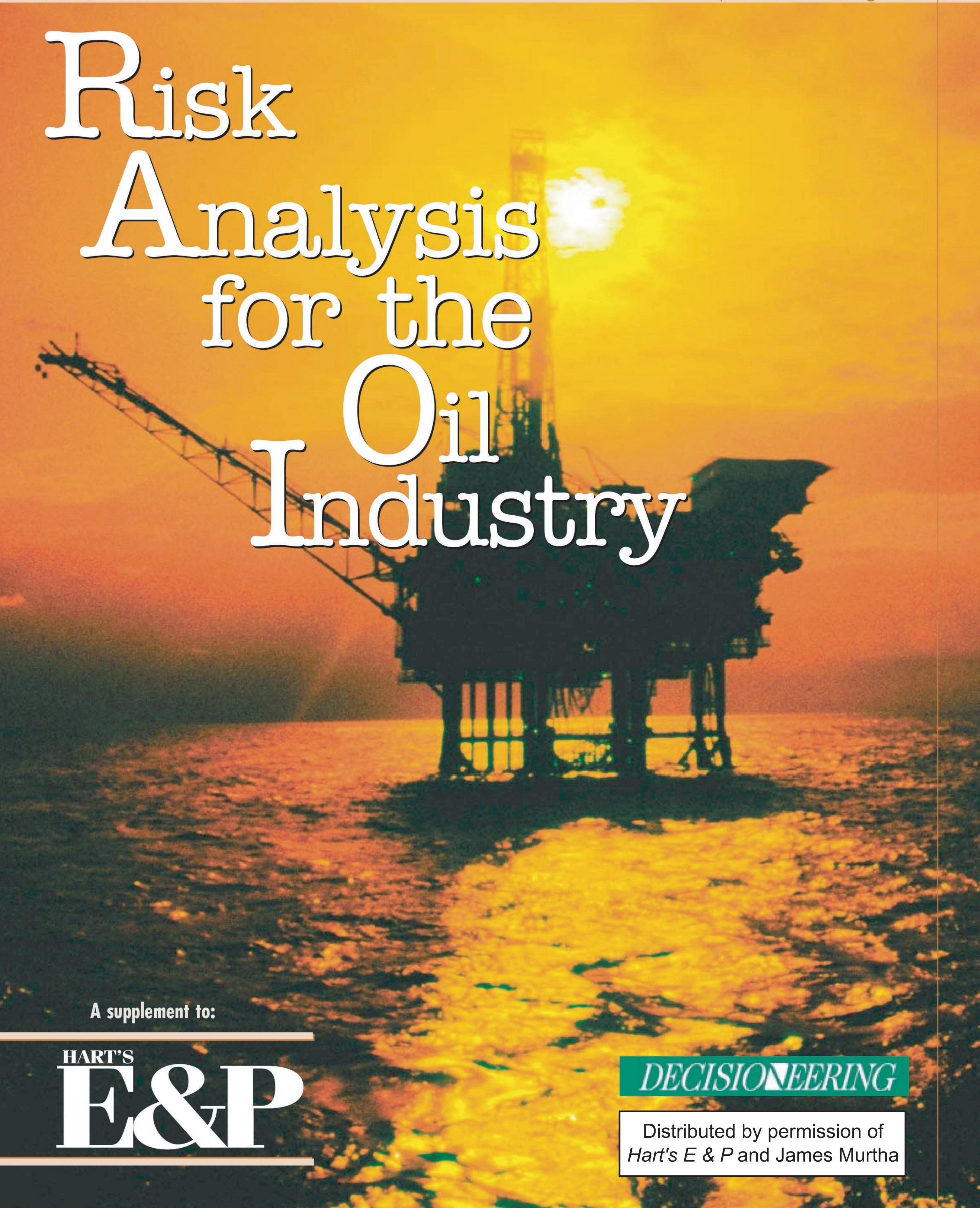
A Comprehensive
Study of Risk Management

Risk Analysis for the Oil Industry



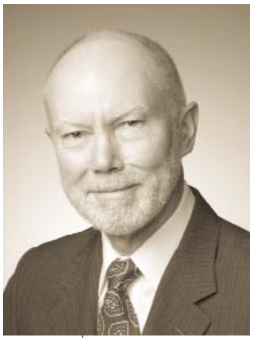
A supplement to:

HART'S E&P

DECISIONEERING

Distributed by permission of *Hart's E & P* and James Murtha

Biography



Jim Murtha, a registered petroleum engineer, presents seminars and training courses and advises clients in building probabilistic models in risk analysis and decision making. He was elected to Distinguished Membership in SPE in 1999, received the 1998 SPE Award in Economics and Evaluation, and was 1996-97 SPE Distinguished Lecturer in Risk and Decision Analysis. Since 1992, more than 2,500 professionals have taken his classes. He has published Decisions Involving Uncertainty – An @RISK Tutorial for the Petroleum Industry. In 25 years of academic experience, he chaired a math department, taught petroleum engineering, served as academic dean, and co-authored two texts in mathematics and statistics. Jim has a Ph.D. in mathematics from the University of Wisconsin, an MS in petroleum and natural gas engineering from Penn State and a BS in mathematics from Marietta College. ◆

Acknowledgements

When I was a struggling assistant professor of mathematics, I yearned for more ideas, for we were expected to write technical papers and suggest wonderful projects to graduate students. Now I have no students and no one is counting my publications. But, the ideas have been coming. Indeed, I find myself, like anyone who teaches classes to professionals, constantly stumbling on notions worth exploring.

The articles herein were generated during a few years and written mostly in about 6 months. A couple of related papers found their way into SPE meetings this year.

I thank the hundreds of people who listened and challenged and suggested during classes.

I owe a lot to Susan Peterson, John Trahan and Red White, friends with whom I argue and bounce ideas around from time to time.

Most of all, these articles benefited by the careful reading of one person, Wilton Adams, who has often assisted Susan and me in risk analysis classes. During the past year, he has been especially helpful in reviewing every word of the papers I wrote for SPE and for this publication. Among his talents are a well tuned ear and high standards for clarity. I wish to thank him for his generosity.

He also plays a mean keyboard, sings a good song and is a collaborator in a certain periodic culinary activity.

You should be so lucky. ◆

Table of Contents

Bayes' Theorem – Pitfalls

Do you know how to revise the probability of success for a follow up well?

Consider two prospects, A and B, each having a chance of success, P(A) and P(B). Sometimes the prospects are independent in the sense that the success of one has no bearing on the success of the other. This would surely be the case if the prospects were in different basins. Other times, however, say when they share a common source rock, the success of A would cause us to revise the chance of success of B. Classic probability theory provides us with the notation for the (conditional) probability of B given A, P(B|A), as well as the (joint) probability of both being successful, P(A&B).

Our interest lies in the manner in which we revise our estimates. In particular, we will ask:

- how much better can we make the chance of B when A is successful?

That is, how large can P(B|A) be relative to P(B); and

- if we revise the chance of B upward when A is a success, how much can or should we revise the chance of B downward when A is a failure?

As we shall see, there are limits to these revisions, stemming from Bayes' Theorem.

Bayes' Theorem regulates the way two or more events depend on one another, using conditional probability, P(A|B), and joint probability, P(A&B). It addresses independence and partial dependence between pairs of events. The formal statement, shown here, has numerous applications in the oil and gas industry.

Bayes' Theorem

1. **P(B|A)= P(A|B)*P(B)/P(A)**
2. **P(A) = P(A&B$_1$) + P(A&B$_2$) + … + P(A&B$_n$)**, where B_1, B_2, …B_n are mutually exclusive and exhaustive

We can rewrite part 2 if we use the fact that:

P(A&B) = P(A|B)*P(B)

2′. P(A) = P(A|B$_1$)*P(B$_1$) + P(A|B$_2$)*P(B$_2$) + …+P(A|B$_n$)*P(B$_n$)

Part 1 says that we can calculate the conditional probability in one direction, provided we know the conditional probability in the other direction along with the two unconditional probabilities. It can be derived from the definition of joint probability, which can be written backward and forward.

P(B|A)P(A) = P(A&B) = P(B&A) = P(A|B)P(B)

Part 2 says that if A can happen in conjunction with one and only one of the Bi's, then we can calculate the probability of A by summing the various joint probabilities.

There are numerous applications of Bayes' Theorem. Aside from the two drilling prospects mentioned above, one well-known situation is the role Bayes' Theorem plays in estimating the value of information, usually done with decision trees. In that context, the revised probabilities acknowledge the additional information, which might fall into the mutually exclusive and exhaustive categories of good news and bad news (and sometimes no news).

Further, we can define P(~A) to be the probability that prospect A fails (or in a more general context that event A does not occur). A rather obvious fact is that

Bayes' Theorem regulates **the way two or more events depend on one another, using conditional probability, P(A|B), and joint probability, P(A&B).**

$P(A) + P(\sim A) = 1$, which says that either A happens or it doesn't.

The events A and ~A are exhaustive and mutually exclusive.

Almost as obvious is the similar situation

$P(A|B) + P(\sim A|B) = 1$, which says that once B happens, either A happens or it doesn't.

Armed with these simple tools, we can point out some common pitfalls in estimating probabilities for geologically related prospects.

Pitfall 1: Upgrading the prospect too much

Suppose we believe the prospects are highly dependent on each other, because they have a common source and a common potential seal.

Suppose $P(A)=.2$, $P(B) = .1$, and $P(B|A)= .6$

This is the type of revised estimate people tend to make when they believe A and B are highly correlated. The success of A "proves" the common uncertainties and makes B much more likely.

But, consider the direct application of Bayes' Theorem:

$$P(A|B) = P(B|A)*P(A)/P(B) = (.6)*(.2)/.1 = 1.2$$

Since no event, conditional or otherwise, can have a probability exceeding 1.0, we have reached a contradiction, which we can blame on the assumptions.

What is the problem?

When two prospects are highly correlated, they must have similar probabilities; one cannot be twice as probable as the other. Another way of looking at this is to resolve the equations:

$P(A|B)/P(A) = P(B|A)/P(B)$, which says that the relative increase in probability is identical for both A and B.

Decision tree interpretation

Conditional probabilities arise naturally in decision trees. For any branch along which there is a sequence of chance node, choice node, chance node, the branches emanating from the second chance node represent events with conditional probabilities. Figure 1 shows a simple example of a two-prospect tree.

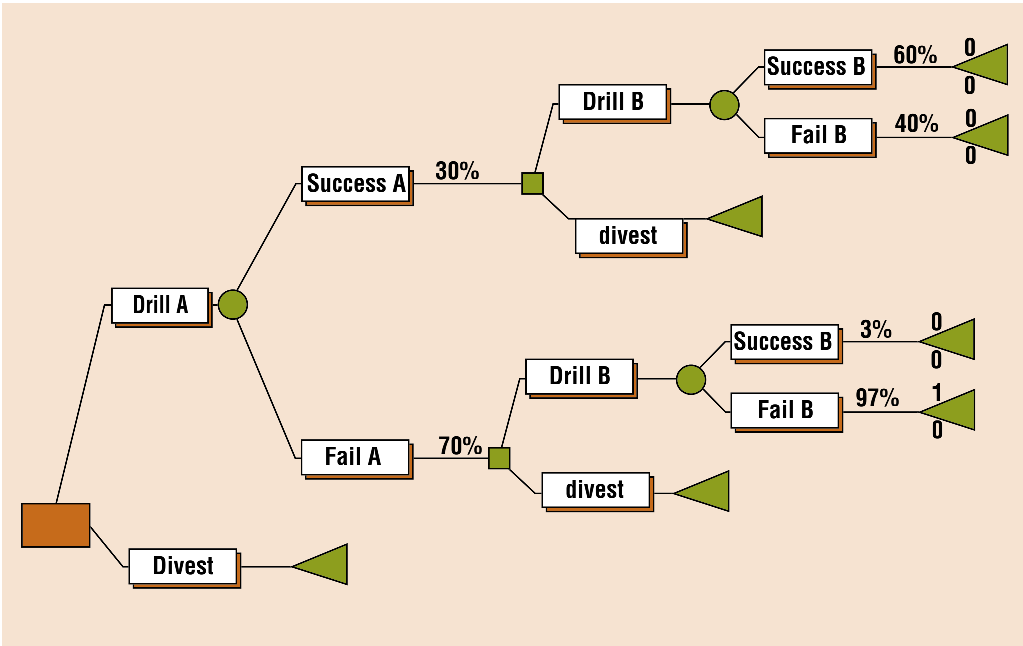


Figure 1. Decision tree showing conditional probabilities

Pitfall 2: Estimating revised P(S) after bad news

Suppose $P(A)=.3$, $P(B) = .2$, and $P(B|A)= .6$

This is precisely the situation depicted in Figure 1.

In other words, we believe the two prospects are dependent and upgrade the chances for B once A is a success.

But what if A fails? Then, what is $P(B|\sim A)$? Clearly, one should instead downgrade B's chance of success when A fails. Some people assign a value to $P(B|\sim A)$, but they should exercise great caution when they do so. Here is why.

Bayes' Theorem says $P(B|\text{-}A)=P(\text{-}A|B)*P(B)/ P(\text{-}A)$

But $P(\sim A) = 1\text{-}P(A) = 0.7$

and $P(\sim A|B)=1\text{-}P(A|B) = 1\text{-}P(B|A)*P(A)/ P(B) = 1\text{-}.6*.3/.2 = .1$

Thus $P(B|\sim A) = (.1)*(.2)/.7 = .03$

The point is that this value already is completely determined from our other assumptions; we cannot just assign a value to it – we are too late. Not only that, but .03 is probably much less than most people would guess.

In summary, these two common examples point out both the power of Bayes' Theorem to provide us with informed decisions, and the ease with which casual estimates of our chances can lead us. ◆