An option pricing approach to evaluating petroleum projects

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Option pricing applications in capital budgeting decisions are still in the development stage. This paper uses concepts and methods from option pricing theory to evaluate real projects rather than financial ones. Within a petroleum projects context such real options include development and operations of satellite fields, break-even values of incremental capacity, and flexibility value with compound development and operations options. The option pricing results may be quite different from those obtained by traditional capital budgeting and decision tree approaches. Some related papers on real options are also briefly noted.

Keywords: Option pricing; Valuation; Petroleum projects

This paper applies an option pricing approach to capital budgeting exemplified by evaluation of petroleum projects. The purpose is to show that concepts and methods from option pricing theory and the related contingent claims analysis may be quite useful additions to the analysts' tool kit for real investments.

As evidenced by the bibliography of Cox and Rubinstein [5], the option literature is both extensive and impressive. In a recent survey Mason and Merton [16] classify the option applications into three categories:

(i) 'past': traded financial options, by now firmly established in financial practice;
(ii) 'present': corporate securities interpreted as options, representing the state-of-the-art in financial practice; and
(iii) 'future': real options aspects of investments projects, still being in the development stage within academic research, but holding promise as a part of financial practice in the future.

This paper falls within the 'future applications' category. Without purporting to report on major theoretical advances of its own, it builds on an applied research programme aimed at giving analysts in oil companies as well as in government a rudimentary insight into option-based project evaluation.

Compared to traditional discounted cash flow methods, the option framework relies more heavily on market-based input data and yields improved estimates of the expected cash flow elements. Moreover, the approach avoids the need to specify appropriate risk-adjusted discount rates. From interpreting a real investment opportunity as an option, the investment may be worthwhile (the option value is positive) even if its expected cash flow is worth less than the investment cost (the option is out-of-the-money). Similarly, if making the investment is interpreted as exercising that option, a positive net present value is not a clearcut go signal (in-the-money American options may be worth more alive than dead). Thus, conventional standard results are refuted, as a traditionally calculated positive net present value is neither a necessary nor a sufficient condition for a project to be profitable.

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When superimposed on decision trees, the option pricing approach is far superior when evaluating time-distributed, uncertain outcomes. This is an important feature, as proper and consistent evaluation is necessary for pruning the tree to determine the optimal decision.

Throughout the paper the option approach is illustrated by a highly stylized general example from a petroleum environment. Suppose that a minor reservoir has been discovered close to a major oil field which is operating at its current capacity (see Figure 1). The minor reserve contains such a small amount of hydrocarbons that development and extraction of that field separately is out of the question. However, the minor field may be linked up to the main field as a satellite. If so, the satellite may then draw upon the processing and transport facilities of the main field, provided the latter one is operating and has idle capacity. By increasing the processing capacity of the major field, oil may be extracted from the satellite reserve while the main field is still producing at its plateau level (see Figure 2).

A binomial oil price process

A stochastic output price is the source of uncertainty in the examples. For expositional purposes, assume that the oil price follows a binomial multiplicative random walk. Let

S_t = oil price per barrel at time t
H = the event of higher price at t than at $(t-1)$
L = the event of lower price at t than at $(t-1)$
q = probability of a price increase
u = multiplicative upward factor of oil price
d = multiplicative downward factor of oil price

The price relative S_{t+1}/S_t is the gross holding period return per barrel of oil. For consistency with a continuous oil price movement later in the paper, the logarithmic return or exponential growth rate

$$g \equiv \ln(S_{t+1}/S_t) \quad (1)$$

is more important.

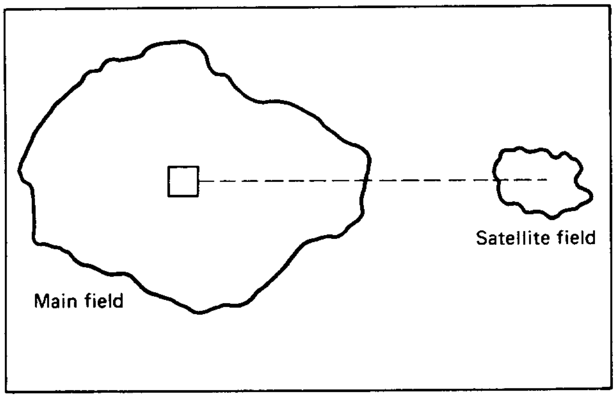

Figure 1. General example.

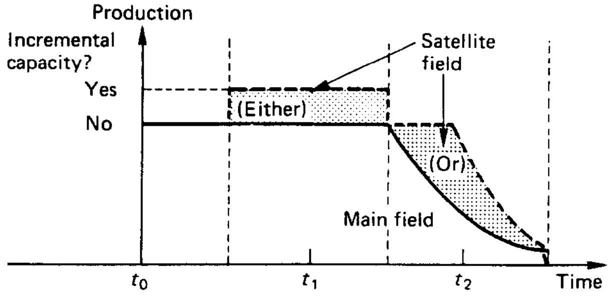

Figure 2. Production and incremental capacity.

Figure 3 illustrates the assumed oil price movement in time. Assume that the parameters are stationary and take on the exogenously given numerical values $q = 1/2$, $u = 3/2$ and $d = 2/3$. Also, let the initial oil price be $S_0 = 18$.

With these parameters, at $t = 1$ an oil price of $S_1 = 27$ or $S_1 = 12$ will be equally likely. At $t = 2$, the possible oil prices are 40.5, 18 and 8, with probabilities 0.25, 0.50, and 0.25, respectively. As of $t = 0$, the expected oil price in one year is $E_0(S_1) = 19.5$, whereas in two years it is $E_0(S_2) = 21.125$.

The exponential growth rate g (the logarithmic price relative) is stochastic and takes on the values $g = \ln u$ and $g = \ln d$ with equal probability q. Hence, with $q = 1/2$ and $d = 1/u$, its expectation is $E(g) = 0$ and its standard deviation is $\sigma(g) = \ln u = 0.405465$.

Suppose the risk-free rate of interest is $R_F = 1/6$ (=16.67%), corresponding to a risk-free factor $r \equiv$

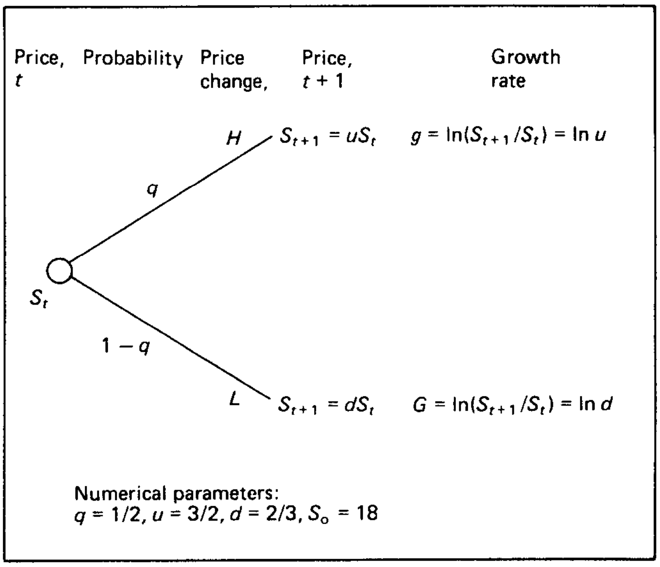

Figure 3. Binomial oil price process.

$1 + R_F = 7/6$. Thus, the growth rate of the oil price follows a symmetric random walk without drift. The expected oil price rises over time, but its rate of 1/12 is less than the risk-free interest rate of 1/6.

Suppose further that oil has no convenience yield, ie the owner of the physical commodity and the owner of a contract for future delivery receive equivalent returns. For a risk-neutral individual, the expected oil price at $(t + 1)$ discounted one period at the riskless rate of interest must then equal S_t, as there by assumption is no other return to holding oil. In general, this equality will not hold for the assumed true probability of q ($= 1/2$) for an upward price movement. It is well known (see eg Cox and Rubinstein [5], p 173]) that the condition will be satisfied for a probability

$$p \equiv (r - d)/(u - d) \tag{2}$$

This so-called hedging probability p can be derived by a replicating portfolio argument, ruling out favorable arbitrage opportunities. It is the value q would have in equilibrium if all investors were risk-neutral.

Plugging in the values for r, u and d, Equation (2) yields an imputed 'probability' $p = 0.6$, for which the current oil price at any point in time equals the risklessly discounted expected future prices.

The imputed or option probability p can also be interpreted in terms of prices of elementary state-contingent claims, or state-prices for short. Suppose the price change of oil is just about to be revealed. Consider a claim that pays off one dollar at time $(t + 1)$ if and only if the oil price increases from time t. That claim will have a value of p a second before the new price becomes known. The corresponding state-price at t of an upward price change between t and $(t + 1)$ is p/r, ie the risklessly discounted hedging probability. In the next section it is shown that the state-price interpretation may be useful for making qualitative statements about the relation between risk-adjusted and risk-free discount rates.

Valuation of a satellite field

For simplicity, consider a two-period model where development and extractions, if any, will take place in period two. The decisions may be made after knowing the oil price of that period. Also, the satellite field will either be fully operated or not at all. Hence, all cost data may be given on a unit (barrel) basis.

Let the following cost data be given:

development cost, $t = 2$	7
operating cost, $t = 2$	15.5
total unit cost, $t = 2$	$V_2 = 22.5$

Recall that the expected oil price at $t = 2$ is $E_0(S_2) = 21.125$, which is less than the known unit cost $V_2 = 22.5$. Superficially, the expected profit is negative, and the satellite seems unprofitable. But this is a fallacy, as the option aspects of development and operation have been ignored.

Figure 4 shows, in a decision tree format, how the satellite field may be evaluated by discounting 'expected' cashflows at the riskless rate. The crucial feature is that the imputed hedging probability $p = 0.6$ for higher prices is used for calculating expected values rather than the 'real' probability $q = 0.5$

Taking the combined development and operations options into account, the satellite field has a value at $t = 0$ of $C_0 = 4.76$. Alternatively, the same answer could be obtained by constructing a sequence of self-financing replicating portfolios.

If the 'real' probability of $q = 0.5$ were used in the decision tree, with a riskless discount factor $r = 7/6$, the value would decline to 3.31. This appears to be a truly risk-neutral evaluation, but it is not consistent with the assumed price process for oil.

A decision tree analyst, sticking with $q = 0.5$, might consider introducing a risk-adjusted discount rate (RADR) to incorporate risk aversion. Suppose she selects a RADR which is higher than $R_F = 1/6$, such as RADR $= 1/4$ ($=25\%$). The field value then decreases further to 2.88.

Actually, from the state-price interpretation of the hedging probability p, it follows that this imputed probability equals the true probability q of a price increase multiplied by the ratio of marginal utility given price increase to expected marginal utility [6]:

$$p = q \cdot u'_H / E(u') \tag{3}$$

Consequently, as $p = 0.6 > q = 0.5$, for a representative individual in market equilibrium the marginal utility

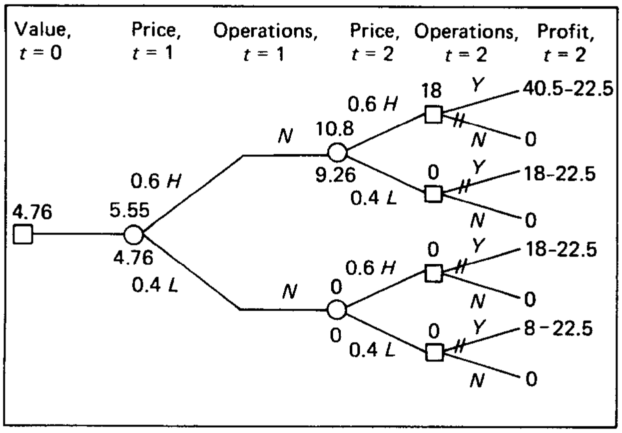

Figure 4. Valuation of satellite field by decision tree.

u'_H given an oil price increase is higher than the expected marginal utility $E(u')$. A higher marginal utility conditional on increasing oil prices is consistent with higher oil prices being associated with lower market returns. Thus, the assumed market parameters correspond to oil prices moving counter-cyclical in the world economy. Therefore, as in the case of a negative beta in a CAPM framework, the appropriate RADR is less than the risk-free rate of interest. An analyst using $q = 0.5$ would, however, get the correct field value 4.76 only if she by chance picks RADR $= -1/36$ ($= -2.78\%$)

A different market parameter scenario could be consistent with higher oil prices being associated with upswings in the economy. Keeping q, u and d at their previous values, assume a zero risk-free rate of interest. As $R_F = 0.0$ implies $r = 1.0$, Equation (2) yields $p = 0.4 < q = 0.5$.

Repeating the calculations for the adjusted risk-free discount factor $r = 1.0$, it turns out that the field value with the imputed probability $p = 0.4$ is now less than with the real probability $q = 0.5$.

Table 1 summarizes the results of the value of the satellite field, with panels (a) and (b) applying to risk-free rates of interest of 1/6 and 0, respectively.

Rather than using decision trees, the appropriate field value may be found by the Cox–Ross–Rubinstein binomial option pricing formula ([5], p 178)

$$C = S \cdot B(a;\, n,\, p') - Kr^{-n} \cdot B(a;\, n,\, p) \qquad (4)$$

where

C = current call value
$B(';\, ',\, ')$ = complementary binomial distribution
S = current price
K = the option's exercise price
n = number of periods to the option's expiration
p = the hedging probability, given by Equation (2)
p' $\equiv (u/r) \cdot p$
a = the minimum of n and the smallest integer greater than $\ln(K/(Sd^n))/\ln(u/d)$

In words, Equation (4) says that the call value equals the current asset price multiplied by a probability, less the present value of the option's exercise price multiplied by another probability.

For the satellite field example, with $u = 3/2$, $d = 2/3$, $r = 7/6$, $S = 18$, $K = V_2 = 22.5$, $n = 2$, $p = 3/5$, $p' = 27/35$, the critical ratio becomes $\ln(22.5/8) = 1.034$ implying $a = 2$. Hence,

$$\begin{aligned} C &= 18 \cdot B(2;\, 2,\, 27/35) - 22.5 \cdot (7/6)^{-2} \cdot B(2;\, 2,\, 3/5) \\ &= 18 \cdot 0.5951 - 22.5 \cdot 0.7347 \cdot 0.36 \\ &= 10.71 - 5.95 = 4.76 \end{aligned}$$

which is exactly the desired value, obtained by plugging into an option formula rather than relying on decision trees with imputed probabilities.

If the periods are divided into shorter subperiods, the decision tree quickly becomes quite complicated. Use of the closed-form pricing formula Equation (4) then becomes even more attractive.

Table 1. Valuation of satellite field.

Probability	Risk-free interest rate	Discount rate	Value
(a) $p = 0.6$	$R_F = 1/6$	$R_F = 1/6$	4.76
$q = 0.5$	$R_F = 1/6$	$R_F = 1/6$	3.31
$q = 0.5$	$R_F = 1/6$	$RADR = 1/4$	2.88
$q = 0.5$	$R_F = 1/6$	$RADR = -1/36$	4.76
(b) $p = 0.4$	$R_F = 0$	$R_F = 0$	2.88
$q = 0.5$	$R_F = 0$	$R_F = 0$	4.50

(c) The expected price (21.125) is less than unit cost (22.5), which superficially might suggest a negative value.

Value of incremental capacity

By installing incremental processing capacity at the main field, suppose that the satellite field may, if so desired, be developed and operated already in period 1. Operations in period 1, if any, preclude operations in period 2. Even with incremental capacity, there is no obligation to develop and/or operate in any period. The development and operations options may still be exercised after obtaining information about the oil price of the period. The problem at hand is to determine the maximum amount k_0 to invest at $t = 0$ in such incremental capacity.

The oil price data are as before and the risk-free interest rate is still $R_F = 1/6$. However, the cost data are now:

	$t = 1$	$t = 2$
development cost	6	7
operating cost	9	15.5
total unit cost	$V_1 = 15$	$V_2 = 22.5$

A quick and dirty calculation would be to take the expected profit at time $t = 1$ with incremental capacity, $E_0(S_1) - V_1 = 19.5 - 15 = 4.5$, and discount it back one period at the riskless rate, yielding 3.86 as a rough indication of the break-even value of incremental capacity. But there is no obligation to develop and operate at $t = 1$ if incremental capacity has been

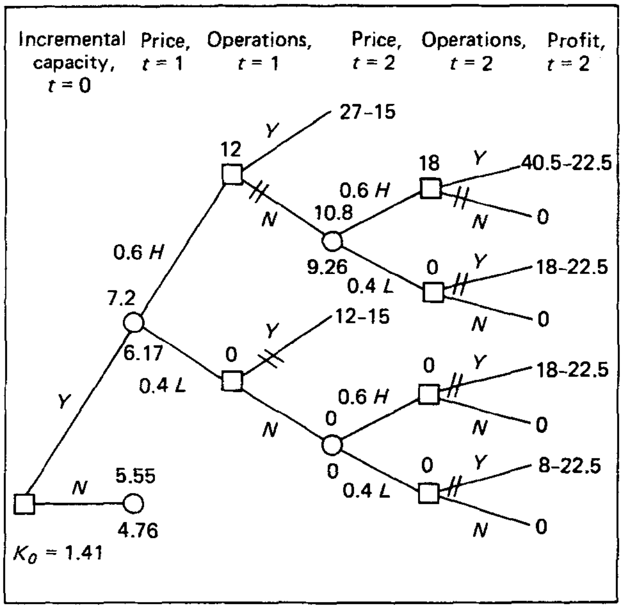

Figure 5. Break-even value of incremental capacity.

procured. Also, without incremental capacity there remain the options to develop and operate at $t = 2$.

The decision tree Figure 5 reflects an option approach to valuing the incremental capacity. Note that the imputed probability $p = 0.6$ as well as the decision options open to management are explicitly entered into the analysis.

Table 2 summarizes the computations for various sets of probabilities, risk-free rates of interest, and discount rates. Its panel (a) is based on a risk-free interest rate of 1/6, whereas a zero risk-free rate is used for panel (b). It is just a coincidence that a constant and common RADR of $-1/36$ happens to yield all the correct values, and in any case it would be most difficult to figure that out in advance.

Value of flexibility with a compound option

The two previous sections considered a joint development and operation option, where the development and operation decisions were simultaneous and identical. This section models these decisions as being taken successively. Operations are only possible when the field has already been developed. Development is then an option on the operations option. Before the development decision is being taken, the satellite field owner or lessee has a compound option, ie an option on an option.

Assume that the development decision has to be made either at $t = 1$ or at $t = 0$. In other words, the European development option expires alternatively at $t = 1$ or $t = 0$. A longer time to expiration may be considered as increased flexibility, with a value of its own.

The development cost at $t = 1$ is 6, whereas it is $36/7 = 5.14$ at $t = 0$. Of course, there is no reason why the development cost should increase at the riskless rate $R_F = 1/6$. As by assumption there is no longer any incremental capacity available at time $t = 1$, any operations decisions will be made at $t = 2$, whether the development option has been exercised at $t = 0$ or $t = 1$. The unit operations cost at $t = 2$ is 15.5 as before.

Figure 6 shows the compound option evaluation, when the European development option expires at $t = 1$ and must be exercised to retain the European operations option expiring at $t = 2$.

A similar tree can be drawn for the case when the compound option's first component (ie the development option) expires immediately. Panel (a) of Table 3 compares the value of the compound option for different expiration times of the development option. In all cases it is assumed that operations can only take place at $t = 2$, and only if the field has been developed.

Panel (b) of Table 3 tries to give an example of a more precise meaning of the slippery term value of flexibility. A decision-maker maintains flexibility with respect to a particular decision as long as she has more than one choice alternative. Being able to postpone the decision of whether to develop yields increased flexibility. The economic value of that flexibility can be found in the panel.

Table 2. Break-even valuation of incremental capacity.

Probability	Risk-free interest rate	Discount rate	Incremental capacity value With	Without	Break-even
(a) $p = 0.6$	$R_F = 1/6$	$R_F = 1/6$	6.171	4.761	1.410
$q = 0.5$	$R_F = 1/6$	$R_F = 1/6$	5.143	3.306	1.837
$q = 0.5$	$R_F = 1/6$	$RADR = 1/4$	4.800	2.880	1.920
$q = 0.5$	$R_F = 1/6$	$RADR = -1/36$	6.171	4.761	1.410
(b) $p = 0.4$	$R_F = 0$	$R_F = 0$	4.800	2.880	1.920
$q = 0.5$	$R_F = 0$	$R_F = 0$	6.000	4.500	1.500

(c) Discounted value of expected profit $t = 1$ with incremental capacity: 3.86.

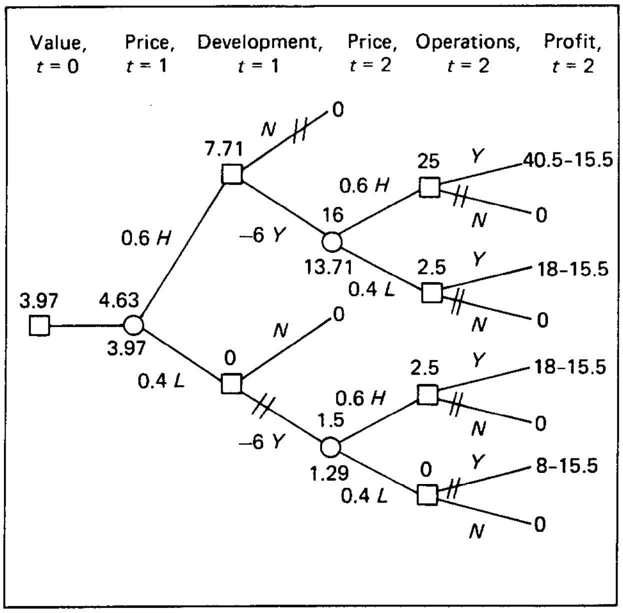

Figure 6. Value of compound option.

A continuous oil price process

From now on it is assumed that the oil price movements in time are smooth and continuous. That is, there is an infinitesimal short interval between each price change, and all price changes are infinitely small. Figure 7(a) is a typical picture of how prices may move over time under the new assumption, which rules out jumps in the price path.

Technically, the oil price is assumed to follow a diffusion-type process involving a standard Gauss-Wiener process (see eg [16]). Therefore, future oil prices are lognormally distributed, as sketched in Figure 7(b).

With a judicious choice of parameters, the binomial price model and the current continuous model become consistent. Appropriate formulas are widely available, as in Cox and Rubinstein ([5], p 200) or, for a somewhat different set, see Jarrow and Rudd ([12], p 186). Recall that the numerical parameters in the discrete case resulted in the growth rate of the oil price having a mean of zero and a standard deviation of $\ln u$.

It can be shown that the results $E(g) = 0$ and $\sigma(g) = \ln u$ carry over to the continuous price dynamics case, but with g now interpreted as an annualized instantaneous growth rate.

The famous Black–Scholes formula is a powerful result for valuation with such a continuous price process. It is stated throughout the option literature (eg [2], p 446 or [5] p 211]) and may be written as

$$C = SN(d_1) - Ke^{-R_F T}N(d_2) \tag{5}$$

where

$$d_1 \equiv [\ln(S/K) + R_F T + \sigma^2 T/2]/[\sigma\sqrt{T}]$$
$$d_2 \equiv d_1 - \sigma\sqrt{T}$$

and $N(\cdot)$ is the standardized cumulative normal distribution.

Replacing the binomial formula (4), the Black–Scholes formula (5) has the same structure. It also yields a recipe for a portfolio duplicating the cashflows from holding one call for one period: the first term shows the amount to be invested in the corresponding stock, partly financed by riskless borrowing the amount given by the second term.

Field values with continuous oil price

As the development and operations decisions at the outset were assumed to be simultaneous and identical, they may be considered as one single option when valuing the satellite field. Therefore, the Black–Scholes formula is applicable. The parameter values to be plugged into Equation (5) are:

$S = 18$ (current oil price)
$K = 22.5 = V_2$ (exercise price = unit cost)
$T = 2$ (time to expiration)
$R_F = \ln(7/6)$ (annualized instantaneous risk-free rate of interest)
$\sigma = \ln(3/2)$ (standard deviation of annualized instantaneous growth rate (return on holding the asset)).

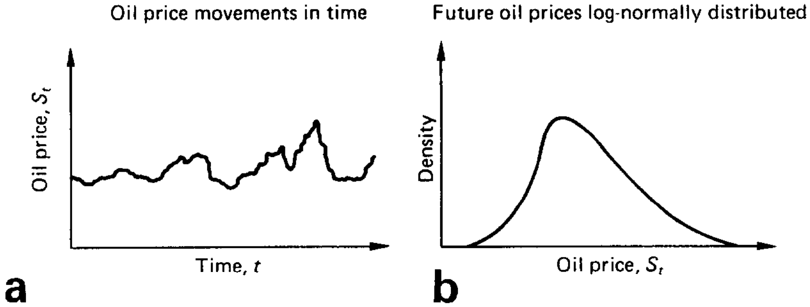

Figure 7. Continuous oil price process.

With these parameters, the Black–Scholes formula shows that the satellite field has a value of 4.67. This continuous price dynamics case value is close to the value 4.76 previously obtained for the discrete price case.

The Black–Scholes formula can no longer be used when the development decision and the operations decision are separated in time. Fortunately, Geske [11] has developed a corresponding formula for a two-stage compound option. This formula is quite complicated and will not be reproduced here (see eg [5], p 414), but it builds on a recursive procedure which will be sketched briefly. First, the conditional value of the second-stage option as of $t = 1$ is computed by the Black–Scholes formula. The important feature is that at $t = 1$ there exists a cut-off value V^* for the price V_1 of the underlying asset (note that the standard notation for compound options differs from that used elsewhere in this paper). The first-stage option will then be exercised if and only if $V_1 > V^*$. Next, further computations involve the current price as well as the present values of the exercise prices of the two constituent options, all terms being multiplied by particular probabilities from bivariate or univariate normal distributions.

Figure 8 presents the general structure of the continuous price case compound option, where development may be decided only at $t = 1$ and operations may be decided upon only at $t = 2$, provided the field has been developed.

Using Geske's formula, it turns out that the critical value $V^* = 18.55$. Thus, the field will be developed at $t = 1$ if and only if the oil price at that point in time exceeds 18.55. Subject to interpolation errors, the compound options is found to be worth 3.67 as of $t = 0$. In panel (a) of Table 3 a corresponding value of 3.97 was found for the binomial price process.

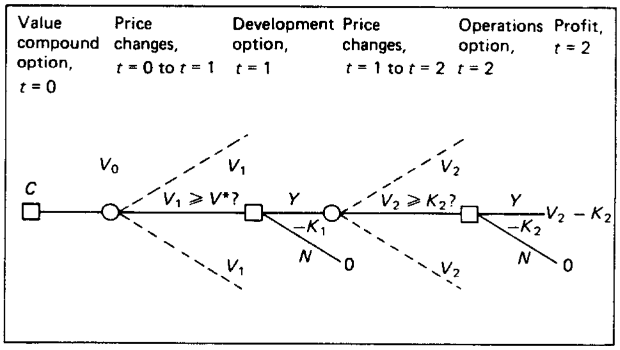

Figure 8. Compound option with continuous oil prices.

Incremental capacity value with continuous price

Figure 9 depicts the problem of valuing incremental capacity for the continous price case. Without incremental capacity, a joint development-operations decision will be made at $t = 2$. In the previous section that option was found to have a value of 4.67. With incremental capacity, the option may be exercised either at $t = 1$ or at $t = 2$. Finding a closed-form solution to that option value appears to be rather challenging.

A cursory literature search indicated some promising sources:

(i) value of an option with uncertain exercise price (Fischer [8]);
(ii) value of an option to exchange one asset for another (Margrabe [15]); and
(iii) value of an option on the minimum or maximum of two risky assets (Stulz [24]).

Unfortunately, it seems as if none of these contributions quite cover the problem at hand. Therefore, one may so far have to settle for either the binomial model

Table 3. Compound option of successive development and operations options.

(a) **Development option may be exercised only at**	**Value**
$t = 2$	4.76
$t = 1$	3.97
$t = 0$	2.35

Assumption: the conditional operations option may be exercised only at $t = 2$.

(b) Value of extra flexibility by postponing development decision:
From $t = 0$ to $t = 1$: $3.97 - 2.35 = 1.62$
From $t = 1$ to $t = 2$: $4.76 - 3.97 = 0.79$

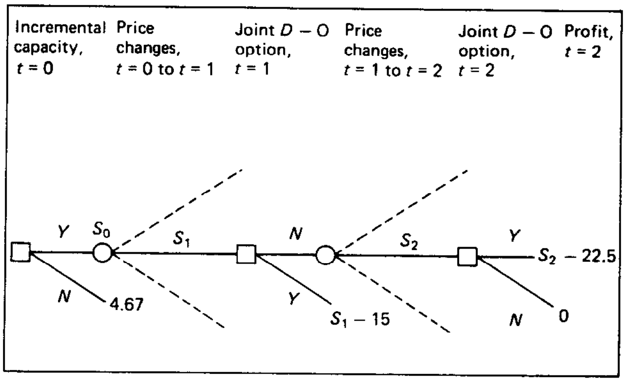

Figure 9. Incremental capacity problem with continuous oil price process.

(possibly with more and shorter time periods) or numerical methods (Jarrow and Rudd [12], pp 202–208).

Some concluding remarks

The admittedly simplified examples of this paper may nevertheless suggest that the option pricing approach to project evaluation may give worthwhile supplemental insight into project profitability. The option analysis may yield results partly conflicting with the recommendations of traditional capital budgeting methods. The paper should not be construed as an attempt to peddle the option analogy to real investments as an all purpose snake oil, curing the various shortcomings of traditional techniques for project assessment. At this stage, a comprehensive and systematic comparison of the pros and cons of the options framework would be somewhat premature, awaiting the outcome of Myers' [19] call to arms for further research on real options.

So far, the option approach is still in its infancy. The challenge remains to develop project evaluation methods which are both applicable in practice and have a sound theoretical basis. Progress in this respect is quite important for capital investment projects where huge amounts of money are at stake, such as in the petroleum industry.

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Appendix

Some previous literature on real options

Galai and Masulis [10] suggested using option pricing theory for corporate investment decisions. Emery *et al* [7] and Rao and Martin [23] disagreed with respect to whether option pricing models would give reasonable capital investment decisions. In a contribution of substantial theoretical interest, Banz and Miller [1] proposed using options to deduce the value of state-contingent claims, which in turn in principle might be used for valuing real projects as well.

In his dissertation Tourinho [25] originated the application of option pricing to valuation of reserves of natural resources. Brennan and Schwartz [3] used the self-financing portfolio approach to evaluate natural resource investments under output price uncertainty, simultaneously obtaining optimal policies for the underlying development, operating and abandonment decisions. Their simplified version [4] conveyed the main ideas in a form suitable to a general management readership. Paddock, Siegel and Smith [21] developed an option pricing methodology for the valuation of claims on a real asset such as an offshore petroleum lease. In a case study Kemna [13] examined input estimates to be used for oil-based contingent claims analysis.

Myers [19] emphasized that corporate financial theory requires extension to deal with real options, in order to reconcile financial and strategic analysis. Majd and Pindyck [14] used option pricing methods to derive optimal decision rules for sequential investment outlays, when a considerable construction period preceeds any project returns. In a related paper McDonald and Siegel [18] also stressed the option value of postponing an irreversible investment. Both Myers and Majd [20] and McDonald and Siegel [17] applied option-pricing techniques to the investment abandonment decision. Petruzzi [22] derived optimal hurdle rates for real investment opportunities using the option analogy.

A discussion of some options aspects of real capital projects has recently been included in textbooks such as Brealey and Myers ([2], p 429 and pp 450–454) and Franks, Broyles and Carleton ([9], pp 562–565).