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Comparing Three Methods for Evaluating Oil Projects: Option Pricing, Decision Trees, and Monte Carlo Simulations

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Abstract

Option pricing, decision trees and Monte Carlo simulations are three methods used for evaluating projects. In this paper their similarities and differences are compared from three points of view:- how they handle uncertainty in the values of key parameters such as the reserves, the oil price and costs; how they incorporate the time value of money and whether they allow for managerial flexibility. We show that despite their obvious differences, they are in fact different facets of a general project evaluation framework which has the static base case scenario as its simplest form. Compromises have to be made when modelling the complexity of the real world. These three approaches can be obtained from the general framework by focussing on certainty aspects.

Introduction

Option pricing, decision trees and Monte Carlo simulations are three methods for evaluating oil projects that seem at first radically different. Option pricing comes from the world of finance. In its most common form, it incorporates Black and Scholes model for spot prices and expresses the value of the project as a stochastic differential equation. Decision trees which come from operations research and games theory, neglect the time variations in prices but concentrate on estimating the probabilities of possible values of the project, sometimes using Bayes theorem and prior and post probabilities. In their simplest form, Monte Carlo simulations merely require the user to specify the marginal distributions of all the parameters appearing in the equation for the NPV of the project.

All three approaches seek to value the expected value of the project (or its maximum expected value) and possibly the histogram of project values but make different assumptions about the underlying distributions, the variation with time of input variables and the correlations between these variables. Another important difference is the way they handle the time value of money. Decision trees and Monte Carlo simulations use the traditional discount rate; option pricing make use of the financial concept of risk neutral probabilities. One of the difficulties in estimating the value of a project is that it is usually a non-linear function of the input variables; for example, because tax is treated differently in years when a profit is made to loss-making years. Starting out from the NPV calculated on the base case, this paper shows how Monte Carlo simulations and decision trees build uncertainty and managerial flexibility into the evaluation methodology. Option pricing starts out by defining the options available to management and then models the uncertainty in key parameters. In fact the three approaches are different facets of a general framework; they can be obtained from it by focussing on certain aspects and simplifying or ignoring others.

First Step – NPV for the base case

The first step in evaluating any project is to set up a base case scenario and to calculate its NPV using the parameter values that have been agreed upon. This assumes that the values of the input parameters are known:

original oil in place,
decline rate,
oil prices for each year,
costs for each year,
discount rate,
tax structure, etc.

Going further, it assumes that the scenario and the project life are fixed and that management will not intervene irrespective of changes in the oil price, new technological developments etc. In the real world, the values of the variables are uncertain and management does react to changing situations, so it is vital to incorporate these two factors into the evaluation procedure.

Ideally, the distributions of all the variables should be modelled together with the correlations over time and the complex links between variables, and this for a wide variety of management scenarios. But as Smith and McCardle[1] have demonstrated, this rapidly becomes very unwieldy and the sheer complexity of the situation forces us to compromise. Monte Carlo simulations, decision trees and option pricing address this problem in different ways, each focussing of certain aspects and simplifying or ignoring others. Starting out with Monte Carlo simulations we show how these methodologies build up from the NPV equation in the base case incorporating uncertainty on the input variables and, for decision trees and option pricing, incorporating managerial flexibility.

Monte Carlo Simulations

Monte Carlo simulations concentrate on the uncertainty in parameter values, using statistical distributions to model this. The equation below shows the NPV for a simplified oil project with a life of N years, and using a discount rate i.

$$NPV = -Capex + \sum_{1}^{N} \frac{Cash_{n} \text{ after tax for } n^{th} \text{ year}}{(1+i)^{n}}$$

The annual cashflow is expressed in terms of its key parameters, typically oil production, oil price, production costs, royalties and taxes etc. Standard statistical distributions such as the normal, the lognormal, the triangular and the uniform distributions, are chosen to represent the variability of each parameter. In many cases the variables are assumed to be mutually independent because this greatly simplifies the calculations. Values are selected at random from each parameter distribution for each year and substituted into the equation to obtain one possible value for the NPV. This is repeated hundreds or thousands of times, giving the histogram of possible projects. From this, the average or expected NPV can be calculated and so can the probability of making a loss or the chances of a bonanza. Figure 1 summarises this procedure. Clear descriptions of the method are given by Newendorp[2] and more recently by Murtha[3]. In general Monte Carlo simulations are carried out using specially designed software such as @Risk or Crystal Ball which are adds-on to spreadsheet programs.

From a mathematical point of view, once the distributions of the input parameters have been specified the expected NPV and the probability of it exceeding any particular value could be calculated by integration. Three factors make it difficult to do this analytically: firstly, multiple integration is required (with as many dimensions as there are input variables); secondly, tax is a non-linear function of revenue (losses are treated differently to profits) and thirdly, correlations or links between variables further complicate the integration. The simulation procedure is just an easy way of integrating numerically in higher dimensional space.

From a technical point of view correlations are important because they have a marked impact on the results of the simulations (Armstrong and Berckmans[4]). At present most commercial software ask the user for the marginal distributions and the correlation coefficients. Except for the normal distribution, this is not enough information to specify the joint distribution. Morever not all correlation coefficients are compatible with any given distribution. For example if the first variable X is uniformly distributed on [1/2,1] and the second one Y is positive, it is easy to show that the correlation coefficient ρ verifies:

$$-\sqrt{3}\,\frac{E(Y)}{\sigma_Y} \le \rho \le \sqrt{3}\,\frac{E(Y)}{\sigma_Y}$$

So if the coefficient of variation of Y is larger than $\sqrt{3}$, the correlation coefficient starts to be restricted and when the coefficient of variation is $n\sqrt{3}$, ρ has to lie in the narrow interval $\left[-\frac{1}{n}, \frac{1}{n}\right]$. In fact the multivariate distribution has to be specified completely to avoid inconsistencies like this.

On the financial side two weaknesses of Monte Carlo simulations are that they assume a fixed project life and that they do not allow for any managerial flexibility. The latter disadvantage can be overcome by running simulations for several scenarios. Gutleber, Heiberger and Morris[5] present a case study where simulations were carried out to compare three alternative deals involving an oil company and the local government. Murtha[3] provides a long list of references to practical applications of this procedure including about ten recent ones carried out during the 90s.

Decision Trees

In contrast to Monte Carlo simulations which evaluate pre-determined project scenarios, decision trees focus on managerial decisions such as whether to drill additional wells, or to develop the field or not. They also take account of uncertainty on important parameters, but they do so in a more rudimentary way, typically, by specifying the probabilities that the reserves fall into broad classes such as “large”, “small” or “zero”.

The simplest way to present decision trees is via an example. Suppose that an exploration well has led to the discovery of a field which could have large or small reserves. In the first case it would be optimal to install a large platform whereas in the second case a small one would be more appropriate. Installing the wrong size platform would be an expensive mistake. So the engineer in charge of the project would prefer to obtain more information on the reserves before deciding but this would be costly. Which is the best decision?

Figure 2 shows the decision tree corresponding to this situation. Decisions are represented by squares. The branches emanating from these correspond to possible decisions (installing a large platform straightaway, installing a small one or getting additional information). Circles represent uncertain events, here large reserves (with a probability of 60%) or small ones (with a 40% probability). At the end of each branch the final NPV is marked. So installing a large platform when the reserves prove to be large generates an NPV of 170 if carried out immediately compared to 165 if additional information is obtained. Similarly if the reserves are really small, opting for a for a small platform leads to an NPV of 130 if carried out straightaway compared to 125 if it is deferred.

In order to compare the decisions the expected profit is calculated at each circular node. For the top branch it is 170 x 0.4 + 110 x 0.6 =134. As the expected profits at the other two nodes are 138 and 141 respectively, the best decision would be to carry out additional drilling before choosing the size of the platform. There are two points to note in these calculations; firstly that we have effectively calculated the maximum expected NPV rather than just the expected NPV, and secondly that calculations are carried out from the terminal branches and are "folded back" to the trunk. These comments also apply when tree structures are used for evaluating options.

Decision trees are a standard tool in management science and so many graduate business school textbooks provide descriptions. The book by Eppen, Gould and Schmidt[6] is particularly clear. It shows how to calculate the sensitivity of the tree to input parameters like the probabilities or the payoffs, how to find the value of new information and how to use Bayes theorem to calculate conditional probabilities. Newendorp[2] and Harbaugh, Davis and Wendebourg[7] provide a wide range of petroleum applications. Smith and McCardle[1] present a case study of a marginal oil project in which the company had the possibility of expanding the project if the field turned out to be large. They developed three tree structures to represent this. The first is a standard simplified tree, the second is a highly elaborate "dream tree" and the third is an intermediate tree with about 50,000 nodes. They comment that dynamic programming can also be used to evaluate highly flexible decision models. The advantage is that it can handle very short time steps. Unfortunately it assumes that the objective function is additive, which is simply not true for taxes and royalties. For more on dynamic programming see Bertsekas[8]. As the results are sensitive to the numerical values used for the NPV in the terminal nodes and as these values are difficult to estimate, particularly for the bonanza category, some authors prefer to replace the NPV by a utility function. Eppen, Gould and Schmidt discuss this. Bruneau, Hatchuel and Moisdon[9] gives an example taken from the oil industry in which utility functions are used. Another interesting point is that he uses Bayesian prior and posterior probabilities to modify the probabilities of the reserves being large, small or zero depending on the results of an additional well. For example at the outset the probability of large reserves was estimated at 20%. It increased to 52% if the extra well gave promising results, but it dropped to 5% if the well was dry.

The tree structure replaces continuous distributions for parameter values by discrete ones (e.g. the possible size of reserves is expressed as large, small or zero). The effect of this discretization can be important, particularly in the case of non linearities implying a threshold. This effect is magnified for complex trees.

Comparing MC simulations and Decision Trees. As we have seen, at circular (uncertainty) nodes the expected value is calculated; then at decision nodes the most favourable branch (i.e. the one with the maximum of the expected values) is chosen. So from a mathematical point of view, a decision tree is a way of evaluating the maximum expected NPV whereas Monte Carlo simulations calculate the expected NPV for fixed scenarios. Unlike Monte Carlo simulations, decision trees of this type do not provide the histogram of possible NPVs. This seems to be the price for incorporating decision choices. Both approaches use the traditional discount rate to take account of the time value of money, and both have problems dealing with correlated variables.The third approach, option pricing, abandons the discount rate and replaces it by "risk-free rate", a concept drawn from the financial world of derivatives.

Option Pricing

From the 1970s onward, the financial world began developing contracts called puts and calls which give their owner the right but not the obligation to sell or buy a specified number of shares or a quantity of a commodity such as gold or oil, at or before a specified date. If the transaction has to take place on that date or never, the options are referred to as European, otherwise they are called American options. This does not refer to the location where the transaction takes place.

The central problem with options is working out how much the owner of the contract should pay at the outset. Hence the name "option pricing". Basically the price is like an insurance premium; it is the expected loss that the writer of the contract will sustain. It is clear that being able to exercise at any time up to the maturity date makes American options more valuable than European ones. But it is less obvious that this apparently minor difference necessitates different procedures for calculating option prices.

As derivatives are very widely used nowadays, a large body of literature has developed on them. Hull[10] is a good basic text on the subject. In addition to describing the types of derivatives that are currently available, he provides an easy-to-read presentation of the common mathematical models and on how these are used to price different types of derivatives.

The standard assumption made in option theory is that the prices follow a lognormal brownian motion. This model was invented by Black and Scholes in the early 70s. Experimental studies have shown that it is a good approximation to the behaviour of prices over short periods of time. If they obey the standard Black and Scholes model, then the spot price S satisfies the partial differential equation:

$$dS_t = \sigma S_t dW_t + \mu S_t dt \tag{1.1}$$

or

$$d\ln(S_t) = \sigma dW_t + \mu dt$$

where S_t is the stock price, W_t is a brownian motion, μ is the drift in the stock prices and σ is their volatility.

Alternative models exist for the prices. Common ones include mean reverting models and jump models. For example, if the prices are mean reverting, the modified Ornstein–Ulhenbeck model is one possibility:

$$dS_t = \alpha(m - S_t)dt + \sigma dW_t \tag{1.2}$$

Using Ito's Lemma the explicit formula for the spot price for (1.1) can be shown to be

$$S_t = S_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma W_t} \tag{1.3}$$

whereas for (1.2), it is

$$S_t = m + (S_0 - m)e^{-\alpha t} + \sigma \int_0^t e^{-\alpha(t-s)} dW_s \tag{1.4}$$

The Black and Scholes model for prices is lognormal; $\log(S_t)$-$\log(S_0)$ is normally distributed with mean $(\mu - \sigma^2 / 2)t$ and variance σt ; that is, S_t is non stationary. It is also easy to show that the variogram of W_t is linear. (See Armstrong[11] for an introduction to variograms and geostatistics)

Similarly for the mean reverting model (1.4) if we assume that S_0 is normal with mean m and variance $\sigma^2/2\alpha$, we see that contrary to the Black and Scholes Model, the Ornstein-Ulhenbeck one gives rise to a stationary model for prices having a normal distribution with mean m and with an exponential covariance:

$$\mathrm{cov}(S_t, S_s) = \frac{\sigma^2}{2\alpha} e^{-\alpha|t-s|}.$$

So this model is compatible with the observation by Markland[12] that the spot price of oil have an exponential vario- gram. These two examples are special cases of diffusion models which are the general models considered for options together with the Poisson process to allow for jumps.

Looking back at the formula for NPV, we see that the discount rate was used to account for the effect of time on the value of money but it is not immediately apparent in the option pricing equations. So how is the time effect of money taken into account? The answer is rather subtle. It is incorporated through the risk-free rate of return and by way of a "change of probability". In our opinion, the clearest intuitive explanations are those given by Baxter and Rennie[13] and Trigeorgis[14]. These are summarised in the Appendix. A similar explanation is given for spot oil prices by Smith and McCardle[1]. These results are well known but are difficult to prove in the general case. See Harrison and Kreps[15] for a proof in the continuous case. Jacod and Shiryaev[16] give a more "elementary" one for the case of discrete time.

It is quite easy to evaluate European options which can only be exercised on a specified date, particularly when the prices follow Black and Scholes model because there is an analytical solution to the corresponding differential equation. This gives the expected value of the max(S exp(-rt) - K, 0) for a call or of max(K – S exp(-rt), 0) for a put. In general the simplest way of getting the histogram as well the expected value is by simulating the diffusion process giving the prices and comparing each of the terminal values to the strike price of the option. Solving the partial differential equation numerically merely gives the option price.

Evaluating the price of an American option is more difficult because the option can be exercised at any time up to the maturity date. From the point of view of stochastic differential equations, this corresponds to a free boundary problem. See Wilmott, Dewynne & Howison[17] for details. An analytical approximation is given by Barone-Adesi and Whaley [18].

The most common way of solving this type of problem is by constructing a binary tree. The life of the option is divided into intervals that are short enough so that only two price movements need be considered: a jump from S up to Su or a drop down to Sd. The size of the jumps depends on the volatility and the size of the time interval. Figure 3 shows a binomial tree for an American call with an exercise price of $50 over 5 months (taken from Hull[10], p339). Two numbers are shown at each node. The top one is the stock price at that node; the lower one is the value of the option. As for a decision tree, the value is calculated by "folding back" from the terminal nodes toward the trunk. At each step, the present value of the expected value of further out nodes is calculated and compared to the value if exercised at that node (i.e. prematurely). So this approach for evaluating American options is very similar to that used in decision trees. One problem with it is that only a limited number of time steps are used, generally less than 50. This can be a problem when options are applied to oil projects which have a life of 20-30 years. If only 50-60 time intervals are used each one would represent 6 months. Using only two prices to model possible oil prices after 6 months is a rather severe limitation.

Extending Option Pricing to Natural Resource Projects. Soon after the theory for pricing stock options was developed, Brennan and Schwartz[19] worked out a way of extending it to valuing natural resource projects, using Chilean copper mines to illustrate the procedure. They reasoned that managerial flexibility should increase the value of a project. To be more specific they allowed for three options: production (when prices are high enough), temporary shutdown (when they are lower) and permanent closure (when prices drop too low for too long). Different costs were associated with changing from one production option to another. They found the threshold copper prices at which it was optimal to temporarily close a producing mine or to open one that had been mothballed. In Galli and Armstrong[20] this approach is discussed and some analytical formulas are given for the case of permanent closure.The key to applying options in practice is in defining the options that are actually available to management. In the introductory chapter of his book Trigeorgis[14] lists a whole range of managerial options (or real options, as they are called) covering R&D intensive industries and capital intens- ive ones, as well as oil and mining. The other classical text on real options, Dixit and Pindyck[21], describes several oil applications. As well as sequential decision making for opening up oil fields, they also present a study on building, mothballing and scrapping oil tankers.

Since Brennan and Schwartz's seminal paper, many others have studied petroleum options. For example, Copeland, Koller and Murrin[22] describe a case study involving an option to expand production. Kemma[23] worked on a timing option for developing a new lease area in which the company is obliged to drill extra wells. The weak point in his argument is that he treats wells like parking meters. You have to put money in to stay there without getting anything in return. Extra wells do provide additional information. The question is how to quantify it. Dias[24] considers several realistic scenarios including sequential decision-making when opening up a new area and a wait-and-see option when a second oil company is exploring an adjoining property. More recently, Copeland and Keenan[25] discuss a case study on a natural gas field where the company has to decide how much production capacity to install. Their problem is that as 40% of the lease has yet to be explored, the reserves are very uncertain.

From a theoretical point of view, the key is in deciding which variable is assumed to follow a Black and Scholes model (or more generally an Ito diffusion process). Brennan and Schwartz[19] assume that the spot price (here the oil price) obeys this model. Paddock, Smith and Siegel[26], Trigeorgis[14] and Kemna[23] have taken a radically different approach; they base their analysis on the hypothesis that the project value itself obeys the model. The difference is important because the theory of option pricing requires that there is a liquid market for the underlying commodity and that there are no transaction costs and no arbitrage. While this is probably true of oil prices, it is doubtful whether there is a large enough market for oil projects. As is shown in appendix 1, the assumptions of no arbitrage and of a replicate portfolio use risk neutral probabilities and the risk-free rate. One consequence of this is that this modifies the probabilities of having the different sized fields. In the example given in the appendix, using these new probabilities and the riskless rate gives the same value for the project as that obtained with the initial probabilities and discount rate, but although this remains constant, all the other statistics change. For example the variance changes from 2500 to 2963. So the hypothesis of a twin security is extremely strong. This can be seen quite clearly in academic examples such as those given in the appendix; it is less obvious in practical case studies.

Another point about options that has to be stressed is that the value of an option is in fact an expectation. More precisely it is the conditional expectation of the value given the initial conditions. So as for decision trees, real options do not give any indications about the uncertainty of the project.

Conclusions

Monte Carlo simulations are a natural extension of the standard NPV base case by allowing for that fact that variables are not known with certainty. Standard statistical distributions such as the normal, lognormal, uniform and the triangular distributions are used to describe the input parameters. While it would be possible to allow for correlations between variables it is more common to find them being treated as independent. A second obvious extension would be to use time series models or models like Black and Scholes to incorporate the correlation between successive values of parameters like the oil price. These extended MC simulations would be very similar to simulations used for pricing European options. In both cases the project life is fixed; the results give the histogram of possible outcomes as well as the expected value. The essential difference lies in the way the time value of money is treated. In classical MC simulations the discount rate is used; in options the risk-free rate is used after a change to risk-neutral probabilities.

In the same way there is a close relation between the binary trees used to evaluate American options and decision trees. In both cases the maximum expected value is calculated by folding the tree back from the outermost branches towards the trunk. Having said that, their treatment of the time value of money differs as was mentioned in the previous paragraph.

Now let us look at the types of distributions that can be used to model input parameters. Virtually any distribution can be used in Monte Carlo simulations. In options, the choice of the stochastic model (Black and Scholes, mean reverting, jump models) implicitly specifies this distribution. For example, for Black and Scholes it is a lognormal with drift. In decision trees the distribution of possible values is modelled in a much more rudimentary way - in general, as two or three broad categories such as “no oil”, “a little oil” or “lots of oil”.

We have seen that although American options and decision trees initially seem quite different, they have many simil- arities. The same is true for classical MC simulations and simulations of European options. In fact they can be considered as different facets of a common framework. There are three main sources of divergence between the three approaches:- (1) the way they handle the time value of money (discount rate versus risk-free rate plus change of probability), (2) how they allow for uncertainty in parameter values and (3) whether they incorporate managerial flexibility. Standard MC simulations focus on modelling the uncertainty in parameter values, often at the expense of serial correlation or correlations between variables, and they ignore managerial flexibility. In contrast to this, decision trees analyse different managerial strategies, choosing the one that maximizes the expected NPV. As designing a reservoir production plan is very time consuming, only a few strategies corresponding to broad groupings (large versus small platforms) can be studied. In a similar way, option pricing actively studies possible management choices but to make the computations tractable, it is limited to certain wellknown models such as Black and Scholes or mean reverting, for the behaviour over time of the variables. Conceptually one could imagine allowing for several possible projects together with any multivariate distribution of variables over time, but the resulting computations would be intractable, to say nothing of the problems in inferring the distibutions for parameter values. This forces us to compromise – to focus on certain aspects, simplifying and ignoring ther others. A key question for real options is the choice of the variable on which to apply option theory: the spot price or a twin security. Moreover in order to be realistic, real options have to incorporate more technical information. For example, they must integrate recent advances in modelling uncertainty and in reservoir characterization, and they need to measure the value of information like a new well in a meaningful way.

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Appendix-How the time value of money is incorporated into option pricing

The aim of this appendix is to show how option pricing incorporates the time value of money into its formalism without using the classical concept of discount rate. The ideas presented here are drawn partly from Baxter and Rennie[13] partly from Trigeorgis[14].

Parable of the Bookmaker. A bookmaker is working out the odds for a two horse race. After studying the form of both horses, he comes to the conclusion that their chances of winning are respectively 25% and 75%. But the punters have a different opinion of the horses. The bets for the first one total $5,000 whereas those for the second one total $10,000. If the second horse wins, the bookmaker makes a profit of $1667 whereas if the first one wins he will lose $5000. His expected profit is zero ($1667x75% - $5000x25% = 0), so he would expect to break even in the long run. But in the short run, he stands to lose money. In order to break even whichever horse wins he must set the odds at 2-1 against and 2-1 on respectively, which is equivalent to probabilities of 33.3% and 66.6%. (Of course in the real world, the bookmaker would set the odds at 9-5 against and 5-2 on, thereby making $1000 whichever horse wins.)

The point to note is that working with the "true" probabilities left the bookmaker wide open to serious losses; by changing to a more suitable set of values he can guarantee to breakeven whatever the outcome of the race. The same sort of problems arise when pricing financial derivatives. Setting the prices to breakeven in the long run could lead to catastrophic short-term losses. As we will see the solution comes from choosing a new set of probabilities so that we break even every time.

Setting up a self-replicating portfolio

The idea is to construct a portfolio by buying of m shares, initially worth S=$20, and borrowing $B at the risk-free rate, so that the portfolio would duplicate future returns on an option whatever the share price does. A discrete binomial tree is used to represent movements in the share price. Suppose that in the next time period, it could rise to S^+=Su =$36 or fall to S^- =Sd=$12 with equal probabilities (0.5). So its expected value discounted at the risk adjusted rate of 20% is precisely $20. During the same time the amount of money owing would grow to B (1+r).

As the option's value f depends on the share price, it would be f(36) with probability 0.5 and f(12) with probability 0.5. Here f stands for any option. In the case of a European option f(S) would be max(S-K, 0), where K is the exercise price.

The problem for a market maker is to propose a price for the option. If he chooses to use this discounted expected value, he could well lose in the short run. Initially the portfolio would be worth 20m+ B. After the time period, its value would depend on whether the stock moved up or down. It would be

$36\,m + B(1+r)$ if the price moves up
$12\,m + B(1+r)$ if it moves down

We want these to be equal to the value of the option in each case. This gives two simultaneous equations. Solving them gives

$$m = \frac{f(36)-f(12)}{36-12}$$

$$B = (1+r)\,\frac{(u\,f(12)-d\,f(36))}{u-d}$$

This portfolio has been constructed to have a value V equal to f(36) if the stock price jumps up, and to f(12) if it drops. Suppose that another market maker proposed a different price P. If P < V, then smart traders would buy the option and sell equivalent quantities of the portfolio knowing that they would make a profit of V-P whatever the final price of the stock. And conversely if P > V. So any market maker proposing the price P would rapidly be driven out of business. The only way of avoiding offering riskless profits is by quoting the price

$$V = 20\left(\frac{f(36)-f(12)}{36-12}\right) + (1+r)\,\frac{(u\,f(36)-d\,f(12))}{u-d}$$

Rearranging this equation gives

$$V = \frac{[(1-p)f(12)+pf(36)]}{(1+r)}$$

where

$$p = \frac{1+r-d}{u-d}.$$

So V is just the expected value obtained by using the new probabilities p and (1-p), discounted at the risk-free bond rate r. This simple example shows how option pricing takes account of the effect of the time value of money by a suitable change of probability and by using the risk-free interest rate. It is based on the construction of a fictive portfolio and on the idea that riskless profits cannot exist in a properly functioning market. Basically the idea is "no free lunches". The technical term for it is "no arbitrage". Several names are used for mechanism of changing the set of probabilities. Some authors refer to "risk-neutral probabilities", while the more mathematically inclined prefer the term "equivalent martingale measures".

Assume now we have a project for which S is a twin security. That is we interpret S as the value of a company rather than a share. In this framework it means that the value V of our project is perfectly correlated with S; that is in one period of time V can be V^+=Vu or V^-=Vd with a probability 0.5. u and d having the same values as previously. The value V of the project is then:

$V = \frac{0.5\times180+0.5\times60}{1.20} = 100$. In the case of a petroleum project the possibility to move up or down can be due to spot prices but also to the fact some uncertainty remains on the reserves, which will be resolved by additional information at time 1. In that case the hypothesis that S is a twin security is really a strong (may be too strong) hypothesis because we see that the evaluation of V will now be made on the new probability system (the risk neutral one). It is easy to verify as indicated in

Trigeorgis[14] p 157 that the value of the project remains the same under this new probability N. That is,

$$V_N = \frac{0.4\times180+0.6\times60}{1.08} = V=100$$

However in this system the probability for the field to be large is now 0.4 instead of 0.5, and it is used in all subsequent computations.

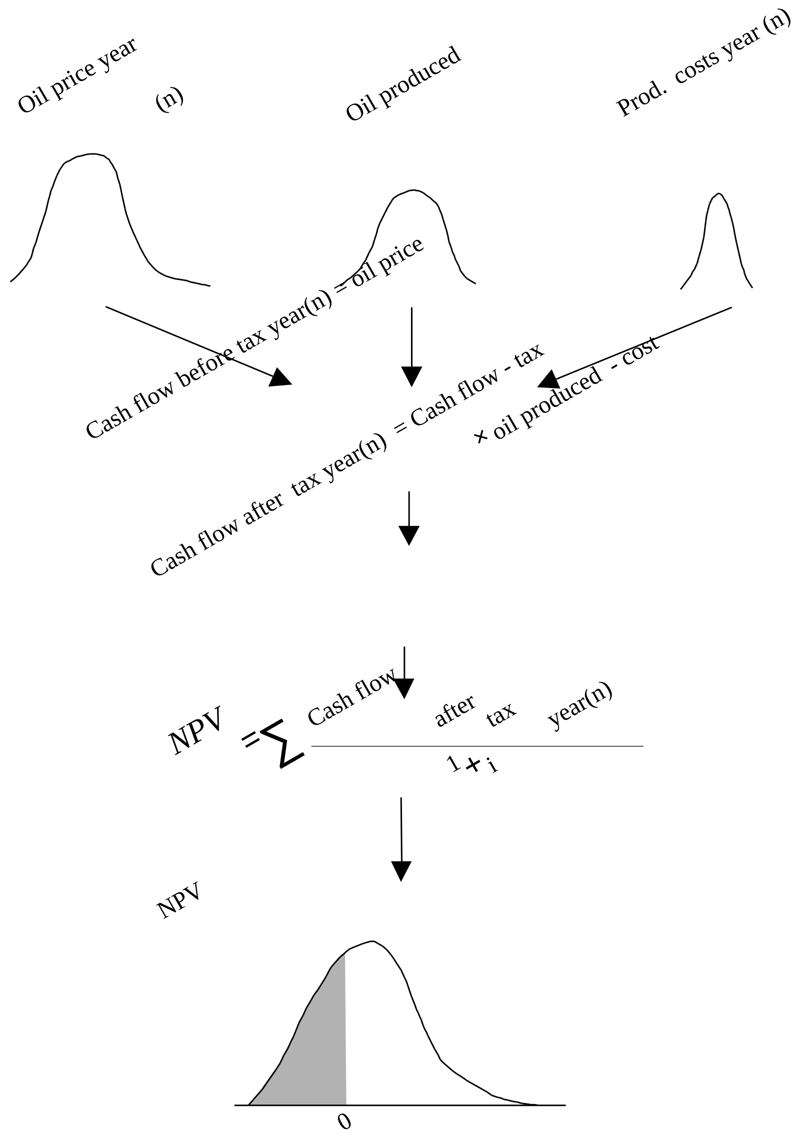

Fig.1-Simplified example of Monte Carlo simulations

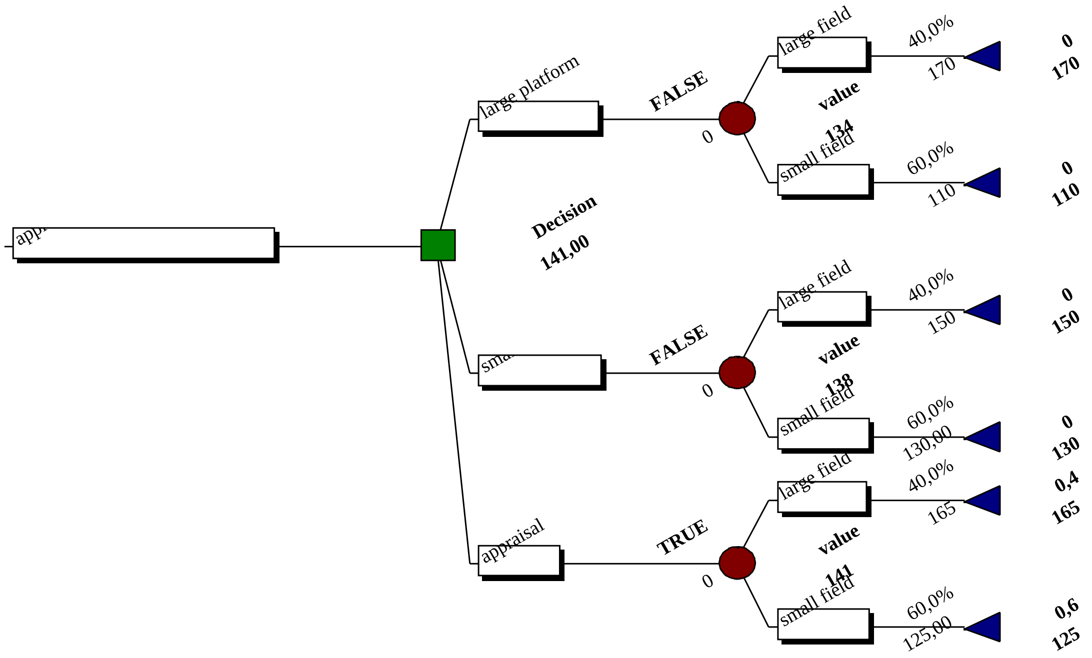

Fig.2- Example of a Decision Tree. At chance nodes probabilities of each branch are indicated in %, the figure below is the value of the branch.

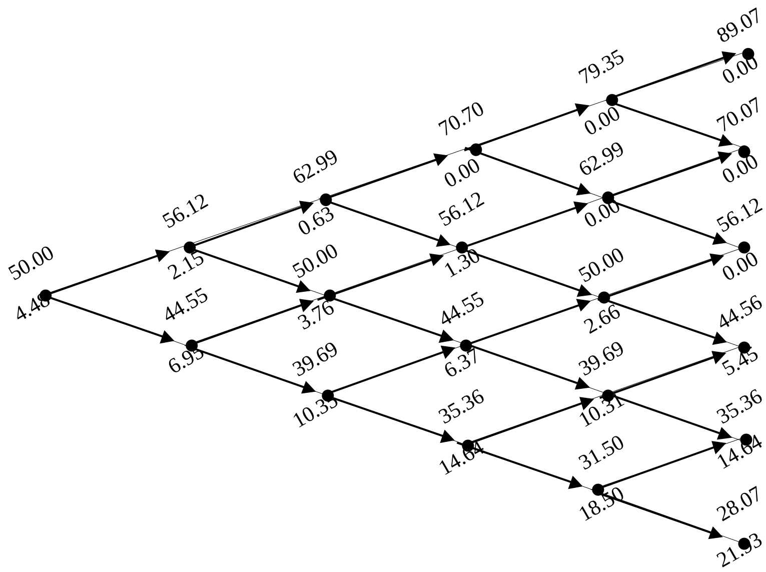

Fig 3. Binomial Tree for American put on Non Dividend paying stock. The figure above the nodes represents the price and the one below the nodes the value of the option if exercised at the time corresponding to the node.