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Incorporating technical uncertainty in real option valuation of oil projects

M. Armstrong[a,*], A. Galli[a,1], W. Bailey[b,2], B. Couët[b,2]

[a] CERNA, Ecole des Mines de Paris, 60 Blvd. Saint Michel, 75272 Paris, France
[b] Schlumberger–Doll Research, 36 Old Quarry Road, Ridgefield, CT 06877-4127, USA

Abstract

Real options are a way of valuing projects such as oil fields, which involve irreversible investment decisions subject to uncertainty. Whereas discounted cash flow (DCF) analysis is based on fixed estimates of costs and revenues, and a predetermined development scenario, real options focus on project flexibility (e.g., being able to defer starting a project, or conversely, to accelerate its development, etc.). Early applications concentrated on flexibility to overcome uncertainty on financial parameters and tended to ignore uncertainty on technical parameters (such as recoverable reserves). In some cases, companies can reduce the latter by acquiring additional information and sequentially updating parameter values and then the estimate of the project's value. This paper addresses the question of how to evaluate the option to acquire more information. The natural way to incorporate the new data is by Bayesian analysis. Choosing a multivariate normal framework for this analysis considerably simplifies the computations but this distribution has symmetric upper and lower tail dependence. To overcome this limitation, we have developed a novel form of Bayesian updating based on Archimedean copulas. We present a case study on production enhancement that shows how to combine Bayesian updating and real options. In this study, the oil company has the option to gather information from a production logging tool (PLT) before carrying out a workover.

Keywords: Real options; Bayesian updating; Archimedean copulas; Production enhancement; Workover; Production logging tool

1. Introduction: a brief history of real options

By the mid 1980s, people had begun to question traditional methods of valuing projects [such as discounted cash flow (DCF)] which are based on fixed estimates of costs and revenues, and a predetermined development scenario. They realised that good managers react to changing circumstances to capitalise on upside potential or to mitigate downside risk by identifying the "options" that are open to them. Typically these include

- deferring or abandoning a project if circumstances are unfavourable or,
- starting it or accelerating its timing if the opposite was true,
- changing the production rate, etc.

According to Brealey and Myers (1991), the first person to have recognised the value of this flexibility was Kester (1984) in an article in the Harvard Busi-

* Corresponding author. Tel.: +33-1-4051-9313; fax: +33-4407-1046.
E-mail addresses: margaret.armstrong@ensmp.fr (M. Armstrong), alain.galli@ensmp.fr (A. Galli), wbailey@slb.com (W. Bailey), couet1@slb.com (B. Couët).
[1] Tel. +33-1-4051-9313; fax: +33-4407-1046.
[2] Tel. +1-203-431-5000; fax: +1-203-438-3819.

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ness Review.[3] The following year, Mason and Merton (1985) reviewed a range of applications to corporate finance and Brennan and Schwartz (1985) applied option pricing techniques to the evaluation of irreversible natural resource investments using the Chilean copper mines to illustrate the procedure. This paper is widely recognised as a key contribution to the subject. At about the same time, McDonald and Siegel (1984, 1985, 1986) developed similar ideas. Other developments followed rapidly: the spring 1987 issue of the Midland Corporate Finance Journal contained several articles on real options and capital budgeting, including one by Siegel et al. (1987).

Applications of real options to the oil industry followed quickly. Paddock et al. (1988) evaluated offshore petroleum leases; Ekern (1988) worked on evaluating oil projects and Copeland et al. (1990) presented a case study involving an option to expand oil production. Kemma (1993) studied a timing option for developing a new lease area in which the company was obliged to drill extra wells and, in order to simplify the problem, he assumed that extra wells provided no useful additional information. Dixit and Pindyck (1994) described several oil-related applications including sequential decision-making for opening up oil fields and a study on building, mothballing and scrapping oil tankers. In a study of two Norwegian fields, Laine (1997) showed that the deferral and abandonment options are most valuable when the field is marginal, whereas the expansion option is most important if the field turns out to be more profitable than anticipated. Walkup and Chatwin (1999) demonstrated that valuing managerial flexibility, corporate synergies and shareholder risk assessment could lead to far more accurate valuations and better strategies.

By the mid 1990's, several important textbooks had been published (Dixit and Pindyck, 1994; Trigeorgis, 1996), and the range of applications had widened to include high-risk, high-tech industries such as research and development (R&D; Smit and Trigeorgis, 1999; Pennings and Lint, 2000), pharmacy and biotechnology (Favato, 1999; Micalizzi, 1999; Ottoo, 1998), information technology infrastructure (Panayi and Trigeorgis, 1998; Campbell, 2002), internet banking (Courchane et al., 2002). Another interesting application is in aircraft sales where manufacturers provided customers with a wide range of purchase options (delivery dates, aircraft type, etc.). Initially, they did not price this flexibility into the contract price (see Stonier and Triantis, 1999; Sick, 1999). In a similar vein, applications have been made to pricing telephone contracts (Choo et al., 2002) and electricity contracts (Oren, 2001).

In many industrial projects, management has the option to acquire additional information in order to reduce uncertainty on technical and/or financial parameters before committing to an irreversible investment decision. Although this option can add value and reduce risk, it attracted little attention until the late 1990s. Gallant et al. (1999) explicitly recognised the importance of "learning models to capture dynamic complexity" in the oil industry. Chorn and Carr (1997) investigated the value of purchasing information to reduce risk in capital investment projects. Chorn and Croft (1998) showed the value of reducing reservoir uncertainty via real options. Dias (1997, 2002) studied the effect of timing in E&P projects and explicitly modelled the value of learning. One common feature of all these papers is that the essential characteristics of the reservoir were not modified as a result of the information gathering process. As Dias explained, the new information "revealed" more about its true state. In our case study, as we will show in Section 4, the information changes the way in which the engineers design the workover, and this materially changes the flow rates.

From a statistical point of view, Bayesian analysis is the natural way to update distributions (see Pratt et al., 1995). Hatchuel and Moisdon (1997) and Lund (1999) presented typical case studies of Bayesian updating where the reserves are divided into two or three broad categories (high, medium, low) and where the probabilities for each category are conditional upon the true reservoir volume. Once the prior probabilities for each reserve category are known, Bayes theorem can be used to deduce the probability of high/medium/low reserves given new observations. Hatchuel and Moisdon used decision theory (without explicitly mentioning real options) while Lund evaluated real options using stochastic dynamic programming.

By the end of the 1990's, Bayesian analysis had started to be incorporated into real option evaluations. Herath and Park (2001) considered the relationship between Bayesian decision-making methods and real

[3] It seems that Brealey and Myers missed Tourinho's unpublished thesis (1979).

options. Galli et al. (2001) studied a small satellite field in the North Sea where the company had to decide whether to drill one or two more producers after recompleting the discovery well as a producer. Management also had the option to reappraise the reservoir after one more well had been drilled. Based on the seismic profiles, the reservoir was considered as a tilted block bordered on one side by a sealing fault. Because of the simple geometry, if the top of the reservoir at the new well is high above the gas–water contact, then the reservoir is probably relatively large; and conversely, if the reservoir is small, there may be little point in drilling a second supplementary well. Therefore, knowing the height of the top above the contact provided important information on the reservoir's size and its economic potential.

In order to model complex situations in the real world, simplifications have to be made. In real option evaluations of the impact of new information on oil projects, two types of simplifications are common:

1. assuming that the new information reduces the variance but does not change the type of distribution;
2. splitting possible values of the technical parameters into a few discrete categories.

Our aim in this paper is to present a more general approach to Bayesian updating. Firstly, the new information can radically change the distribution of the parameters. Secondly, although we assume that the technical parameters have continuous distributions, no particular parametric distributions are specified. As the data have a continuous distribution, they can be transformed to have any desired marginal distribution (e.g., normal or uniform). Details are given in Appendix 1. The normal distribution provides a very convenient framework for Bayesian analysis because the computations are much simpler (otherwise, MCMC methods are required). This framework can be used if the joint distribution of the variables is multivariate normal or if it becomes multivariate normal after the marginal distributions have been transformed to normality. Despite its advantages, the multivariate normal has the disadvantage of having symmetric tail dependence. Copulas provide an alternative way to model multivariate distributions. In contrast to the normal distribution, they can have different types of upper and lower tail dependence. As Archimedean copulas are one of the simplest families of copulas, we have used these to develop a new type of Bayesian updating.

The paper is structured as follows. Section 2 briefly reviews financial options and their pricing, and shows how this has been extended to evaluating projects. Section 3 is devoted to Bayesian updating. A simplified case based on a small gas reservoir is used to illustrate the concepts. In Section 4, we present a case study where this analysis was used to update estimates of the reservoir parameters, and hence to evaluate the option to obtain additional information. Workovers are designed to reduce water influx in a well. Logging the well with a production logging tool (PLT) prior to the workover provides useful information on inflow location, phase distribution and rate, which can lead to improved engineering design and subsequent economic viability. We assess the value of this PLT information. The conclusions follow in the last section.

2. From financial options to real options

Classical financial options are contracts which give their owner the right, but no obligation, to buy (or sell) a specified number of shares or a quantity of a commodity such as gold or oil, at or before a specified date. If the transaction has to take place on that date, the options are referred to as European; if they can be exercised at any date prior to that date, they are referred to as American options. The central problem with financial options is working out how much the owner of the contract should pay for this buy/sell flexibility. Hence, the name "option pricing". Basically, the price is like an insurance premium; it is the expected loss (under a risk-neutral probability) that the writer of the contract will sustain.

The standard model (Black and Scholes, 1973) assumes that the spot price follows a geometric Brownian motion. That is, it satisfies the stochastic partial differential equation:

$$\frac{\mathrm{d}S_t}{S_t} = \mu \ \mathrm{d}t + \sigma \ \mathrm{d}W_t \tag{1}$$

where S_t is the stock price; W_t is a Brownian motion; μ is the drift in the stock prices; and σ is the volatility (i.e., standard deviation). One advantage of this model is that closed-form solutions to Eq. (1) exist for European options (puts and calls). This makes it quicker and simpler to evaluate them. It is more difficult to evaluate American options because they can be exercised at any time up to the exercise date. In general, binomial (or trinomial) trees are used in such cases. For more complicated types of options (often referred to as 'exotics'), Monte Carlo simulations are used to generate realizations of the spot prices, and hence to evaluate the option value. For more information on this model and financial options in general, readers can consult Hull (2003).

In addition to Black–Scholes, other models have been proposed for commodities, such as oil, which are affected by supply and demand. Several recent papers (Cortazar and Schwartz, 1997; Cortazar and Casassus, 1998; Spahr and Schwebach, 1998; Slade, 2001) have advocated using a mean reverting model such as Ornstein–Uhlenbeck. Another factor that affects commodity prices is convenience yield. Brennan (1991) defines it as the flow of services which accrues to the owner of a physical inventory but not to the owner of a contract for future delivery. The best-known model that incorporates convenience yield is Schwartz's (1997) two-factor model. An extension by Lautier and Galli (2001) introduces asymmetry not available in the former. Both models rely on latent (nonobservable) variables whose parameters have to be inferred, e.g., using a Kalman filter (Lautier and Galli, 2001; Javaheri et al., 2003). As our primary aim is to model the impact of new information, we will use a Black–Scholes model with no convenience yield.

In mathematical terms, the payoff from a call option with a strike price, K, is the $\max(S_t-K,0)$, and the corresponding premium, C, is its expected value (averaging over all possible prices):

$$C = E_Q[\max(S_t - K, 0)] \tag{2}$$

where the subscript $_Q$ denotes the expectation calculated using risk-neutral probability and not the historic one. This is a consequence of the nonarbitrage assumption.

One point about financial options is that, as their owner knows the true price at the time when he/she must choose whether to exercise, their value is always positive. In contrast to this, the owner of a project rarely possesses perfect technical and financial information (e.g., recoverable reserves, recovery factors and commodity prices) when making decisions. These have to be based on the expected value of the project based on parameter estimates rather than on true values. Consequently, there is a real risk of a subsequent loss. This uncertainty has other follow-on consequences: it can lead to suboptimal development plans for an individual reservoir and to suboptimal management of a portfolio of assets.

A key step in extending option pricing to evaluating projects is determining what level of flexibility is available to the company as the project proceeds. That is, to what extent can the initial development scenario be changed. This requires an in-depth analysis of each project. At the outset, case studies were carried out by specialists in finance who focused on financial flexibility and tended to ignore the technical (or material) aspects of the project. As real options are now applied to a wider range of projects, more emphasis has been placed on defining the available flexibility and evaluating its impact. In many cases, additional flexibility comes from being able to react to information on key technical parameters which will be better known as time passes.

Suppose we wish to evaluate a project subject to uncertainty on one or more technical parameters θ (such as STOIIP, permeability, water saturation, porosity, etc.) as well as the oil price S_0. If the parameters θ were known, then standard option pricing techniques could be used. If the distribution of θ is known (but not its precise value), then the expected value of the project can be obtained by a two-stage procedure: by averaging firstly over all possible prices for a given set of technical parameters, then averaging this over all possible values of θ (i.e., deconditioning), thus:

$$\text{Value} = E_\theta[E_{S_0}[V(S_0,\theta)\,|\,\theta]] \tag{3}$$

where the $V(S_0,\theta)$ is the value of the project for known values of S_0 and θ. Galli and Armstrong (1997) show how to incorporate uncertainty on reserves into Brennan and Schwartz's model by deconditioning over possible reserve sizes. In cases where more information becomes available sequentially, the project's value (or the distribution of possible values) can be updated using Bayesian analysis. The distribution at the outset is called the *prior* distribution; after having been updated, it is called the *posterior* distribution. At the

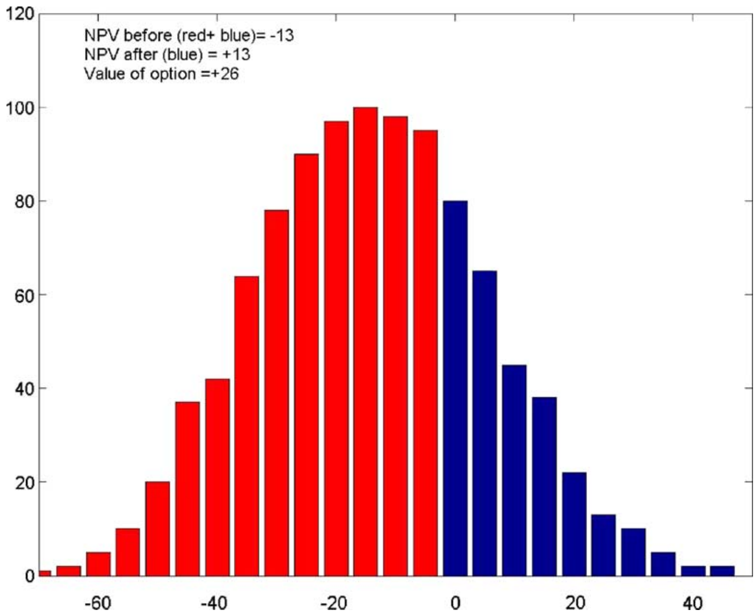

Fig. 1. Histogram of the expected values (EV_1) of the small gas field given new information from the addition well plus the long-duration well test. As the initial oil price was low, the average expected value (EV_0) without additional information was − 26. The mean EV_1* of all the projects where EV_1 was positive (shown in blue on right), turns out to be + 13 units. Therefore, the option to drill a new well and carry out a long well test has a net value of 26 in this case.

outset (time = 0), we compute the expected value of the project using the prior distribution. This value depends on the initial oil price S_0 and is denoted by

$$\mathrm{EV}_0 = E_\theta^0[V(S_0,\theta)] \tag{4}$$

When some new information, I, becomes available, the updated value given the new data is

$$\mathrm{EV}_1 = E_I[V(S_0,\theta)\,|\,I] \tag{5}$$

From an economic point of view, the project will go ahead if the expected value is positive (or higher than a threshold value). The fact that EV_0, or even EV_1, is positive does not necessarily guarantee that the project value will be positive in all cases because the information on the reservoir is still incomplete and because prices can change. One difficulty in using Eq. (5) is that, at time 0, the new information is not yet known, and as such, we cannot evaluate its true worth. However, the nature of the information-gathering procedure is usually known (e.g., how many wells are to be drilled, where and whether they will be logged), and so it may be possible to express the distribution of the updated value as a function of I. This allows us to compute expectations, e.g., the expected value, when this new value is positive:

$$\mathrm{EV}_1{*} = E[\mathrm{EV}_1 \; I_{\mathrm{EV}_1>0}] \tag{6}$$

where I denotes the indicator function. The option value of the information is then max(EV_1*− EV_0,0). To illustrate these concepts, we present a simplified example of a small gas field.

2.1. Simplified example of a small gas field [4]

The available information indicates that the field is small so that only limited work is warranted. The distribution of the gas initially in place (GIIP) has been calculated using stochastic simulations; the internal architecture of the reservoir has been modelled, and some dynamic flow simulations have been carried out. Moreover, at the time when the project was being evaluated, the oil price was relatively low. This, to-

[4] This case study was presented at the 2002 SPE Applied Technology Workshop on Risk Analysis Applied to Field Development Under Uncertainty, held 29–30 August 2002, in Rio de Janeiro, Brazil, and is available at www.cerna.ensmp.fr.

gether with the high level of uncertainty on both the reserves and the productivity, made the project unattractive. Its value was negative (say, − 13 units). As such, management was considering drilling a new well and carrying out a long-duration well test. The production was assumed to decline exponentially after the plateau. The length of the plateau and the productivity of the wells were strongly correlated with the new information gathered. The forward gas price was modelled as a Brownian motion with a volatility of 20% and a convenience yield of 2%. The expected value EV_1 has been evaluated as a function of the result of this new information. Its histogram is shown in Fig. 1. The projects where EV_1 is negative are shown in red whereas those where it is positive are shown in blue. The mean EV_1* computed for the projects where EV_1 was positive turned out to be + 13 units. Therefore, the option to drill a new well and carry out a long well test has a net value of 26 in this case, and the project can now be considered in a different light.

3. Bayesian updating

The first step in a Bayesian analysis is to specify the prior distribution of the parameter θ of interest and

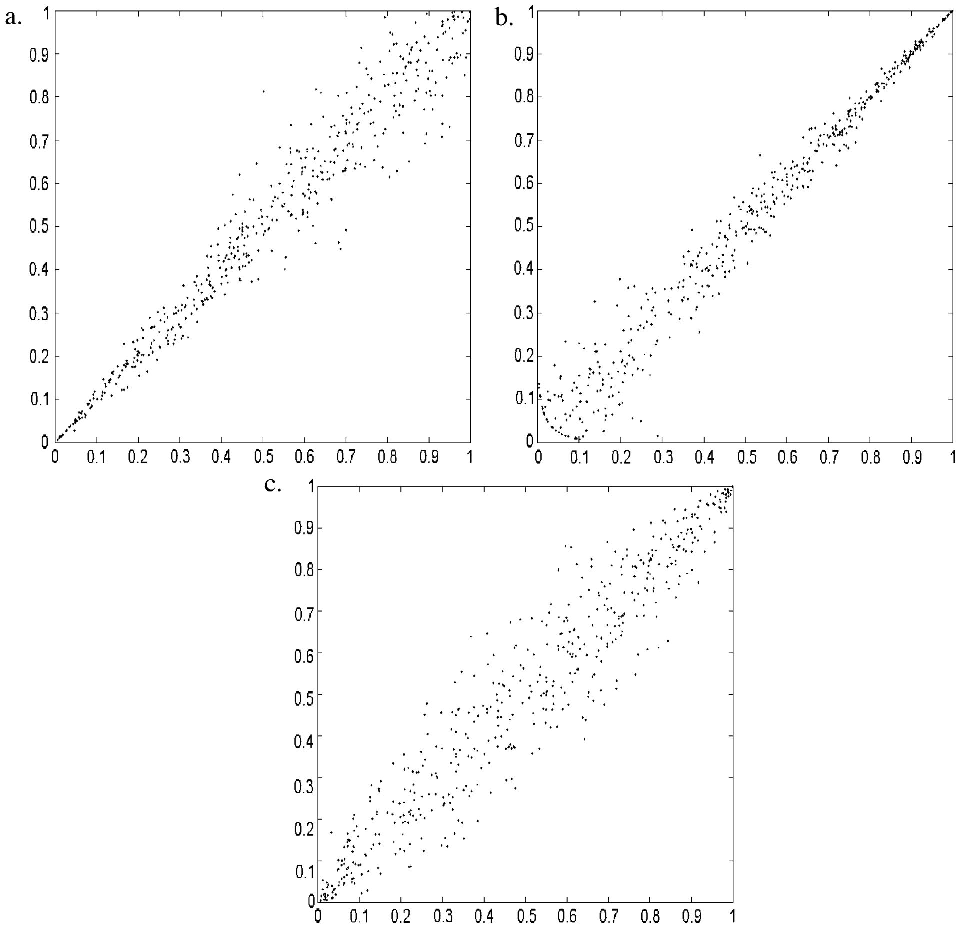

Fig. 2. (a), (b) and (c): Five hundred realizations of the two 1-parameter Archimedean copulas: Nelsen No. 1 Clayton with $\alpha = 15$ (top left); Nelsen No. 2 with $\alpha = 19$ (top right) and the normal distribution (below). The Clayton copula shows lower tail dependence; conversely, the Nelsen No. 2 copula shows upper tail dependence. In contrast to these two, the normal distribution has symmetric upper and lower tails. Note that the normal variates have been transformed to have uniform margins for comparison purposes.

the conditional distribution of the new information x given θ. Let $\pi(\theta)$ and $\pi(x|\theta)$ denote these two distributions. The aim is to find their joint distribution, and hence the conditional distribution of the updated parameter $\pi(\theta\,|\,x)$. (If we were interested in several parameters, then θ would be the vector of parameters and similarly for x).

3.1. Standard approach using conjugate distributions

Where possible, statisticians choose conjugate priors to make the mathematics tractable. The best-known pair is normal–normal. We will use it here. As information on Bayesian analysis is readily available (see for example, Robert, 1994), we will not go into details here. The key results are that the joint distribution is bivariate normal, the posterior distribution $\pi(\theta|x)$ is normally distributed and its mean is a linear combination of the prior mean and the observed value x (expressed as a normal score). Secondly, the correlation coefficient between θ and x can be expressed in terms of the prior standard deviation σ_0 and that of x, σ. This will be used in the case study to choose suitable values for the standard deviations.

In practice, variables are rarely normally distributed. Therefore, they have to be transformed to the equivalent normal scores. The Bayesian updating is carried out in the normal framework, then the results are transformed back to their original scale. Details are given in Appendix 1.

3.2. Bayesian updating using copulas

An alternative way of modelling multivariate distributions is by using copulas. In order to focus on the correlation structure without the effect of the marginal distributions, the variables are transformed to have uniform margins on (0,1). Archimedean copulas are one family with a wide range of tail dependence. The ones used here depend on a single parameter α. To illustrate their properties, we simulated 500 pairs of points from two such copulas (Fig. 2a and b). The first one (top left) shows lower tail dependence, and the second (top right) shows upper tail dependence. In the first case, the tail dependence is higher for low values than for higher ones, and conversely for the second. This contrasts sharply with the normal distribution which is symmetric in its upper and lower tails (Fig. 2c).

A new type of Bayesian updating based on Archimedean copulas has been developed. The interesting types of tail dependence flow through to the updated estimates first of parameters, and then to the project. Details of the mathematics are given in the Appendix 2. Readers can consult Nelsen (1999) or Joe (1997) for comprehensive reviews of copulas and their properties.

4. Case study on oilfield production enhancement

Ideally, oil wells should produce oil with little or no water. As the quantity of water entering the well tends to increase while that of oil declines, some wells benefit from periodic workovers to maintain hydrocarbon production at satisfactory economic levels. Information about the location of fluid inflow, the quantity of production and the water cut (i.e., the proportion of water produced in the total fluid stream) is vital to petroleum engineers, especially when planning any intervention to increase the productivity of a well. Such useful information can be obtained from a production logging tool (PLT). When commissioning such an intervention, the asset holder may have to decide whether the additional information gained from a PLT is justified or not from a financial standpoint. The aim of this study was to develop an objective procedure for evaluating the financial potential of this information.

In our example on a single well (which is based on work by Bailey et al., 2002), the asset holder can choose between three possible scenarios (operational alternatives):

- J_0: continue production as is (no workover or production enhancement),
- J_1: production enhancement without running the production logging tool,
- J_2: production enhancement after running the production logging tool.

Here, we focus on the option to run the PLT in the well. That is, we are interested in the relative value of J_2 compared to J_1. We assume that engineering, planning and rig mobilization will take 6 months. The financial impact of these alternatives will be compared over a 1-year period (the 6 months preparation time plus 6 months production after the intervention).

After the well intervention, the asset holder can decide whether to stop production if the oil price or certain technical parameters turn out to be unfavourable and the operation loses money. One extreme case would be for the company to turn the flow-control valves off (on) as soon as the instantaneous net revenue becomes negative (positive). The other extreme is to continue until the end of the full 12-month trial period (that is 6 months worth of actual production after intervention). In the latter case, the value of the difference between scenarios J_2 and J_1 could be negative for certain realizations even if its expected value at the outset is positive. Many intermediate stopping rules can also be imagined. Although the first definition is equivalent to standard financial options, it does not correspond to industry practice. The second case (i.e., continuing till the end of the trial period) is more realistic. In many respects, it is similar to an Asian option on the oil prices.

For both scenarios, the income at time t is the product of three terms: the current oil price, the initial oil flow rate (ML) and an exponential decline term (which is assumed to be known through analysis of historical field production data). The production cost at time t depends on the initial rate of oil production (and its decline factor) and the water cut (and its associated increase over time). The initial oil flow rates are assumed to have triangular distributions (see Table 1). The initial values of the water cut are assumed to be known (65% for J_1 and 50% for J_2); afterwards, these increase at different rates (10% and 15% per annum, respectively). The rig costs include a fixed cost per day plus fixed and variable costs for products and services. A geometric Brownian motion (i.e., Black–Scholes model) with a volatility of 10% and a risk-free rate of 3% was used to model oil price. Monte Carlo methods were used in computing the impact of new information.

Table 1
Parameter values for the distributions for the initial oil production rate for scenarios J_1 and J_2

	Minimum oil production (bbl/day)	Maximum oil production (bbl/day)	Modal oil Production (bbl/day)	Water cut (fraction of total fluid produced)
J_1	3500	5100	3900	0.65
J_2	3250	4000	3500	0.50

Reservoir engineers have determined that predicted oil production rates could be represented by a simple triangular distribution. The oil production rates given below reflect the fact that after J_2 information, engineers can better locate points in the well where there is excessive water production. While the production rates in this specific example are lower after a J_2-type intervention, the associated NPVs are likely to be greater as less water is being produced as unit variable cost is reduced.

From reservoir simulation and geological analysis, petroleum engineers have determined that the total flow rates for a J_2-type workover would be lower on average and would have a lower variance than J_1. As the reduced oil production in J_2 is accompanied by a corresponding decrease in water production due to the reduced water cut, the cost savings due to producing less water more than compensate for the smaller gross revenues from reduced oil production observed here.

One key difficulty turned out to be in finding a meaningful way of comparing scenario J_2 with scenario J_1, so as to reveal the impact of the new information. To decide this, we carried out a simple ‘‘thought experiment’’. Suppose that the well was relogged but that the results were not given to the engineering team. A high-quality reservoir (one where deliverability of hydrocarbons was assured and plentiful) would still be a high-quality reservoir even if the engineers did not know this. As such, our working hypothesis is that relogging the well will not change the inherent quality of the reservoir; it merely reveals reservoir quality and deliverability to the engineers undertaking the production enhancement design. In mathematical terms, we assume that the quantiles of the ML distributions are invariant from one scenario to another.

4.1. Results of Bayesian updating of the oil flow rate

In order to specify the bivariate normal distributions, we need an estimate of the correlation between x and θ. As we expect it to be high (e.g., 0.95), the values of the standard deviations, σ_0 and σ, were set to $\sigma_0 = 0.95$ and $\sigma = 0.3122$ to achieve this. Similar considerations were used to set the parameter value for the two Archimedean copulas. The posterior distributions were computed for these three cases. Then, the marginal distributions were transformed to the appropriate triangular distribution.

The three posterior means turned out to be very similar but the predicted quantiles were quite different. Fig. 3 shows the 10% quantile (lower curve), the mean

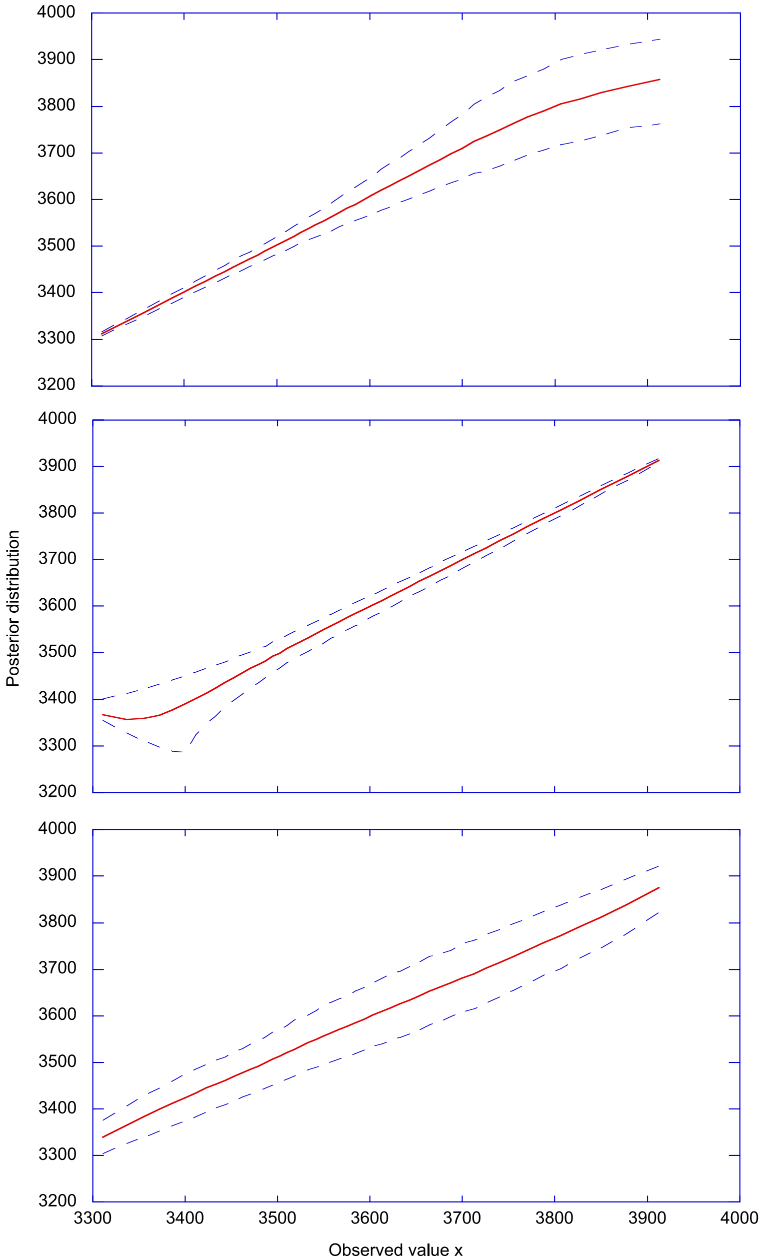

Fig. 3. The 10%, 50% and 90% quantiles of the posterior distribution plotted as a function of the observed value x for the Nelsen copula No. 1 (top) and No. 2 (middle), as well as for Bayesian updating (lower). The 10% quantile is shown below the median, the median is shown in black and the 90% is shown above the median curve. In all cases, the margins are triangular distributions.

(middle curve) and the 90% (upper curve) of the posterior distribution plotted as a function of the new information x for both copulas and for Bayesian updating with a normal prior (lower figure). For the first copula (top), the three quantiles are, as expected, tightly grouped for low values but spread out for high ones because of its lower tail dependence, and conversely for the second copula (middle) which is upper tail dependent. The clear barrier seen in Fig. 2 for this copula is the root cause of the curious behaviour of the quantiles for very low values: namely instead of decreasing with x the curves rise. This shows how important the choice

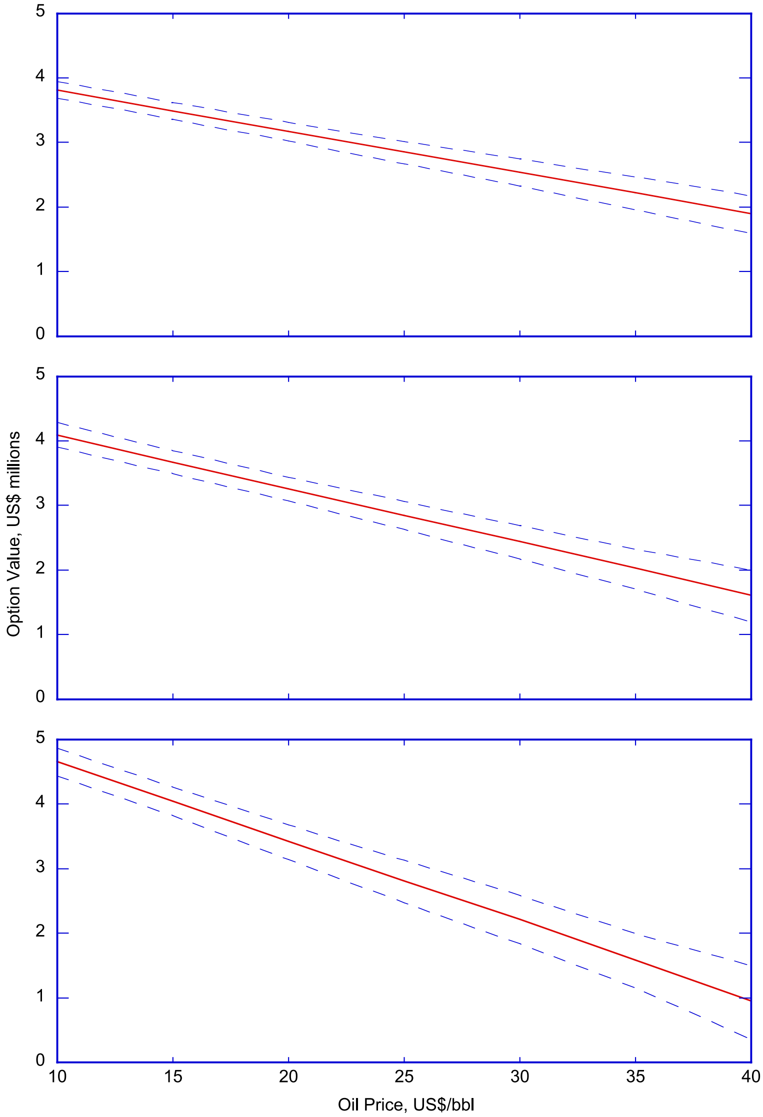

Fig. 4. The solid lines represent the average value of the option as a function of the oil price while the dotted ones represent the 10% and 90% quantiles, all for the case of Bayesian updating. The figure in the centre corresponds to the case where the sample value obtained by relogging the well was the median; the upper and lower figures correspond to samples at the 10% and 90% quantiles of the triangular distribution. Initially, we were surprised to see that the value decreased with increasing oil price. This is because we were plotting the incremental difference $J_2 - J_1$ rather than scenario J_1 or J_2.

of the model for the joint distribution is even if it is hidden within the updating procedure.

4.2. Evaluation of the option to relog the well to obtain additional information

The incremental value ($J_2 - J_1$) of obtaining new production information from running a PLT in the well was evaluated for a wide range of oil prices ranging from US$10 to US$40 per barrel, using the posterior distributions calculated by Bayesian updating. While the average value will be used to decide whether to go ahead with the option, it is also important to know what sort of variability to expect around the mean, for example, if the new information turns out to be better or worse than expected. Fig. 4 shows the 10% and 90% quantiles as dotted lines above and below the mean (solid line). The figure in the centre in black corresponds to the case where the sample value obtained by logging the well with the PLT was the median oil flow rate; the upper figure corresponds to a worse than expected flow rate, and conversely for the lower figure (at the bottom). (In fact, the flow rates were set at the 10% and 90% quantiles of the flow rate distribution.) The results shown in Fig. 4 were obtained using Bayesian updating based on the normal distribution. As the results for copulas were very similar, these are not shown here.

At first, we were surprised that the value of the option reduces as oil price increases. However, these apparently counter-intuitive findings correctly represent the incremental value of logging the well before carrying out the intervention, not the value of the final J_2 scenario itself. The value of all scenarios, including J_2, increases with the oil price, as expected. Note that this result is specific to this case. Under other circumstances (i.e., a different well with a different response and under a different fiscal regime), the option value could well increase as oil price rises. Furthermore, although the financial option value of undertaking this additional production logging run diminishes as the oil price rises, the usefulness of such information from an operational/engineering point of view remains very high—regardless of the price of oil. This is because it provides insight into the reservoir response which assists the decision-making process for similar interventions in neighbouring wells as well as other engineering issues in general.

5. Conclusions

In many industries, information that will become available in the future, or that can be acquired, can add considerable value to projects. This paper addresses the question of evaluating its potential worth in the real option framework of decision under uncertainty. Bayesian analysis is widely used for assessing the value of new information. We have developed a novel type of Bayesian updating based on Archimedean copulas. Whereas the multivariate normal distribution has symmetric upper and lower tail dependence, copulas can provide either upper or lower tail dependence.

The posterior distributions for the fluid flow given the new information were computed using two Archimedean copulas in addition to the normal model. The posterior means were similar in all three cases but the quantiles were quite different. This highlights the importance of the underlying multivariate structure and begs the question as to which one is the most appropriate: how to choose the most suitable copula? There are two ways of answering this question. The first is by collecting a large set of experimental data and analysing it. We hope that companies, having realized the importance of this data, will make the effort to collect it. Until sufficient experimental data becomes available, it is possible to select more promising copulas from those that are clearly unsuitable by looking at their bivariate plots. A catalogue showing the shapes of the 22 one-parameter Archimedean copulas listed by Nelsen (1999) is available on the Cerna's website: http://www.cerna.ensmp.fr/Documents/MA-CopulaCatalogue.pdf.

The most interesting copulas are those which provide a full range of possible correlations from -1 to $+1$ (i.e., in copula terminology, from W to M). If the variables can only have zero or positive correlation, then the copula need only range from Π (independence) to M (perfect correlation). This criterion effectively eliminates about half the candidate copulas. The desired tail dependence can also help guide the choice. Finally, the presence of the "hard" barrier as was seen in Fig. 2 for the second copula, is a third factor.

The new updating procedure based on copulas and the existing Bayesian analysis based on normal priors were both applied to value the option to obtain additional information before carrying out an oil production enhancement project. The posterior distributions were incorporated into a real option evaluation of the operational alternative to log the well this way. A geometric Brownian motion was used to model oil prices. The results showed that, contrary to initial expectations, the value of the option was less under conditions of high oil prices than for lower oil prices. In fact, the option value represents the incremental value of updating the in situ flowing profile of the well (and associated geological inference on the reservoir) and not the value of the oil produced, which definitely rises as oil price increases. While the option to carry out the production enhancement procedure is strongly in-the-money, the option to collect more information turned out, in this instance, to be less favourable from a purely financial standpoint as the price of oil increases. It should be stressed that this does not apply to all wells.

One of the aims of this case study was to develop an objective procedure for evaluating the financial potential of the additional information. This is of benefit to both the operator and service company alike as the former can properly determine ultimate economic viability of the PLT intervention while the latter can better value-price the service. Going further, this example demonstrates that Bayesian analysis coupled with real option provides a general framework for evaluating the option to obtain additional information sometime in the future.

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Appendix A. Transforming the triangular distribution

In both the Bayesian analysis and the updating based on copulas, we need to transform the distributions of individual variables from their real marginal

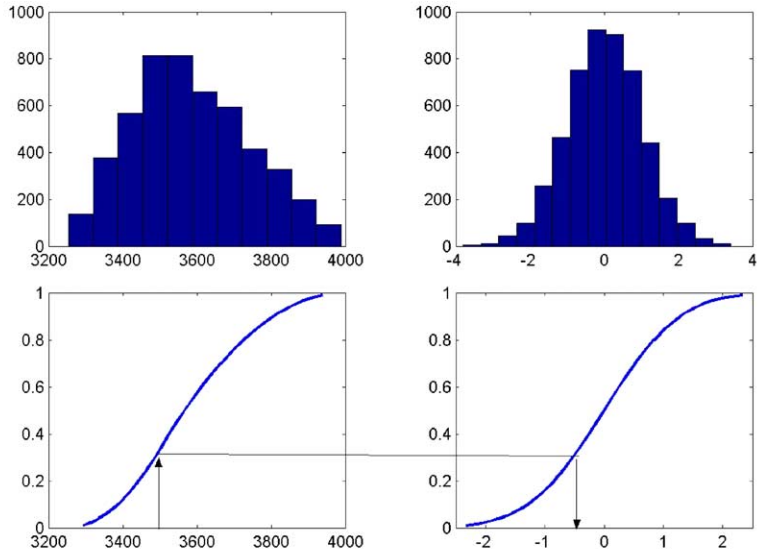

Fig. 5. Triangular distribution with a minimum of 3250, a maximum of 4000 and a mode of 3500 (top left); standard normal distribution (top right), with their cumulative distribution functions below. The arrows show that a value of 3500 in the triangular distribution is equivalent to a normal score of -0.44.

distributions to normality (or to the uniform distribution) and back again. The procedure is widely used in many fields of statistics. In geostatistics, it is called an *anamorphosis*, from the Greek word for distortion. Suppose that we want to transform a continuous random variable, x into its equivalent normal score y. Let $F(x)$ and $G(y)$ denote the cumulative distributions of the data and the standard normal distribution, then the normal score y corresponding to x, is given by $F(x)=G(y)$.

To illustrate this concept, consider the case J_2 from Table 1, where x has a triangular distribution with a minimum value of 3250, a maximum of 4000 and a mode of 3500. Fig. 5a shows this distribution; Fig. 5b shows the $N(0,1)$ distribution. Their cumulative distribution functions are given below each one in Fig. 5c and d. The simplest way to visualize the equivalence $F(x)=G(y)$ is by adjusting the scales on the y-axes on the lower diagrams to the same levels. For example, for the value $x=3500$, $F(3500)=0.333$. To find the value y with $G(y)=0.333$, follow the horizontal arrow to the right till it hits cumulative distribution for the $N(0,1)$ and then read off the value (-0.4308). Similarly, the normal scores equivalent to the minimum (3250) and maximum (4000) are $-\infty$ and $+\infty$ respectively. Fig. 6 plots this transformation function with the normal score on the x-axis and the triangular distribution value on the y-axis. As it is a $1-1$ function, the back transformation is also well defined. The same procedure is used to transforming the triangular to the uniform when using copulas.

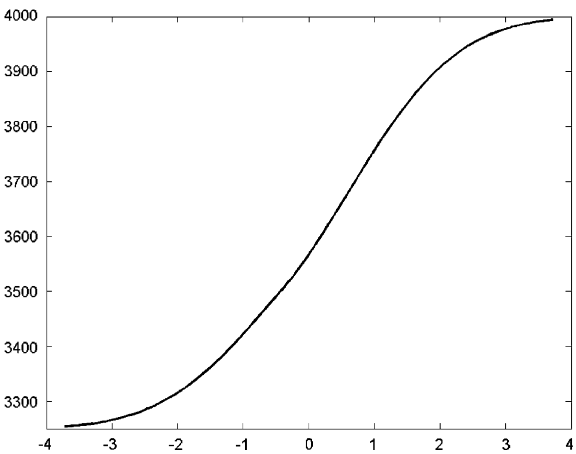

Fig. 6. Transformation function giving the equivalence between a standard $N(0,1)$ distribution on the x-axis and a triangular distribution with a minimum of 3250, a maximum of 4000 and a mode of 3500, on the y-axis.

Appendix B. Mathematics of Bayesian updating

The key step in any form of Bayesian analysis is the choice of the joint distribution of the parameters, here denoted by $\pi(\theta,x)$. In the standard analysis, it is specified via the prior distributions of θ and of the conditional distribution of x given θ. The marginal distribution of x is then found by integration, and the posterior distribution of $\theta|x$ is deduced from this. If conjugate priors are used, its form is simple; otherwise numerical procedure are required. In either case, once the joint distribution is known, the posterior distribution is effectively determined. The same is true if copulas are used to define the joint distribution. To start with, we summarise their main properties.

A copula is a function that links univariate marginal distributions of two or more variables to their multivariate distribution. As the case under consideration involves two variables, x and θ, we only need to consider bivariate copulas but the results can be extended to multivariate cases with little difficulty. Nelsen (1999) defines bivariate copulas as a function $C(u,v)$ having the following properties:

- Dom$C$=$[0,1]\times[0,1]$
- $C(u,0)=0=C(0,v)$
- $C(u,1)=u$ and $C(1,v)=v$
- C is a 2-increasing function. That is, for any rectangle $[u_1,u_2]\times[v_1,v_2]$

$$C(u_2,v_2)-C(u_1,v_2)-C(u_2,v_1)+C(u_1,v_1)\geq 0$$

Here, u and v are defined on [0,1]. As the original variables are assumed to have a triangular distribution, they have to be transformed to have uniform marginals in the same way that they had to be transformed to normality for the standard Bayesian updating Appendix 1. Alternatively, u and v can be viewed as the cumulative distributions of x and θ under study. In that case, their marginal distribution functions $F(x)$ and $G(\theta)$ are linked to their joint distribution function $H(x,\theta)$ by

$$H(x,\theta) = C(u,v) = C(F(x),G(\theta))$$

The marginal distributions of x and θ are $C(x,1)$ and $C(1,\theta)$, respectively. The density c associated with the copula is obtained by differentiating

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u\ \partial v}$$

From this, it is not difficult to demonstrate that the conditional copula of $v|u$ is

$$C(v\,|\,u) = \frac{\partial}{\partial u} C(u,v)$$

That is, the (cumulative) posterior distribution is just the first derivative of the copula in question. In many ways, this is even simpler to obtain than for standard Bayesian analysis. As we will see in the next paragraph, the specific form of Archimedean copulas further simplifies this.

B.1. Bivariate Archimedean copulas

Archimedean copulas, which are one of the most important families, are of the form:

$$\varphi(C(u,v)) = \varphi(u) + \varphi(v)$$

where φ is a continuous, strictly decreasing function from [0,1] to $[0,\infty]$ such that $\varphi\ (1)=0$. It is called the generator of the copula. Table 4.1, pages 94–97, of Nelsen (1999) lists 22 one-parameter bivariate Archimedean copulas. The equations for the first two of these are given in Table 2. More general types of Archimedean copulas also exist (e.g., families with two parameters and families of rational copulas). As the production enhancement options considered were evaluated numerically by Monte Carlo methods, we needed to simulate values from the posterior distribution. Nelson describes several simulation algorithms in Exercises 4.13 to 4.15 (p. 108).

Table 2
Two Archimedean copulas

No.	Name	Equation	Parameter value
1	Nelsen No. 1 Clayton's copula	$C(u,v)=\max([u^{-\alpha}+v^{-\alpha}-1]^{-1/\alpha},0)$	$\alpha=15$
2	Nelsen No. 2	$C(u,v)=\max(1-[(1-u)^{\alpha}+(1-v)^{\alpha}]^{1/\alpha},0)$	$\alpha=19$

The standard text on copulas (Nelsen, 1999) lists 22 Archimedean copulas. Two of these with radically different tail dependence behaviour were selected for this study on Bayesian updating. The equations of the copulas $C(u,v)$ are given below together with their common names and the parameter value α used here.

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