Economic Risk and Decision Analysis for Oil and Gas Industry
CE81.9008

School of Engineering and Technology
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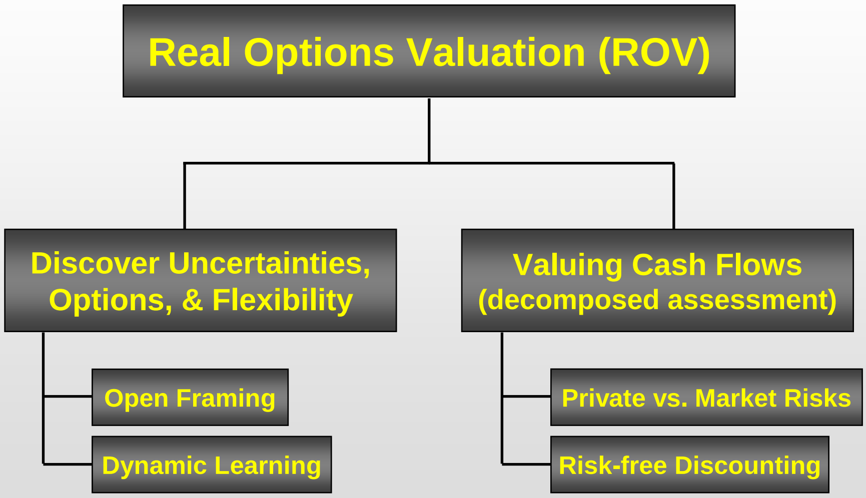
Real Option Valuation (ROV)

Two dimensions of Real Options Valuation



Modeling and Valuing Real Options

Two sets of issues:

- **Valuing risky cash flow streams**
  - What are the uncertain cash flows worth?
  - How should we forecast and discount cash flows?
- **Modeling flexibility**
  - What options are associated with a project?
  - How can we model these flexible options?

Modeling cash flows

- In the **case of certainty**, the cash flows are discounted to reflect the time value of money
- In the **case of risk**, the cash flows are adjusted to reflect the risk aversion of the company and/or the market
- In general, there are three ways to make the adjustment for risk:
  - **Modify the discount rate** by adding a risk premium (risk-adjusted)
  - **Modify the probabilities** (risk-neutral)
  - **Modify the outcomes** by using Certainty Equivalents (utility functions)
- The first two approaches can be related to market values, while the third is related to personal (or company) values

Modeling Flexibility

- **Decision trees** are the most commonly used approach to modeling flexibility
- Requires careful thought and modeling of complexity
- The decision tree can become extremely large, and other techniques (dynamic programming) may be required for computational purposes
- **Combines decision analysis** with **pricing approaches from options pricing theory**

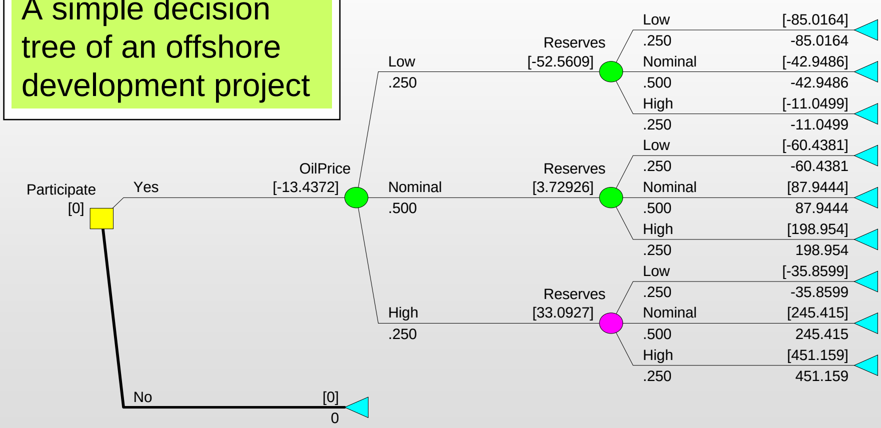
Other useful ideas from decision analysis

- ***Value of information*** —obtaining new information before making a decision to start a product is often an obvious project "option".
- Quantifying uncertainty through the use of subjective probabilities may be important in many applications.
- Assessing a ***risk tolerance*** may be particularly relevant for smaller firms or for risk averse managers—can be used as the basis for determining risk sharing strategies.

Modeling Flexibility with Decision Trees

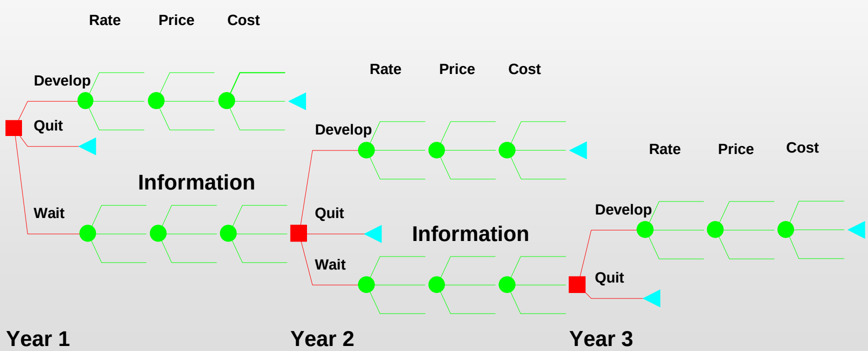
Decision analysis may overlook options

A simple decision tree of an offshore development project



Defer Option

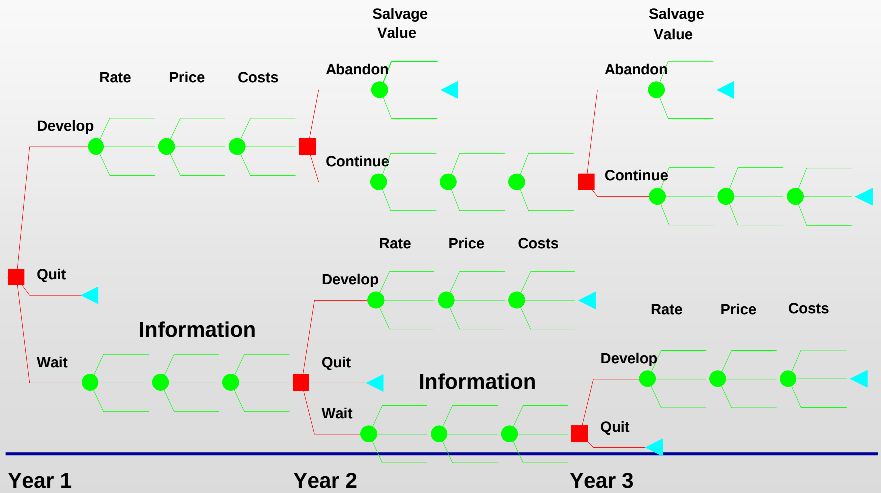
Add Flexibility to Defer Project (Call Option)



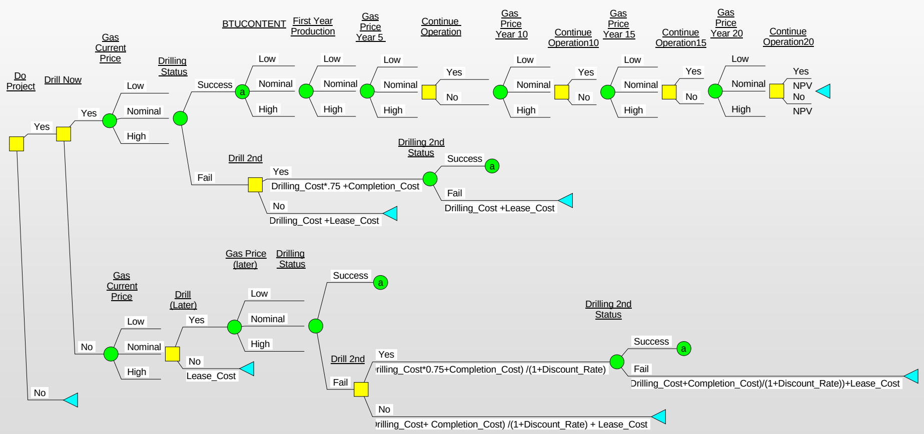
Lease expires in 2 years

Abandon Option

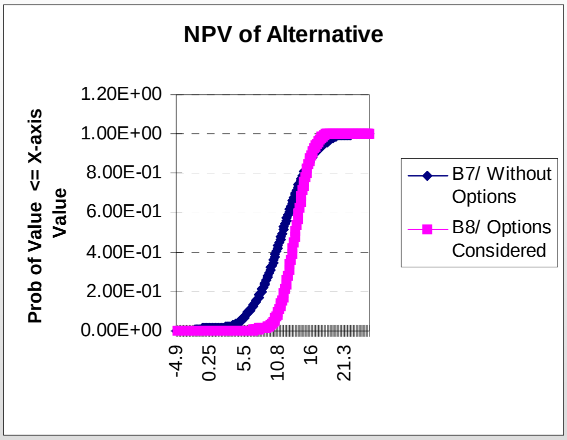
Add Flexibility to Abandon Project (Put Option)



A decision tree with options



Options leads to a reduction in downside risk



Example: the option to abandon after a test well may reduce the downside risk

"Free" options can only increase the value of an alternative

Benefits of Modeling Flexibility

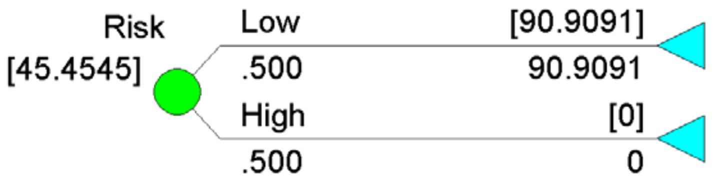
- Improving overall project valuation.
  - Options are difficult to value in an intuitive manner.
  - Modeling flexibility improves the accuracy of valuations on complex, long-term projects.

Valuing Risky Cash Flow Streams

Note: Most of these examples are from T. Copeland and V. Antikorov, Real Options, Texere, 2001.

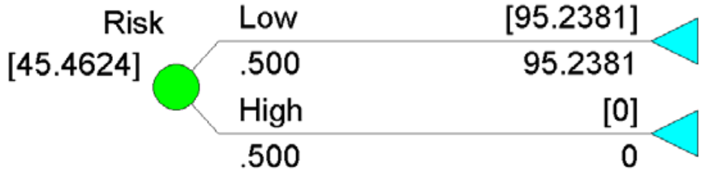
How should we adjust for risk?

Discount the outcomes?



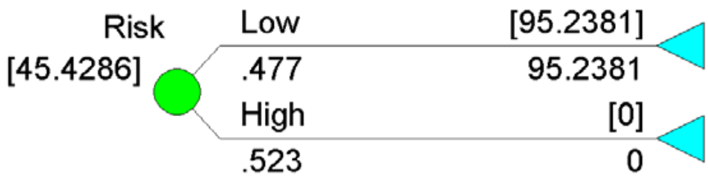
The discount rate is 10%

Discount at the risk free rate and use a risk tolerance?



The discount rate is the risk free rate of 5% and the risk tolerance is $525

Discount at the risk free rate and change the probability?



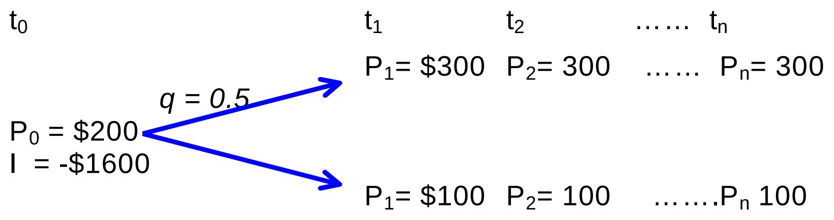
The discount rate is the risk free rate of 5% and the probability of success is .477

Example: A Project With a Deferral Option

Invest in $1,600 project now, or defer until the end of the year

- Irreversible decision
- Yearly returns are $200 now
- Will go up to $300 or down to $100 at end of year, permanently, with equal chance of either
- Cost of capital is 10%

Project With Deferral Option



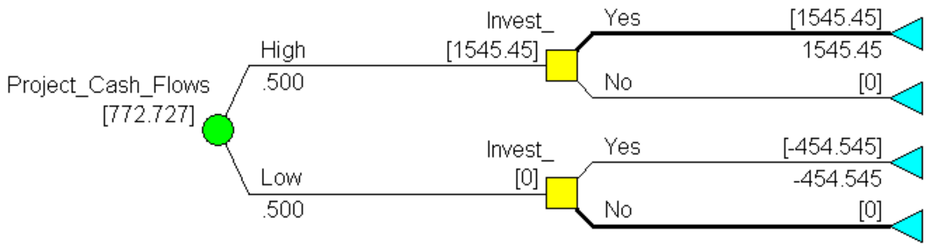
Standard NPV Analysis

- Expected annual cash flows = $200

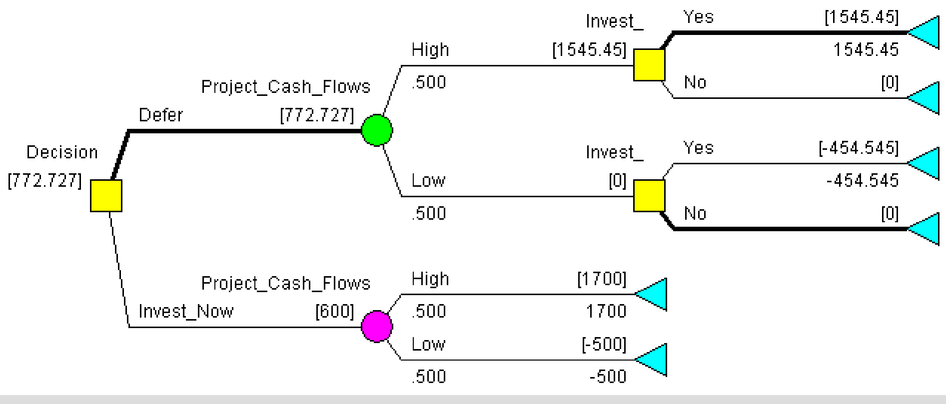
$$NPV = -1{,}600 + \sum_{t=0}^{\infty} \frac{200}{1.1^t} = -1{,}600 + 2{,}200 = 600$$

The Option to Defer

(-1600+300+Project_Cash_Flow/.1)/1.1



The Complete Analysis



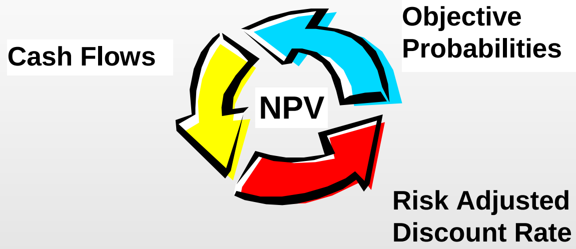
Option Value

- The opportunity to invest now has a NPV of $600
- Since NPV>0, DCF analysis suggests that the firm should take up this project
- But assume the firm has an option to defer the decision to invest until next year, when the price uncertainty will be resolved
- This alternative has a value of $772.73

Option Value

- The increase in value from $600 to $772.73 is due to the presence of the option to defer.
- This option has a value of $772.73 - $600 = 172.73.
- *This assumes that 10% is the appropriate discount rate, but where did that come from?*

An Important Relationship

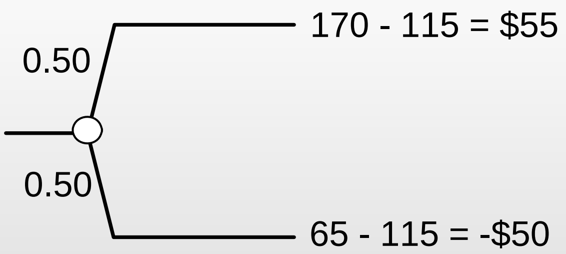


- The four variables above must be consistent with each other
- Given any three of them we can determine the fourth

NPV, Decision Trees and ROA

- Should you commit now to a project that will cost $115 next year, but with an equal chance of cash flows of $170 or $65?
- The risk free rate is 8%.
- Note that we have two pieces of information: the payoffs and the probabilities.
- We need a third one (the discount rate) in order to find the NPV of this project.

NPV, Decision Trees and ROA



- One way to determine the **discount rate** is to find a "twin" security that has a known market price.

"Twin Security" method

- A twin security is one whose cash flows (payoffs) are perfectly correlated with the ones of the project in each and every state.
  - If this twin security exists, then its risk (and thus its discount rate) is the same as the project.
  - From market information we can determine the implied discount rate of this security (and thus, of the project)
- Suppose we find a security that will pay $34 or $13 (1/5 of the project payoffs) and verify that it is selling for $20

Traditional Approach: Find a "Twin Security"

- With this information we can determine the implied discount rate of this security

$$\$20 = \frac{.5(\$34) + .5(\$13)}{1 + k}$$

$$k = 17.5\%$$

- Since this twin security has the same risk as the project, the risk adjusted discount rate must also be the same

Solve for NPV of the Project

$$PV = \frac{.5(\$170) + .5(\$65)}{1.175} = \$100$$

Discount cost of $115 at the risk free rate

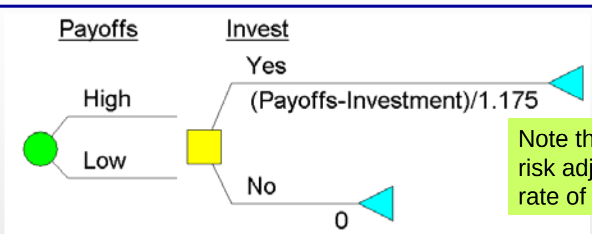
$$\$115 / 1.08 = \$106.48$$

Calculate NPV = $100-$106.48=-$6.48

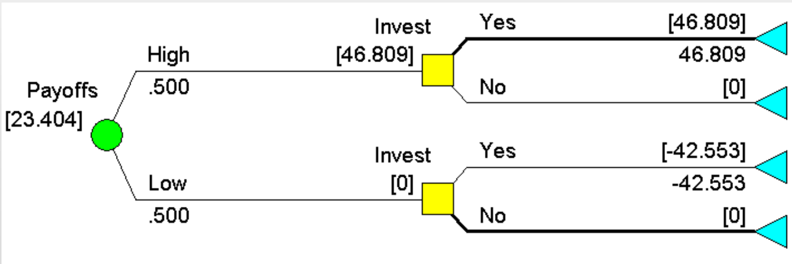
NPV, Decision Trees and ROA

- Now consider this project with an option to defer
- This means that for a price of C_0, we can wait until the end of the year before committing to the project
- This allows the uncertainty to be resolved before we make our decision
- What is the value of the project now?

Naïve Decision Tree Analysis



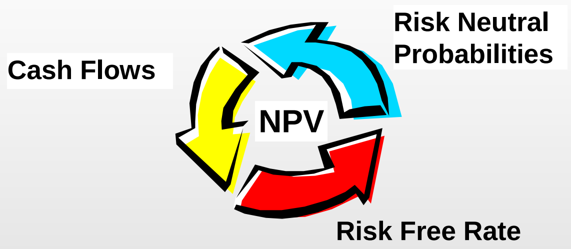
Note the use of the risk adjusted discount rate of 17.5%



Why is this wrong?

- The value of the deferral option is \$23.40-(-\$6.48) = \$29.88 using this approach.
- However, the cash flows of the deferral option are not perfectly correlated with those of the project, so the 17.5% discount rate is wrong.
- Previously, the payoffs were \$175 and \$65, and the twin security had payoffs of \$34 and \$13.

Risk Neutral Probabilities method



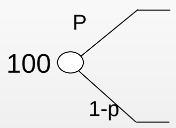
- If we discount the cash flows with the risk free rate instead of the risk adjusted discount rate, we would arrive at a different NPV

Risk-neutral Probabilities

- On the other hand, we could maintain the same NPV as before if we simultaneously adjust the probabilities.
- These “adjusted” probabilities, the one that give us the same NPV as before, are named risk neutral probabilities, as opposed to the real “objective probabilities”.

Risk-neutral Probabilities

- Ex: using the assumption, our twin security is the project itself, without the option:



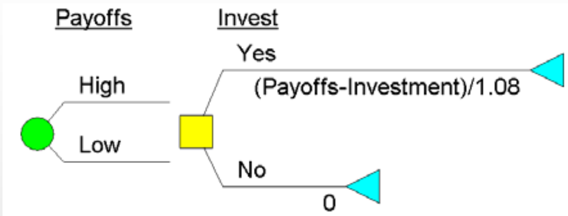
$170 = 100\,u$

$65 = 100\,\mathrm{d}$

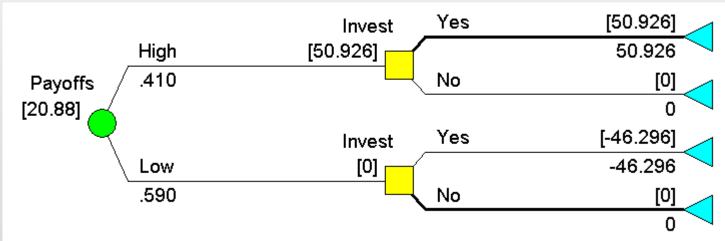
$u = 1.70$ $d = 0.65$ $r = 8\ \%$

$$p = \frac{(1+r)-d}{u-d} = \frac{1.08-.65}{1.7-.65} = 0.410$$

Naïve Decision Tree With Risk Neutral Probabilities



Note: the naïve decision tree with risk neutral probabilities provides the correct answer



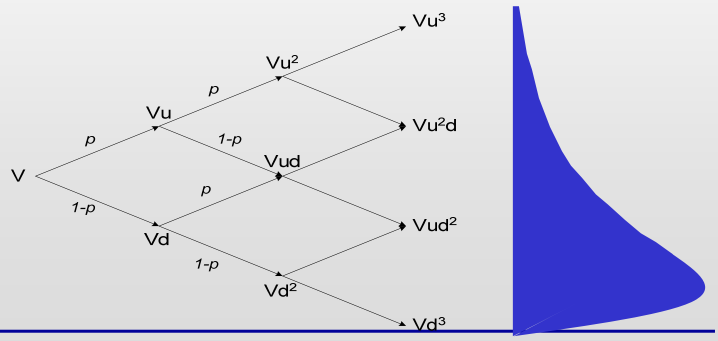
Comments

- Use of replicating portfolios is technically correct, but may be tedious in practice
- Use of risk neutral probabilities with decision trees may provide a practical alternative for calculating project valuations

Black and Scholes and the Binomial Lattice Model

Binomial Approximation

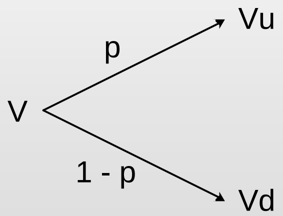
- A lognormal stochastic process can be modeled with a binomial lattice.
- This allows us to use a simpler, discrete model



Binomial Model

- This requires the following parameters:

$$u = e^{\sigma\sqrt{\Delta t}} \quad and \quad d = e^{-\sigma\sqrt{\Delta t}}$$



$$p = \frac{(1+\mu) - d}{u - d}$$

$$p = \frac{e^{\mu t} - d}{u - d}$$

Lattice Models

- Both the **Binomial Lattice model** and the **Black and Scholes formula** can be used to value options.
- The advantage of the lattice approach is that is allows us to solve a much wider range of problems than B&S.
- Although the B&S approach can be used to value financial options, it is not appropriate for real options, which are much more complex than financial options.
- B&S requires a very restrictive set of assumptions that do not usually hold for project valuation
- We use the lattice to model the uncertainty of the project value and a decision tree to model project real options.

Comparison Chart

Black and Scholes	Real Option Lattice and Tree
European Option	Both European and American
Single source of Uncertainty	Allows Multiple Uncertainties (Rainbow Options)
Single Option	Allows Multiple (Compound) Options
No dividends	Allows Dividends
Current Stock Price	PV of Project
Underlying (Stock Price) follows GBM	Underlying (Project Value) follows GBM
Exercise Price	Investment Cost
Risk free rate	Risk free rate

Steps for Solving Real Option Problems

Solving the Real Options Problem

- The valuation of a project with Real Options can be solved with a four step process

  Step 1: Model the underlying asset

  Step 2: Build the corresponding DPL model

  Step 3: Model the project's real options.

  Step 4: Solve the Decision Tree

Solving the Real Options Problem

- **Step 1: Model the underlying asset**
  - The underlying asset it the project without the options
  - We assume that the project value follows a Geometric Brownian Motion stochastic process.
  - We use a binomial lattice to model the value of the project.
  - We use the Cox, Ross and Rubinstein (CRR) binomial approximation model to get the necessary parameters for this.

Solving the Real Options Problem

- **Step 2: Build the corresponding model**
  - We implement the model of the lattice for the GBM process of the underlying project.
  - At this point the model consists only of binary chance nodes
  - Note that CRR model is recombining
  - We compute the risk neutral probabilities and as a check, and verify if our model provides the same PV as we had initially.
  - The lattice shows the evolution of the project value in time.

Solving the Real Options Problem

- **Step 3: Model the project's real options.**
  - Once we have modeled the stochastic process for the underlying asset, we can now model the project's real options.
  - This is done by inserting decision nodes that represent the opportunities the manager has to maximize the value of the project in each period.
  - These decision nodes represent the projects real options, or management flexibility.
  - Since we are using risk neutral probabilities, no further adjustment to the discount rates are necessary.

Solving the Real Options Problem

- **Step 4: Solve the Decision Tree**
  - The corresponding decision tree can get quite large at times, specially if the project has many periods and/or many options.
  - Note that a ten period project with an American type option will have 2^{10} end nodes.
  - These very large trees will slow down the computation speed considerably.
  - DPL has computational shortcuts that allow us to solve this very large trees with a fraction of the usual time at the cost of a small loss in precision.

Example: Problem 4.1 (C&A 4Q1.da)

- A project runs for two years and has a value of $30M.
- Initial investment is $20M and project volatility is 15%.
- At the end of the second year there is an option to expand.
- This would require an investment of $5M and would increase the value of the project by 20%.
- The WACC is 12% and the risk free rate is 5%.

Solution: Step 1

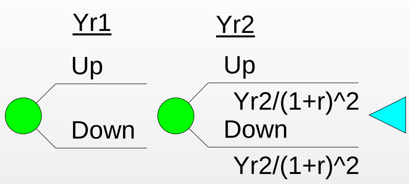
- Model the underlying asset, which is the project without the option
- To do this we first compute *u*, *d*. and the risk neutral probability *p*:

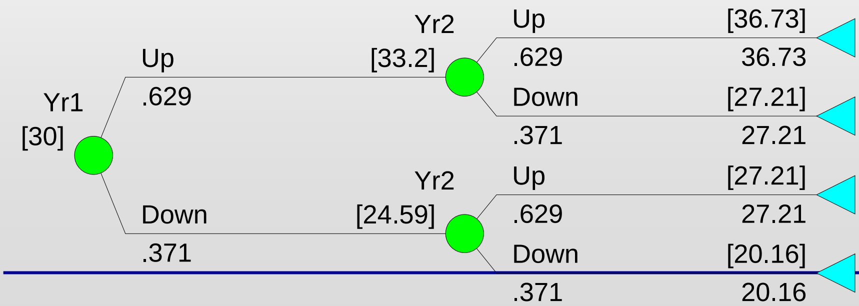
$$u = e^{\sigma\sqrt{t}} = e^{0.15} = 1.16$$
$$d = 1/u = 0.86$$

$$p = \frac{(1+r)-d}{u-d} = \frac{1.05-0.86}{1.16-0.86} = 0.63$$

Solution: Step 2

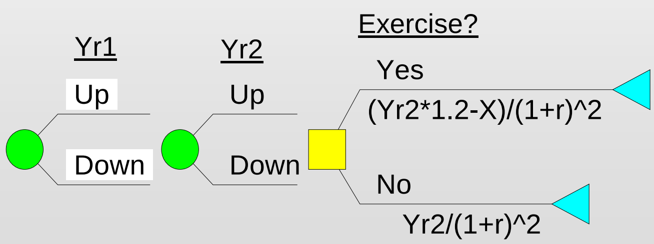
- Build the corresponding model
- The tree shows the evolution of the project value in time



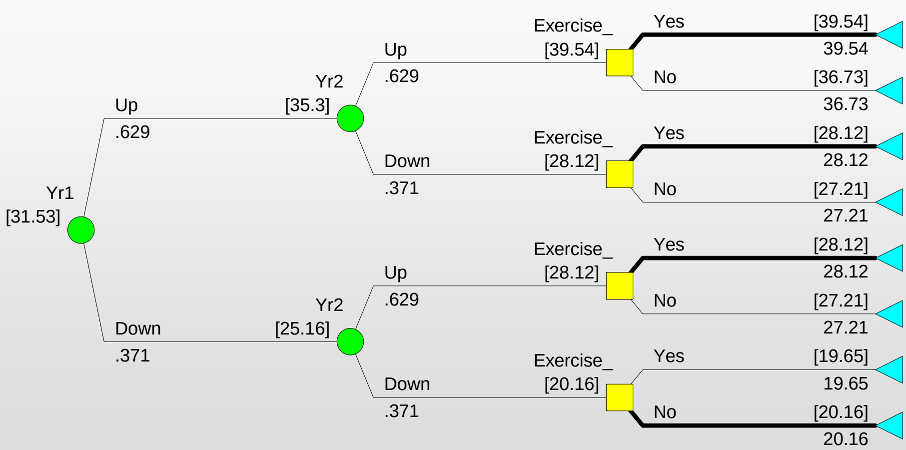


Solution: Step 3

- Once we modeled the stochastic process for the underlying asset, we can now model the project's real options.
- To do this we insert the option as a decision node in year two.



Solution: Step 4 – Solve the Tree



Ex: Question 9, Chapter 4

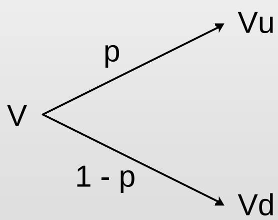
- A company is considering two alternatives to enter a foreign market.
- **A**: Greenfield approach
  - Build its own facilities at a cost of $43 million
- **B**: Buy local producer
  - Purchase local producer sometime during the next two years at a cost of $40 million
- Risk free rate is 5%

(A) Greenfield Alternative

- Present Value of Project is $45 million
  - How was this arrived at?
- Project Volatility is 17 %
  - What exactly does this mean?
  - How was this value arrived at?
- Required Investment is $43 million
  - What is the Project's NPV?
- Project can be sold in two years for $20 million
  - What kind of option is this?
  - What is the value of this option?

Solution

- **Step 1:** Model the underlying asset (the project without the option) using a binomial lattice.
- This is done by determining the binomial parameters below



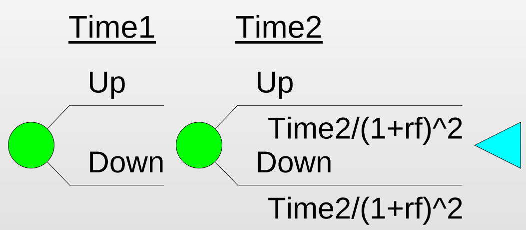
$$u = e^{\sigma\sqrt{\Delta t}}$$

$$d = e^{-\sigma\sqrt{\Delta t}}$$

$$p = \frac{(1+r)-d}{u-d}$$

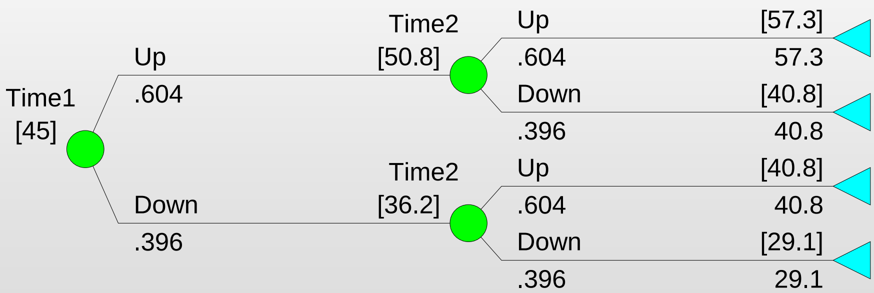
Solution

- **Step 2**: Build the corresponding model



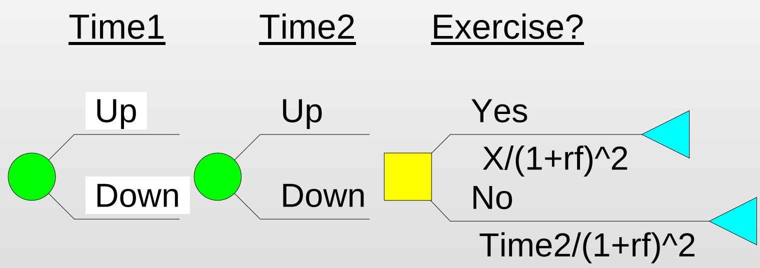
Solution

- The resulting decision tree shows how the underlying project value evolves in time.



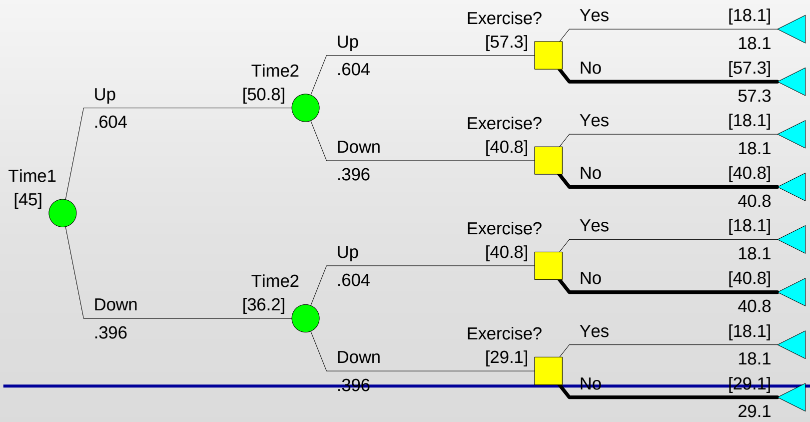
Solution

- **Step 3**: Incorporate the option to sell (Put option) by adding a decision node at year 2.



Solution

- **Step 4**: Compute the value of the option. In this case the decision tree shows us that this option is worthless – it will never be exercised.

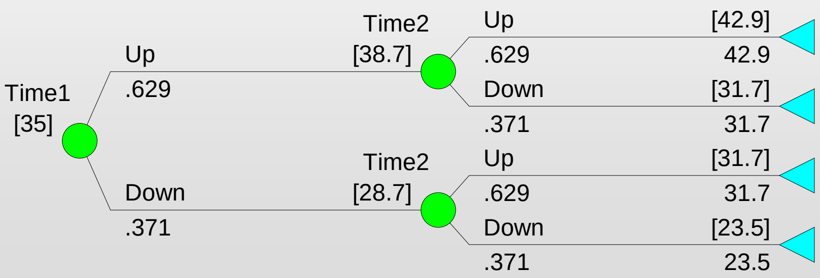


(B) Buy Local Producer Alternative

- Present Value of Project is $35 million
- Project Volatility is 15%
- Purchase price (the required investment) is $40 million over the next two years.
- The firm has a call option on the local producer

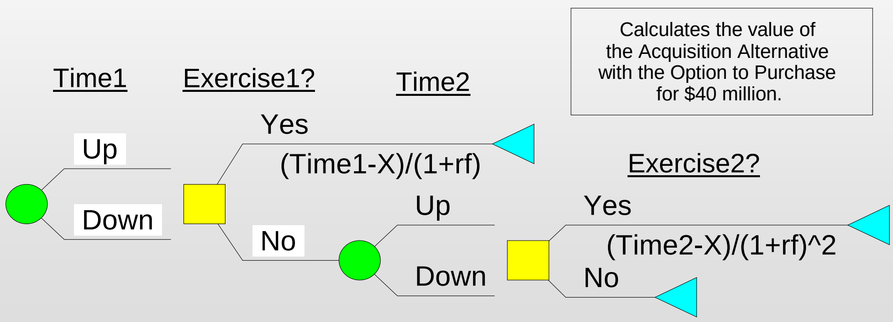
Solution (file C&A 4 Q9b.da)

- **Step 1:** Model the underlying asset (the project without the option) using a binomial lattice.
- **Step 2**: Build the DPL model. The model will be same as before but the parameters will be different.



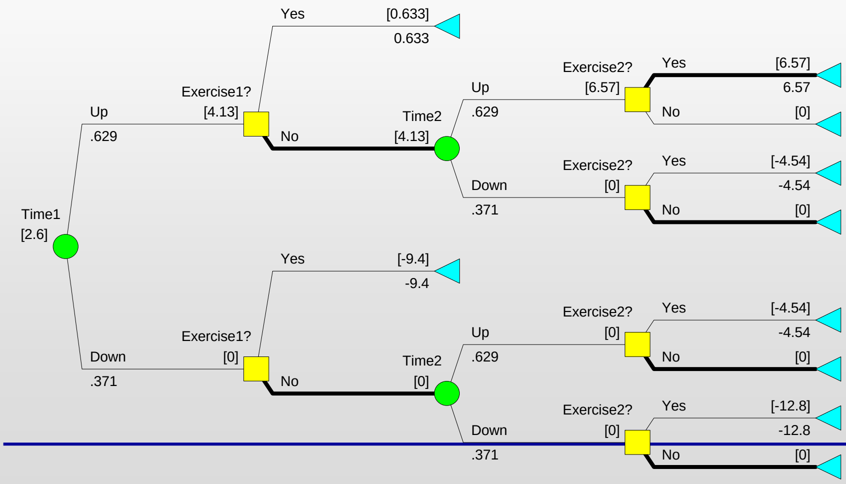
Solution

- **Step 3**: Incorporate the option to buy by adding decision nodes in year 1 and 2.(American Option)



Solution

- **Step 4**: Compute the value of the option.



Results

- The Greenfield alternative had a NPV of $2 million. The put option created no additional value.
- The "Buy local producer" alternative has a NPV of zero as the cost of $40 million is greater that the current value of $35 million. But the fact that this decision can be postponed till year two makes this option be worth $2.6M.
- This is greater than the $2M value of the Greenfield alternative, so the firm should go for alternative B.