Investment Opportunity	Call Option	Variable
Present value of a project's operating assets to be acquired	Stock price	S
Expenditure required to acquire the project assets	Exercise price	X
Length of time the decision may be deferred	Time to expiration	T
Time value of money	Risk-free rate of return	r_f
Riskiness of the project assets	Variance of returns on stock	σ^2

Option Value Metrics



Locating the Option Value in Two-Dimensional Space

We can locate investment opportunities in this two-dimensional space.

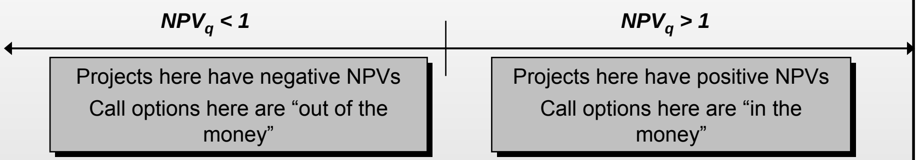


Relating DCF Valuation to Option Value

- **NPV** is simply a measure of the difference between how much an asset is worth and what it costs.
- Put another way, NPV = PV(expected net cash flows) - PV(capital expenditure).
- When a project is worth more than it costs, NPV > 0 and the corporation goes ahead and invests.
- Notice that we can consider **NPV as a quotient** rather than a difference (also known as **Profitability Index**), where:
  - $\mathbf{NPV_q}$ = PV(expected net cash flows) ÷ PV (capital expend.).
  - Restate the decision rule as "invest if $NPV_q > 1$".
  - Reject those projects where $NPV_q < 1$.

Using NPV_q for Projects and Options

$$NPV_q = \frac{PV(\text{expected net cash flows})}{\text{PV(capital expenditure)}} = \frac{S}{PV(X)}$$



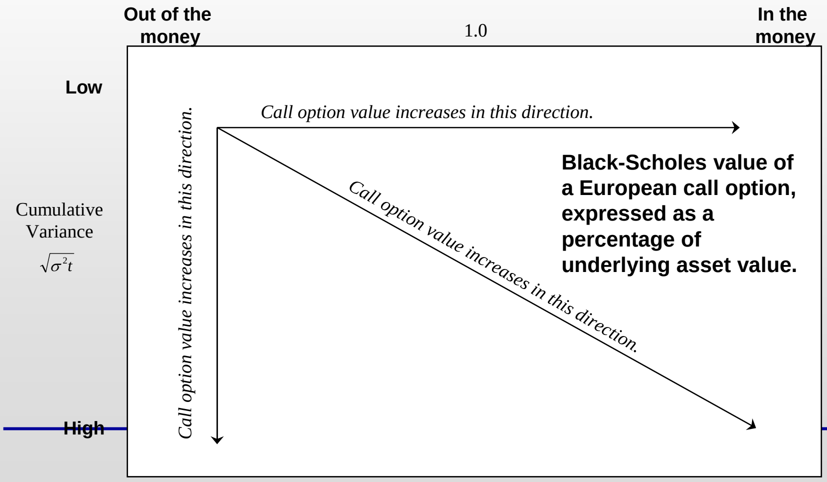
- The traditional approach to **deciding to invest in a project** is identical to deciding whether to **exercise a call option** at expiration.
- Notice that NPV_q combines four of the five determinants of option value: S, X, r, and t.
- Note also that **call option value** is an **increasing function of NPV_q**: the higher NPV_q, the higher the call value.

What About the Variance of Returns?

- **Variability, per unit of time, of returns on the project** is measured by the **variance of returns, σ^2.**
- Multiplying the variance per unit of time by the amount of time remaining gives the ***cumulative variance***, **$\sigma^2 t$.**
  - ***Cumulative variance*** is a measure of how much things could change before time runs out and a decision must be made.
  - The greater the cumulative variance, the more valuable the option.
- **Cumulative variance** and **NPV_q** are sufficient to value a European call option.
- Options (or projects) for which either σ^2 or **t** is **zero** have **no cumulative variance** and can be evaluated with standard dicounted cash flow techniques.
- When σ^2 or **t** are **non-zero**, then DCF may lead to the wrong exercise decision.

Pricing Options: NPVq and Cum. σ^2

$$NPV_q = \frac{\text{Underlying Asset Value}}{\text{PV(Exercise Price)}}$$



Example I: Value of Investment Opportunities

Issue: Should we invest in the Mark I project?

	Year					
	2009	2010	2011	2012	2013	2014
After-tax CFs	-200	+110	+159	+295	+185	0
CAPEX	250	0	0	0	0	0
Δ NWC	0	50	100	100	-125	-125
Net CFs	-450	+60	+59	+195	+310	+125

NPV at 20% = -$46.45, or about -$46 million

The Mark I

•CFs of the Mark I yield a negative NPV.
•r = 20% (because of the large exploration expenses).
•$450 M total investment required.
•NPV = -$46 Million.

→ Reject Project

The Mark II

1. Invest in Mark II can be made after 3 years

2. The Mark II **costs twice as much** as Mark I.
   Total investment = $900M

3. Total **CFs are also twice as much** as Mark I.
   PV = $463M today.

4. CFs of Mark II have a std. deviation of 35% per year.

Cash flows: The Mark II

	2009		2012	2013	2014	2015	2016	2017
After-tax CFs				+220	+318	+590	+370	0
CAPEX				100	200	200	-250	-250
Δ NWC				+120	+118	+390	+620	+250
PV@ 20%	+467	←	+807					
Investment, PV @10%	676	←	900					
Forecasted NPV in 2015			-93					

The Mark II

Translation: The Mark II opportunity is a 3 year call option on an asset worth $463M with a $900M exercise price.

Call value = $55.5M

Value of Call Option

2 parameters approach:

$$\sigma\sqrt{T} = .35\sqrt{3} = .606$$

$$\frac{S}{PV(EX)} = \frac{467}{900/(1.1)^3} = 0.691$$

Table Value = 11.9%

Call Option Value = (.119)(467) = $55.5 M

Total Value of Mark I Project

V = std. NPV + call value

= value w/o flexibility + value of flexibility

= -46+55.5

= 9.5 M

Example II

Consider a simple project that requires an investment of $100, in return for which the company would receive an asset that is currently worth $90.

There is risk associated with this asset: we assume the returns on the asset have a standard deviation of about 40% per year.

Note that the company ***can wait up to three years*** before deciding to invest.

Assume the **risk free rate is 5%.**

- Viewed conventionally, this project's NPV is $90 - $100 = **-$10**.

Example continued

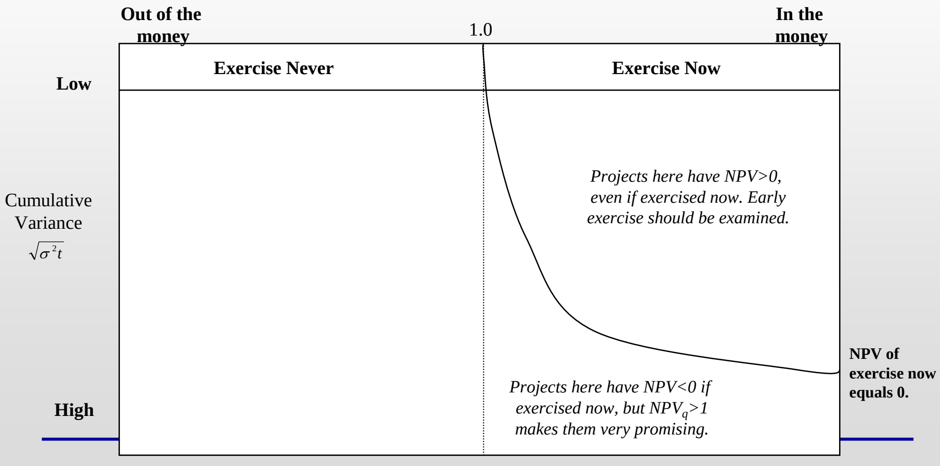
- In effect, however, the company holds **a three-year European call** with an exercise price of $100 on underlying assets worth $90.
- **NPV$_q$** for this option is $90 ÷ [$100/(1.05)^3] = 1.04.
- **Cumulative variance** is 0.40 times $\sqrt{3}$, or 0.69.
- An **option** with these characteristics is worth 28.4% of the value of the underlying asset, or 0.284($90) = **$25.56.**

Managerial Decisions: Optimal Exercise

- In this example the project had an **NPV of -$10**, but an **option value of more than $25**.
  - Are these values contradictory in nature?
  - What should the decision maker do?
- **Although NPV < 0**, the project is very promising because **NPV_q > 1**.
  - Although X > S, these two variables are relatively close to one another because S > PV(X).
  - The values are only separated by the time value of money.
  - By the end of three years, there is a good chance that the NPV will exceed zero and the option will be exercised.
  - At expiration, the option's worth is the greater of 0 or S - X.

Mapping Projects in Call-Option Space

$$NPV_q = \frac{\text{Underlying Asset Value}}{\text{PV(Exercise Price)}}$$



Mapping Projects in Call-Option Space

- All options that fall in the right half of the diagram have $NPV_q > 1$.
  - Not all of these are "in the money".
  - The NPV of an "exercise now" strategy is positive for some and negative for others.
  - Options that fall above the curve have both NPV > 0 and $NPV_q > 1$: they are **in the money**.
  - Those options that fall below the curve have $NPV_q > 1$ but NPV < 0; they are **out of the money**.
- All options that fall in the left half of the diagram have $NPV_q < 1$

Managerial Prescriptions

- Managerial prescriptions for **options with NPV_q >1.**
- At the very top are options with **no cumulative variance** (either no variance or time has run out). These options are **in the money** (NPV >0) and should be ***exercised immediately***.
- Just below these are options in the money but with **some cumulative variance**. The company should **wait, if possible**, to exercise these options.
  - Similar to American call options on a dividend-paying stock.
  - Early exercise prevents the value erosion - the holder gives up the interest on the exercise price; DM must consider the tradeoffs.

Prescriptions Continued

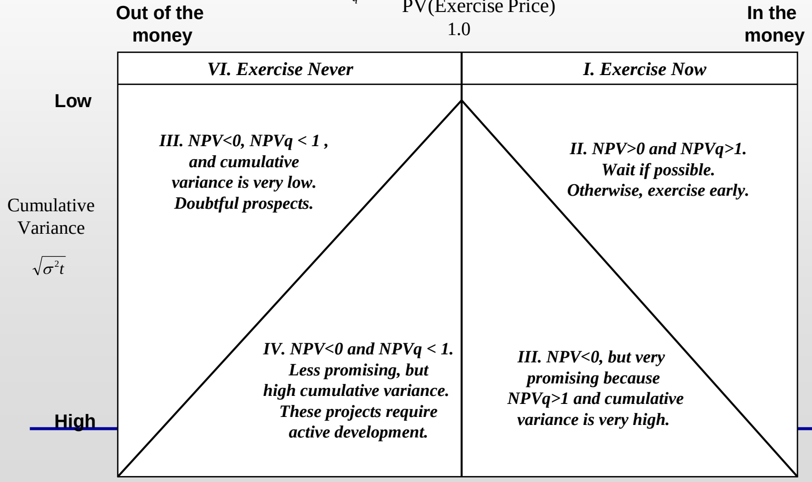
Options at the bottom of the figure are very promising because $NPV_q > 1$, even though NPV <0.

If, as time runs out, neither S nor X changes then these options will expire.

Among a large sample of such projects, we should expect some to end up **in the money**.

Managerial Prescriptions (cont'd)

$$NPV_q = \frac{\text{Underlying Asset Value}}{\text{PV(Exercise Price)}}$$



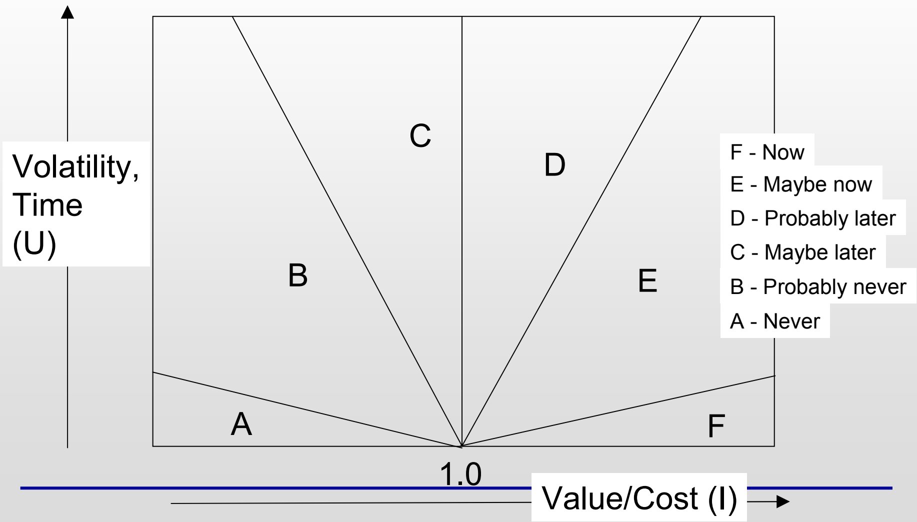
Luehrman (BS analogy) method has pluses and minuses...

- **Pluses**
  - good way to introduce **options thinking** to team by analogy to a financial option
  - conveys the idea that a *negative* or *low NPV project may be worth pursuing if there is lots of uncertainty and time* before large funds must be committed
  - The mental map is a good way of interactively demonstrating the way S/PV(X), T and σ combine to influence an option's value

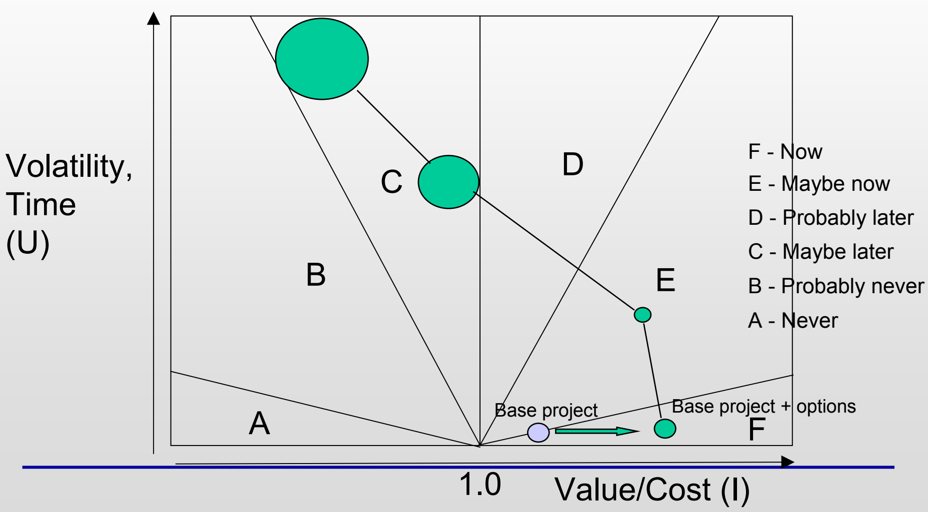
Luehrman (BS analogy) method has pluses and minuses...

- **Minuses**
  - The whole question of whether you can genuinely create a hedged portfolio is somewhat suspect
  - We have negligible information on how we might *estimate the volatility* for any given project

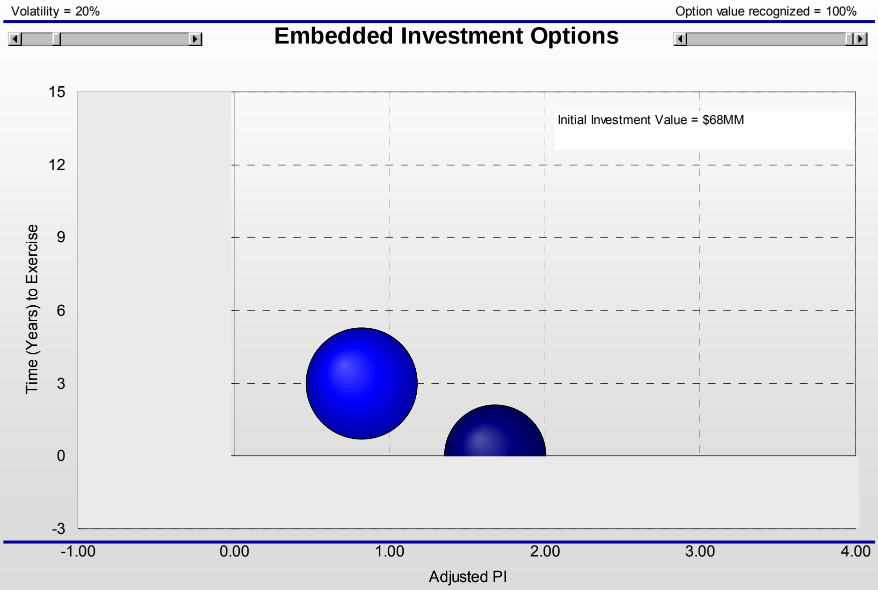
Strong plus: the Luehrman diagram (Mental map)



Options Mental Map applied to a Real Project



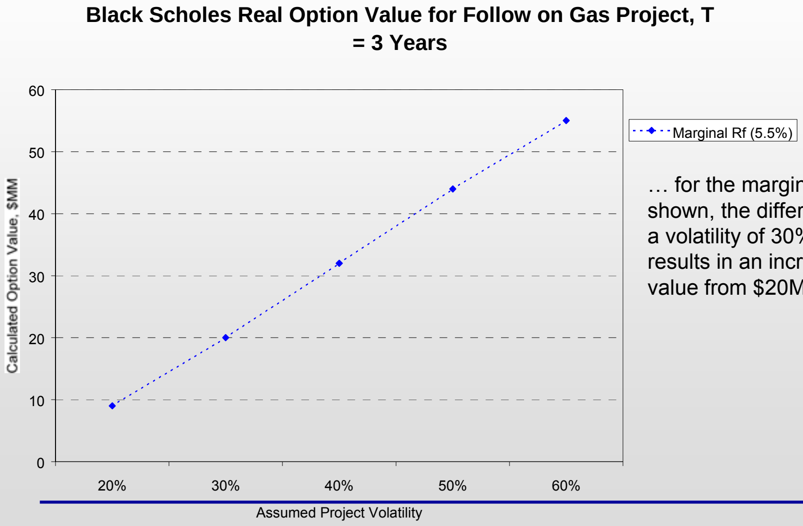
Output Using the Luehrman Frame



Strong minus: the experts aren't always helpful (on volatility)...

- Brealey and Myers: "The future value of the [project] cash flows is highly uncertain. This value evolves as a stock price does with a standard deviation of 35% per year. (Many high-technology stocks have standard deviations higher than 35%)" *(p591, Principles of Corporate Finance 5th edition)*
- Luehrman: "Take a(n educated) guess…individual stocks generally have a higher standard deviation than the market as a whole…individual projects within companies can be expected to have a still higher σ … I begin by examining a range: from 30% to 60% a year" *(p58, HBR July-August 1998)*
- Merck practice: "A sample of the annual standard deviation of returns for typical biotechnology stocks was [used] as a proxy measure for project volatility. A conservative range for the volatility of the project was set at 40% to 60%" *(p92, HBR January-February 1994)*

… and a volatility range from 30% to 60% can make a difference



… for the marginal project shown, the difference between a volatility of 30% and 60% results in an increase in option value from $20MM to $56MM

Some Practical Issues

- **Simplifying Complex Projects**
  - Most strategic projects are combinations of **assets-in-place** and **options** - often a series of nested options.
  - **Simplification** or **decomposition** makes these projects understandable and more discussible with managers.
  - Search for the primary uncertainty against which managers select.
  - Know when it is appropriate to utilize the DCF approach.
  - Construct simplifications such that the simplified project is both priceable and either dominating or dominated by the real complex project. (i.e. European vs. American call option.)

Some Practical Issues (cont'd)

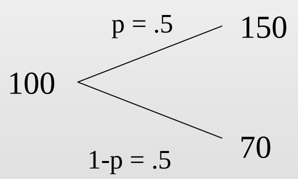
- **Estimating Volatility** - some reasonable approaches
  - ***Gather some data***. ***Historical price trends*** are one obvious approach to ***estimating volatilities***; the length, breadth and quality of data have improved significantly.
  - ***Simulate*** σ^2. Spreadsheet-based projects together with Monte Carlo simulation can be used to synthesize a probability distribution for project returns.
  - ***Make a reasonable guess***. Volatility of 20-30% per year is not remarkably high for a single project. Individual projects have higher volatilities than a diversified portfolio.
- **Interpreting Results**
  - Sensitivity analysis is essential.
  - Layer complexity back into the problem.

II) Binomial Approach

Binomial Approach: one-period binomial tree

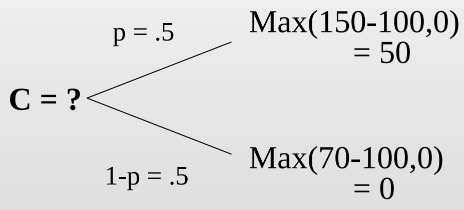
PV(stock price)

T = 0 T = 1



Option Tree

T = 0 T = 1



Volatility = 40%, Exercise price = 100, Risk-free rate = 5%

Replicating portfolio approach

Hedge ratio = Delta $= \frac{\Delta C}{\Delta P} = \frac{50-0}{150-70} = .625$

	P = 70	P = 150
Call option	0	50
.625 shares of stock	43.75	93.75
Repayment + interest	-43.75	-43.75
Total payoff	0	50

Value of call = value of .625 shares of stock - loan

= (.625* 100) - PV(43.75) = $20.83

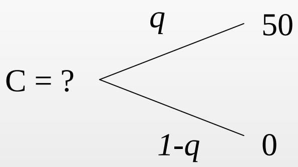
Risk-adjusted Probabilities Approach

$$C = \frac{q(50) + (1-q)(0)}{1.05}$$

1) Risk adjust cashflows downward

2) Use a risk-free rate

Option Tree

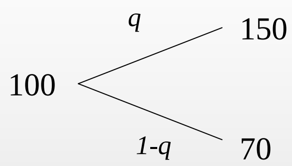


How do we get *q* ?

Risk-adjusted Probabilities Approach

Risk Adjusted Probabilities (*q*, *1-q*)

We can use the underlying asset to derive the risk-adjusted probabilities, *q*



$$PV = \frac{q(uPV) + (1-q)(dPV)}{1+r_f}$$

$$q = \frac{(1 + r_f - d)}{u - d}$$

$$\frac{105}{1+.05} = \frac{q(150) + (1-q)(70)}{1+.05}$$

$$q = \frac{(1 + .05 - .7)}{1.5 - .7} = .437$$

$$C = \frac{(.437)(50) + (1-.437)(0)}{1.05} = 20.83$$

Binomial Lattice Implementation of OPT

- We seek to find the **theoretical value of oil reserves** which have been discovered but not yet developed.
- The theoretical value corresponds to the **break-even cost level** - an amount that if spent acquiring the reserves would yield neither a profit or a loss.
- Begin by considering the *movements over time* in ***the price of developed reserves***. . .

Binomial Lattice Implementation of OPT

- Assume the **market price of developed reserves (in place) today is $1.00/bbl**.
- We first need to know what the market expects the price to be in, let us say, one year.
- The owners of developed reserves derive their required return from two sources, ***operating payouts*** and ***capital gains***.
- Consequently, expected appreciation in reserve values must be just sufficient, when added to operating payouts, to compensate owners for their investment.
- Thus, by observing **the payout rate**, we can infer its complement: **the expected rate of price appreciation**.
- Fortunately, payout rates can be estimated from current commodity quotations.

Consider the following example . . .

Binomial Lattice Example (cont'd)

- Assume today's price of developed reserves is $1.
- Let the **risk-free interest rate, *r*,** be set at **4%** (in real terms).
- Let the **payout rate, δ,** on developed reserves be equal to **13%.**
- Then the expected price of developed reserves in one year's time must **increase by exactly (r - δ)** relative to today's price and the expected price in one year must be equal to 1+ (r - δ) or $1.00 + $0.04 - $0.13 = **$0.91**.
  - If **the expected price were any higher** then it would pay investors to hold larger reserves rather than offer them for sale, which would drive up the current prices.
  - Conversely, if **the expected prices were any lower**, the current price would be driven down.
  - **Today's equilibrium prices** can therefore be used to read market expectations regarding future prices.

Binomial Lattice Example (cont'd)

- What about **price volatility**?
- Suppose we have determined the volatility of oil is **22%.**
  - Current market price of developed reserves, S, will **either move up to 1.22** or current market price of developed reserves, S, will **move down to 0.78**.
  - Let the probability of a price increase be *p*.
  - Then the expected price in one year must be

    $1.22*p* + $0.78(1-*p*)
- Recall that we have shown that the expected price is equal to **$0.91**.
- Therefore, $1.22*p* + $0.78(1 - *p*) must equal $0.91.
- Solving this equation, we find that ***p* = 0.2955** and **(1 - *p*) = 0.7045** - our ***risk-adjusted probabilities***.

Option Value for One Time Period

What is the value today of an option to acquire developed reserves in one year's time?

- Developed reserves are acquired by paying **the development cost** - this is the **exercise price** of the option.
- Assume the **development cost (X)** is \$1.00/bbl. We now have an "at the money" option. The value of immediate exercise is \$0.

Let's investigate the two possible outcomes of holding the option until the end of the year.

- If the price moves up, then the value of the option is \$1.22 - \$1.00 - \$0.22 and if the price has moved down, then the value of the option is \$0.
- The expected value in one year is: (\$0.22 x .2955 + \$0 x 0.7045) = \$0.0650.
- Discounting back to the present at r, we find the **Option Value (OV)** equal to \$0.0650/1.04 = **\$0.0625**.

Option Value

- Since **price movements** occur many more times than one year, let us divide the year into a **number of shorter periods**, each of length ***dt***.
- For example, we now will say the ***price can change at the end of each quarter***.
- To get the **standard deviation** of the rate of quarterly price change we divide by the square root of the number of periods in one year,
- The characteristics of the basic element in the lattice now become:

$$0.22 \div \sqrt{4} = 0.11$$

Upward price change = +11%   Downward price change = -11%

Expected price change = $(r - \delta)dt$,
= (0.04 - 0.13) x 0.25
= -2.25%

Binomial Lattice - Multiple Periods

- But the expected price change is also equal to

  $\$1.11p + \$0.89(1 - p) - \$1.00$
- Therefore, $\$1.11p + \$0.89(1 - p) - \$1.00 = -\$0.0225$.
- Solving this equation, $p = 0.3977$, and $(1 - p) = 0.6023$.
- Generally, as term to expiry is subdivided into finer increments, we would use ***continuous compounding*** rather than discrete leading to these results.

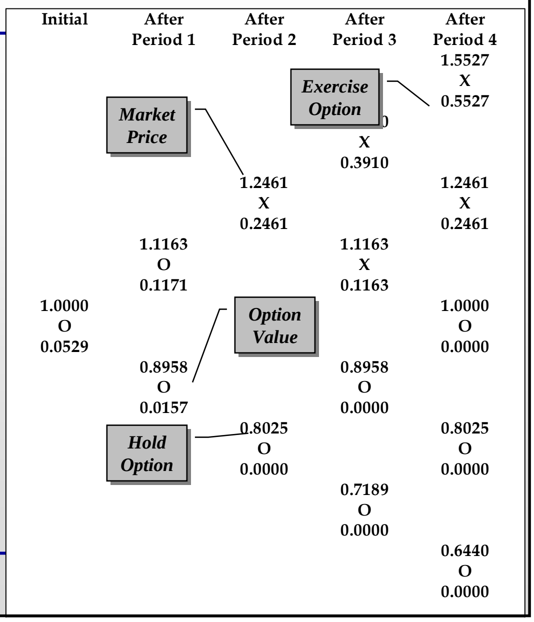
  Upward price change = +11.63 Downward price change = -10.42%

  Expected price change = $\exp[(0.04 - 0.13)dt] - 1 = -2.22\%$

  $p = 0.3716$ and $(1 - p) = 0.6284$
- These results give us the following binomial lattice, including the market price and option value at each node. . .

Binomial Lattice - Multiple Periods (cont'd)

- The number above each node is the market price of developed reserves.
- The number below each node is the value of the option at that point in the lattice and incorporate the impact of preceding price changes.
- The value of the option is found by starting at the extreme right and working backward.
- For example, at the end of period 3, the value of the option will be the discounted EV of the payoffs we have assigned to the end of period 4, unless a greater value can be obtained by immediate exercise.
- "X" indicates the option should be exercised (proceed with development) and "O" indicates the firm should hold the option (or let expire in the final period).



Comments on Input Data for OPT

- **Payout Rate (delta δ).**
  - Recall that it represents the difference between the investor's required ROR and the rate of appreciation earned on developed reserves.
  - Keeping the option open represents an opportunity cost to the firm in that it foregoes the operating income associated with exercise.
  - Assumes that the prices of developed reserves and oil at the wellhead must appreciate at the same rate. Enables us to **use commodity futures prices**, which reflect the rate of payouts generated by crude oil inventories, to estimate the payout rate on developed reserves.
  - Computation of payout considers convenience yields, holding costs and current risk free interest rate.

Comments on Input Data for OPT

- Suppose **futures contract** for oil delivered in March is quoted at $20/bbl and the June contract at $19.70/bbl.
- The net holding cost is -1.5% over the three month period (-6.0% annualized).
- If T-Bill are at 7.0%, then our estimate of delta would be 7.0 - (-6.0) = 13.0%.
- In this market, buyers pay a premium for early delivery.
- Alternatively, if futures prices are increasing with term, the our estimate of delta would be less than the risk-free rate.
- Aside from short-term market fluctuations, the "normal" value of holding cost is 0 and net convenience yield is approximately equal to the r_f nominal interest rate.

Comments on Input Data for OPT (cont'd)

- **Price of Developed Reserves (S)**
  - Identify traded developed reserves of the same type and quality subject to the same or equivalent fiscal regimes.
  - Data sources such as Strevig and Associates (U.S. transactions) and John S. Herold, Inc. for international transactions.
  - Utilize quoted companies whose assets are primarily producing reserves, after making allowances for debt holders.
- **Volatility of Developed Reserves (sigma)**
  - Often derived from the same source as the developed reserves price.
  - For the period 1985-1989 Strevig and Associates reported a volatility of 22.8% on an annualized basis for U.S. reserve prices.
  - For the same period, API reported observed volatility for WTI wellhead prices of 21.7%.

Comments on Input Data for OPT (cont'd)

- **Real Risk-Free Interest Rate (r)**
  - This should be the rate, with inflation removed, at which risk-free investment are available in the capital markets.
  - In the U.S>, yield on a 3-month T-Bill, adjusted for inflation.
- **Present Value of Development Costs (X)**
  - Construct an appropriate development scenario w/investments.
  - Consider after-tax expenditures.
- **Development Delay (delay)**
  - Time in years between onset of development expenditures and first production.
- **Exploration Lead Time (expl. lead)**
  - Time in years required to complete exploration of a prospect before a decision to undertake development can be made.

Four steps valuation approach

In order to calculate a project's value with flexibility we use a four step valuation approach

Calculate static NPV using DCF approach	Frame the problem by building a decision tree	Model uncertainty & calculate volatility	Calculate Dynamic NPV
• Build DCF spreadsheet model. • Discount future cash flows at WACC. • Sum PV of cash in and out flows to arrive at a NPV. • Interpret results.	• Identify alternatives from which to choose. • Identify critical uncertainties that affect the outcomes of the decisions. • Represent alternatives and uncertainties using a decision tree.	• Assess the likelihood of the various technical uncertainties. • Estimate volatility arising from economic uncertainty using Monte Carlo simulation.	• Calculate expected monetary values (EMVs) using the appropriate valuation method. • Calculate dynamic NPV using backward recursion. • Determine if the decision is sensitive to changes in probabilities, valuation of outcomes, or other assumptions.