Structural Engineering

Structure

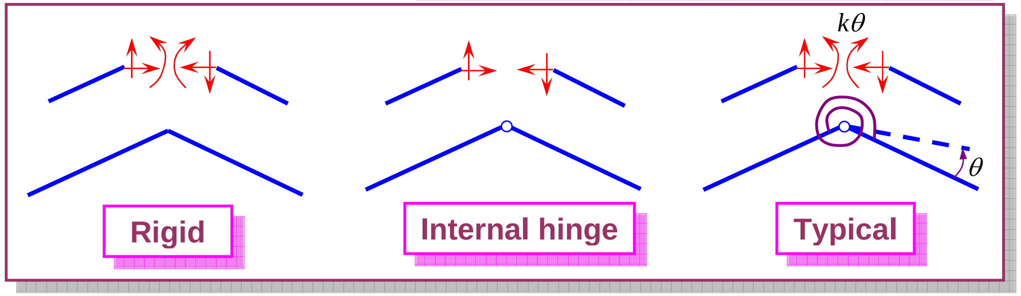
- the action of building: CONSTRUCTION
- a: something (as a building) that is constructed
  b: something arranged in a definite pattern of organization
- manner of construction
- a: the arrangement of particles or parts in a substance or body
  b: organization of parts as dominated by the general character as a whole
- the aggregate of elements of an entity in their relationships to each other

Structural Systems

Structural System

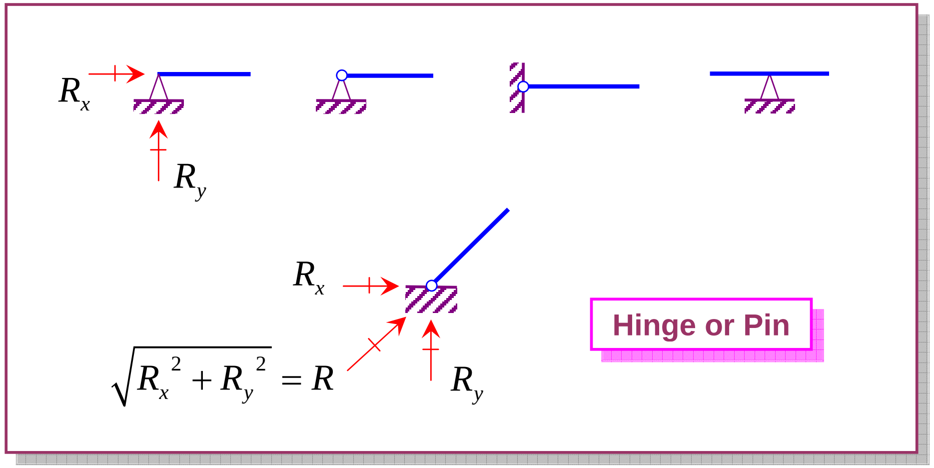
Structural Members/Elements
- Primary load bearing components, e.g. truss and frame members
- Material behavior limited to linear-elastic only

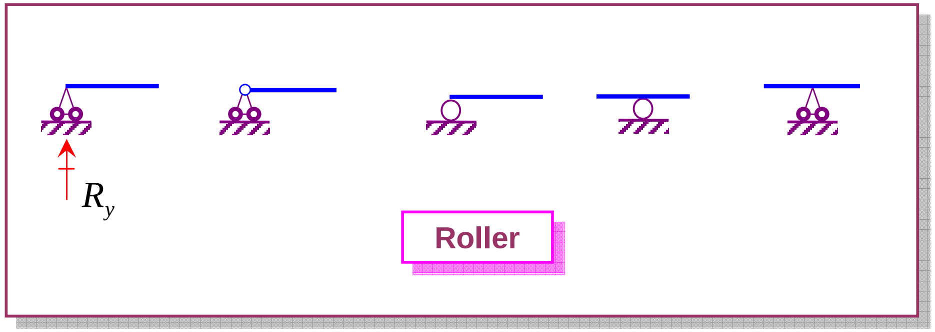
Connections/Joints
- Internal construction conditions where adjacent members are tied to each other
- Types of connections depend on the expected performance of the joining members, e.g. rigid, internal hinge, and typical connection

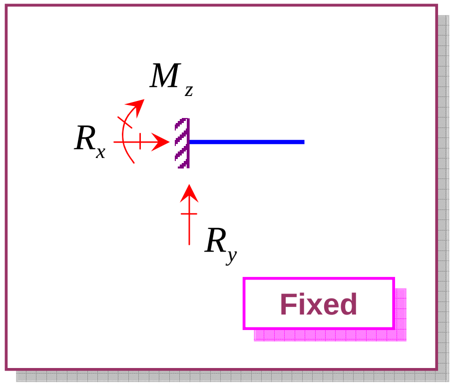


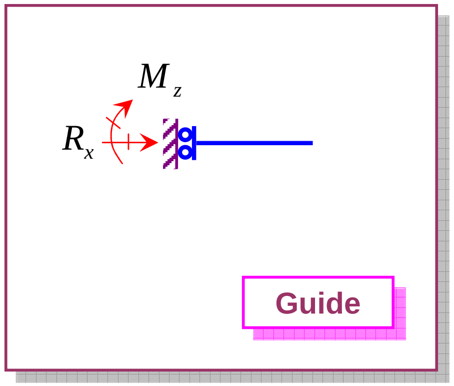
Supports/Boundary Conditions
- Construction conditions at the locations where the applied loads are finally transmitted to
- Resisting forces against translation/rotation at the support are called “support reactions”
- Paths at which the applied loads are transferred to the system to its supports are called “load paths”

Supports/Boundary Conditions









Types of Elements in Structural Systems

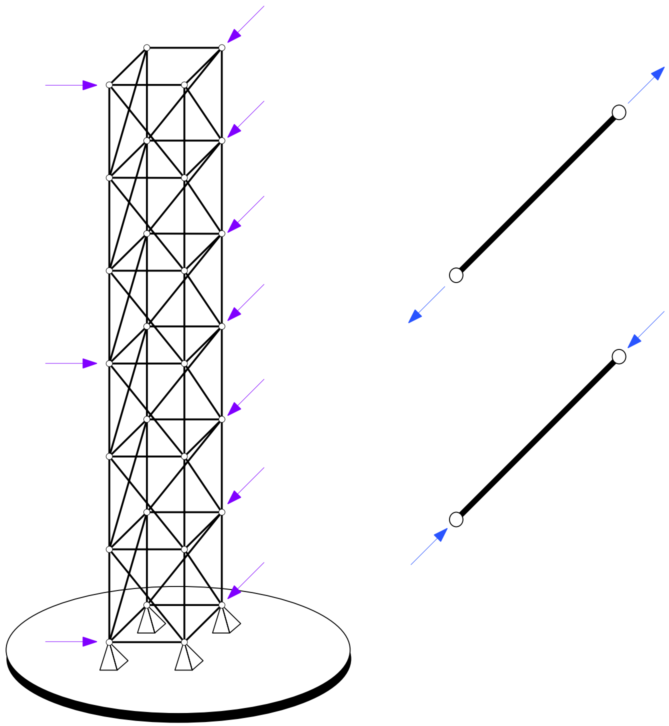
- Classification of structural systems is based on many considerations
- Generally, the most fundamental component or "elements" of a structural system are classified based on their mode(s) of deformation and load-resisting behavior
- A structural system can be composed of various types of elements in order to satisfy the functional requirements
- There is no unique way to classify the structural systems

Structural Elements

- Trusses
- Frames
- Cables
- Arches
- Shells
- Solids

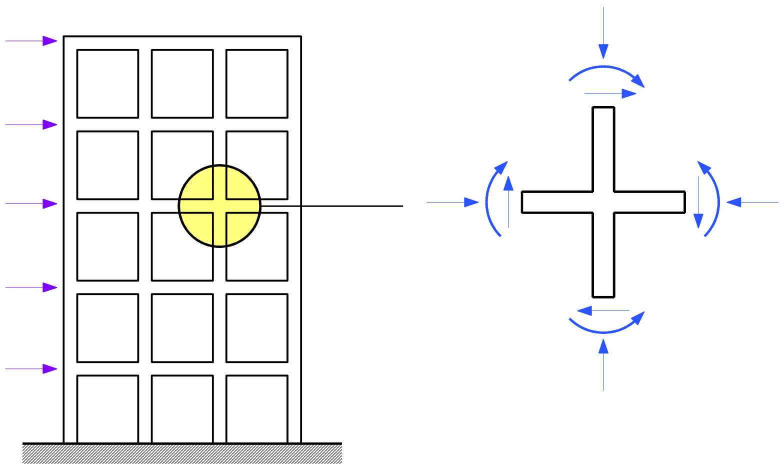
Trusses

- Truss elements, the simplest form of structural elements, carry pure tension and compression
- Truss elements are connected to other elements with pin connections
- The elements, when loaded, are subjected to only axial forces and axial deformation



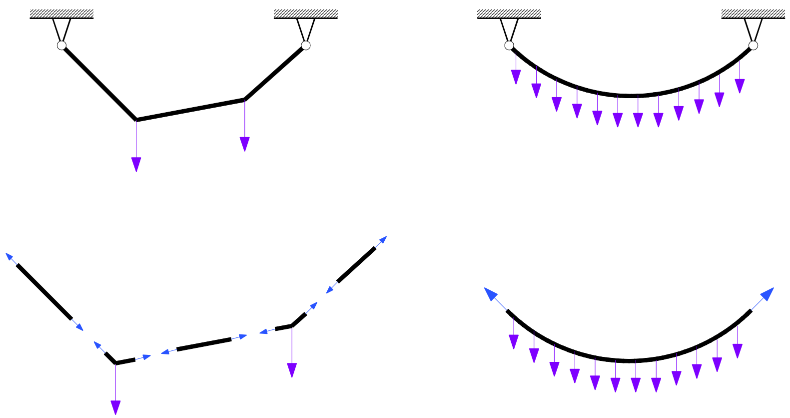
Frames

- Frame elements generally carry shear forces and bending moments in addition to axial forces
- Frame elements are connected to other elements with rigid, semi-rigid, or pin connection
- Pin connections do not allow any moment transfer between the connecting elements (the connecting elements can rotate freely relative to each other)
- Rigid connections imply a complete moment transfer
- Frame elements can have multiple modes of deformation, i.e. axial, shear, and bending



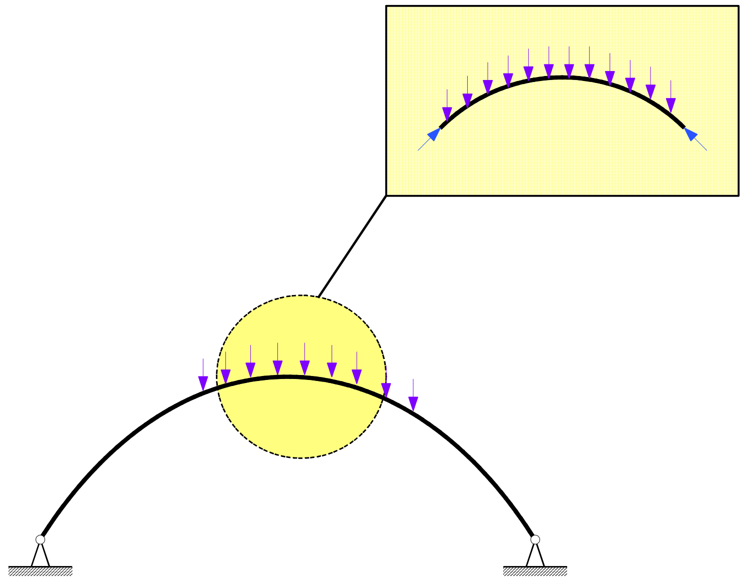
Cables

- Cables are slender members in which the cross-sectional dimensions are relatively small compared with the length
- Primary load-carrying mechanism is axial tension
- The deflected shape of a cable depends on the applied loading
- For discrete loading, the deformed shape of the cable is a series of piecewise straight lines between the points of the applied loads
- For distributed loading, the deformed shape is a curve



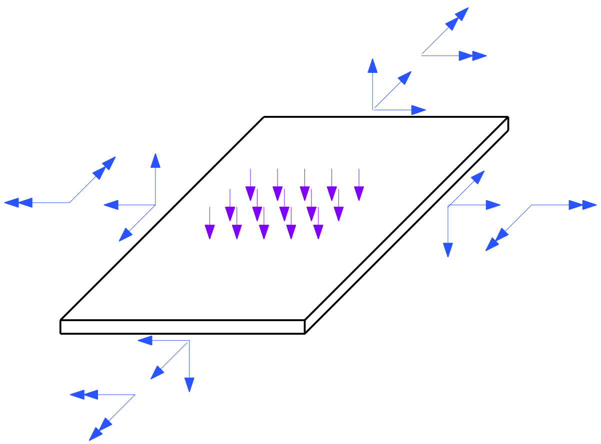
Arches

- Arches are curved members that are supported at each end such that the primary load-carrying mechanism is axial compression
- Arch elements may be subjected to secondary forces such as bending and shear depending on the applied loading and support conditions
- In any cases, axial compression still remains the dominant mode of action



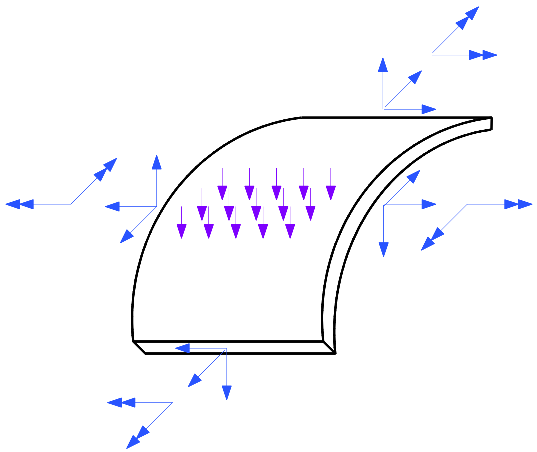
Plates

- Plates are flat structural elements whose thickness is relatively small compared with the lateral dimensions
- The internal actions of plate elements include axial (membrane), transverse shear, and bending
- Bending and transverse shear are carried either normal to or within the plane of the element depending on the applied loading and the element orientation



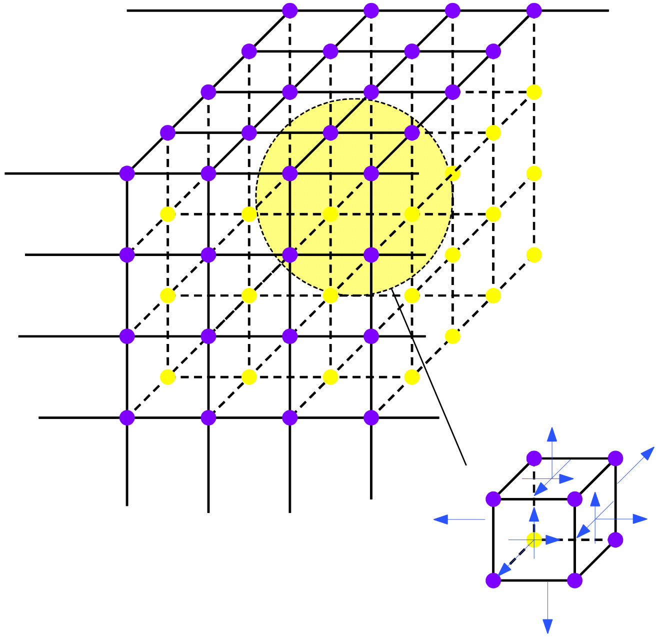
Shells

- Shells, similar to plates, are structural elements whose thickness is small compared to the lateral dimensions
- The difference of shells from plates is that the surface of shell element is not flat but curved



Solids

- Solid elements are three-dimensional structural elements used to model three-dimensional continuum structures such as dams, thick bridge deck, etc.
- The internal action of a solid element is usually described in terms of stress and strain in three dimensions related by constitutive relationship



Structural Analysis

- A systematic study of the response of a structure under the specified actions (external loadings, thermal expansion/contraction of structural members, settlement of supports)
- Usually carried out by establishing the relationships of forces and deformations throughout the structure
- Requires knowledge of element behavior, structural materials, support conditions, and applied loads
- Expressed as mathematical algorithms which can be classified as “classical” methods and “numerical” methods (or matrix methods)

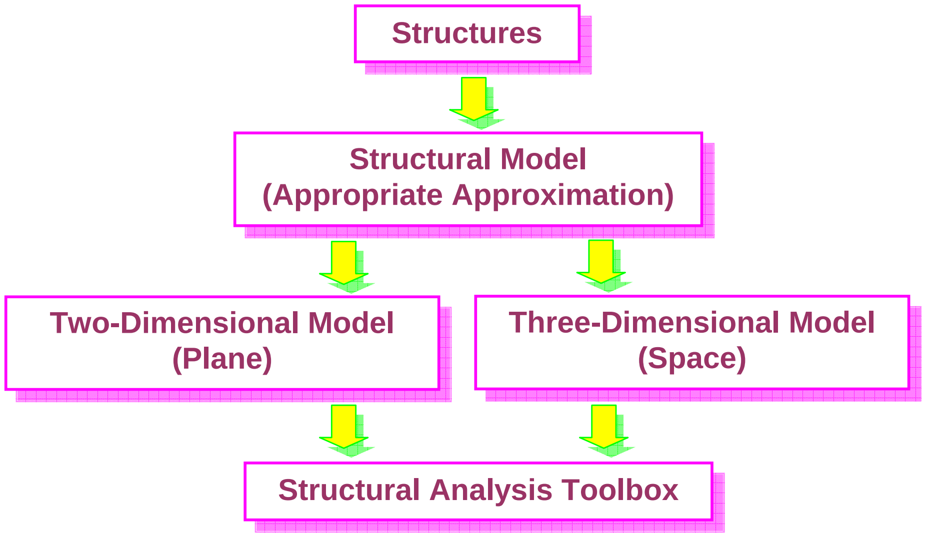
Classical Methods

- Early sophisticated methods invented to analyze specific classes of problems
- Developed based on principles of structural mechanics and suitable approximations
- Essential for understanding the fundamental principles of structural mechanics

Numerical Methods

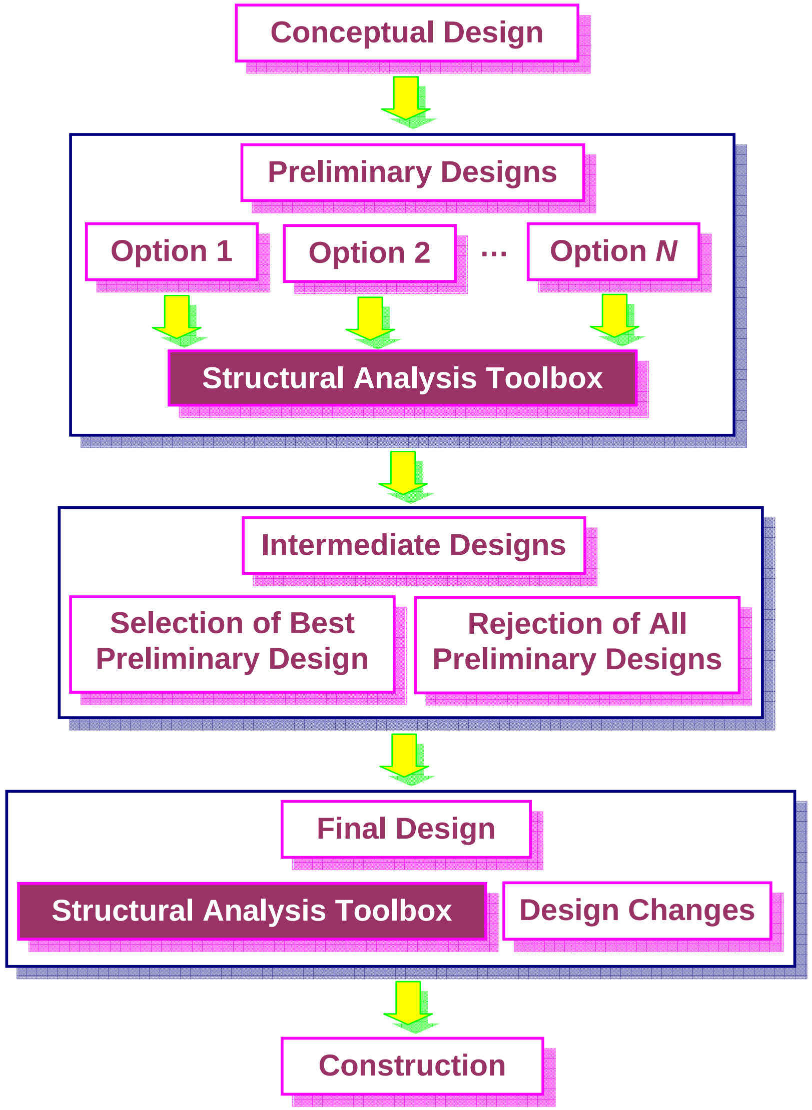
- More generalized matrix-based numerical methods developed to analyze complex systems
- Generally requires the use of computers

Structural Analysis Process



- One of the most important steps in structural analysis is the selection of an appropriate structural model based on the available information of the structure
- In general, the selection of structural model is linked to the determination of the method of analysis

Structural Design Process



Degree of Static Determinacy

Definition:

$r_a =$ **No. of independent (support) reaction components**

$n_c =$ **No. of equations of condition**

Additional static equations from special internal conditions of construction

For planar structures:

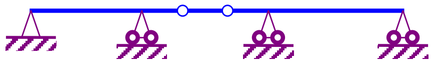
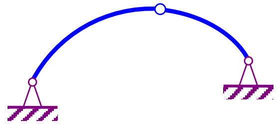
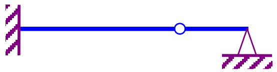
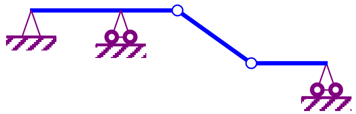
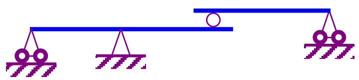
$r_a < 3 + n_c$ → Statically *unstable* "externally"

$r_a = 3 + n_c$ → Statically *determinate* "externally"

$r_a > 3 + n_c$ → Statically *indeterminate* "externally"

Maximum no. of static equilibrium equations that can be written

External Static Determinacy Classification

Structure	Independent Reaction Components r_a	Number of Eqs. of Condition n_c	$r = 3 + n_c$
			
			
			
			
			
			

External & Internal Static Determinacy

Definition:

$m =$ **No. of members in structure**

$j =$ **No. of joints**

For plane frames and beams:

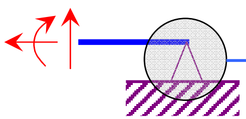
$3m + r_a < 3j + n_c$ → Statically *unstable* "overall"

$3m + r_a = 3j + n_c$ → Statically *determinate* "overall"

$3m + r_a > 3j + n_c$ → Statically *indeterminate* "overall"

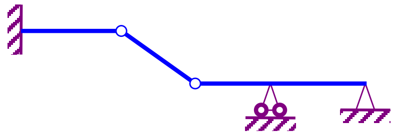
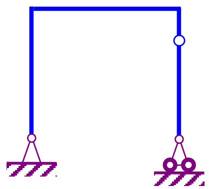
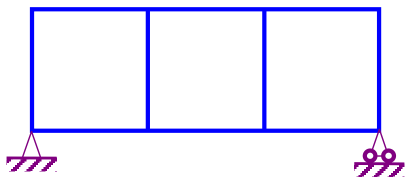
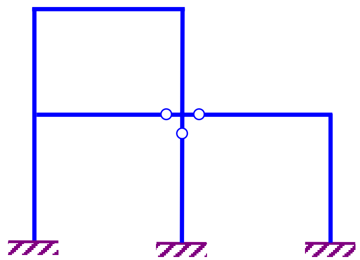
$3j + n_c$: Total no. of static equations that can be written

$3m + r_a$: Total no. of unknown force components



$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_z = 0$$

Overall Static Determinacy Classification

Structure	Structure Characteristics			
	j	n_c	m	r_a
				
				
				
				
				

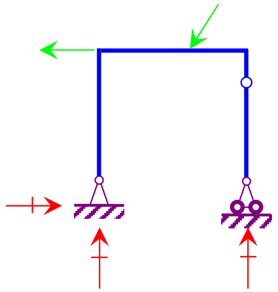
Free-Body Diagram

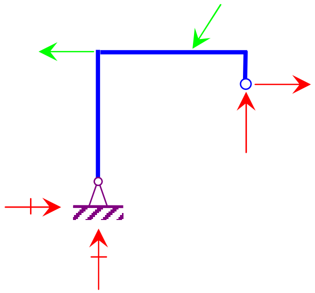
Free-Body Diagram (FBD)

Sketch of a structural component with all of the appropriate forces acting on it

Entire Structure

Portion of Structure

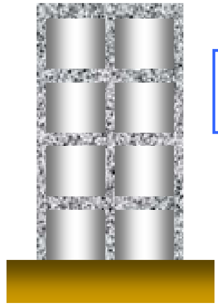




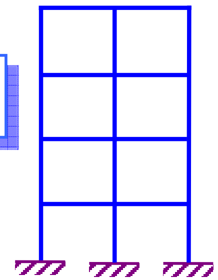
Usually gives relationships between external forces

Usually gives relationships between internal and external forces

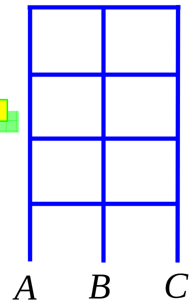
Guidelines to Drawing FBD of A Planar Structure



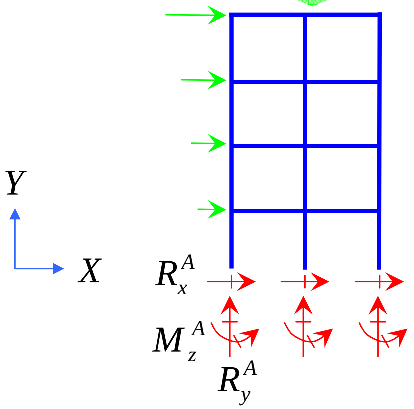
Structural Modeling



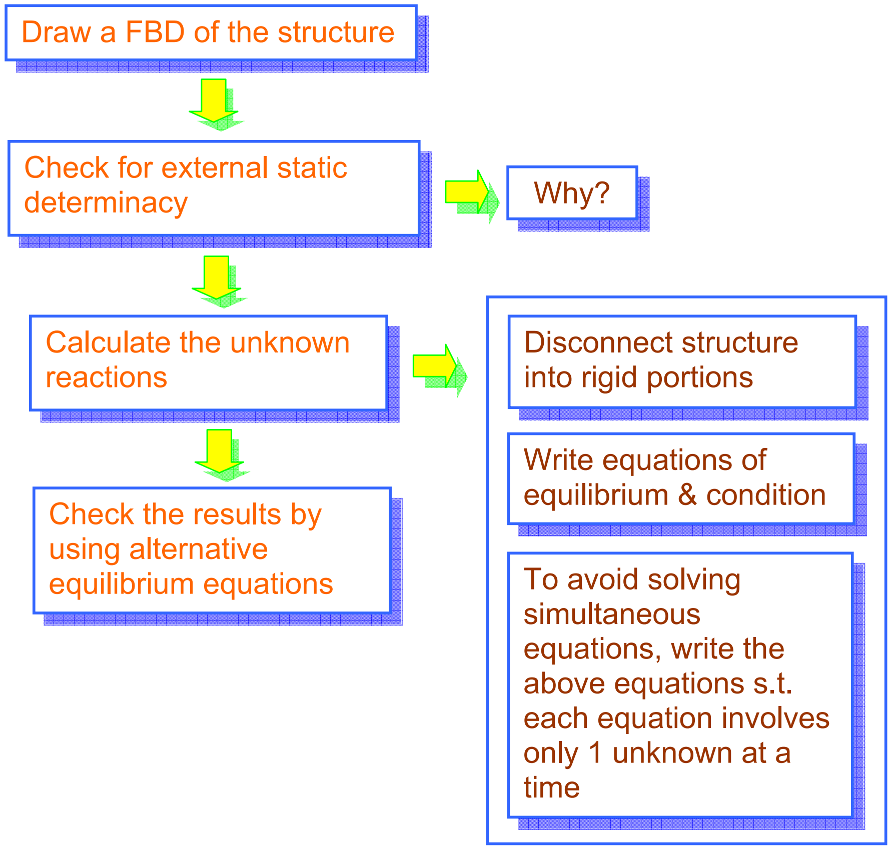
Use line diagrams to represent portion of structure under consideration detached from its supports & disconnected from all other parts of the structure



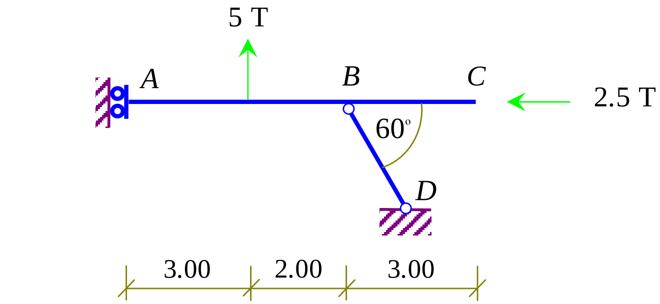
Select orientation of a reference *x-y* coordinate system, and show all force components assuming the positive direction w.r.t. the reference coordinate system

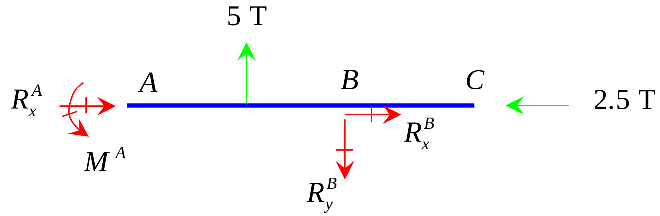


Computation of Reactions



Example 1





$r_a = 3$ **and structure is stable**

$\sum F_y = 0$:

$R_y^B = 5\ \text{T}$ ↓

$R_x^B = \dfrac{R_y^B}{\tan 60^\circ} = \dfrac{5}{\sqrt{3}} = 2.887\ \text{T}$ →

$\therefore R_B = \sqrt{\left(R_x^B\right)^2 + \left(R_y^B\right)^2} = 5.774\ \text{T}$ ↘

$\sum F_x = 0$:

$R_x^A + R_x^B = 2.5$

$R_x^A = 2.5 - 2.887 = -0.387\ \text{T}$ ←

$\sum M_A = 0$:

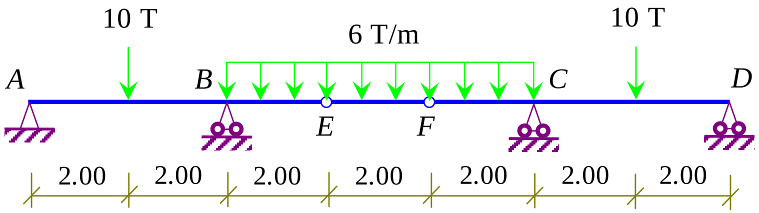
$-R_y^B + 5\times 3 + M^A = 0$

$M^A = 5\times 5 - 5\times 3 = 10\ \text{T-m}$ ↺

Check: $\sum M_C = 0$

$M^A - 5\times 5 + R_y^B \times 3 = 10 - 25 + 5\times 3 = 0$ **O.K.**

Example 2



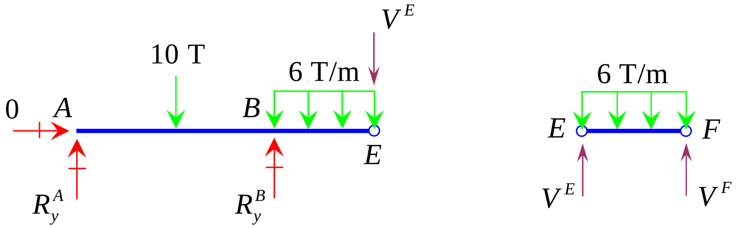
$r_a = 5$ **and** $n_c = 2$

$r_a = 3 + n_c \Rightarrow$ **statically determinate "externally" (stable)**

$\sum F_x = 0$ **results in 4 reaction unknowns**

Symmetry of structure & loading reduces no. of unknowns to 2

$\Rightarrow R_y^A$ **and** R_y^B



$\sum M_A = 0$ **:**

$$-10 \times 2 + R_y^B \times 4 - 6 \times 2 \times 5 - 6 \times 6 = 0$$

$$R_y^B = \frac{1}{4}(20 + 60 + 36) = 29 \text{ T} \uparrow$$

$\sum F_y = 0$**:**

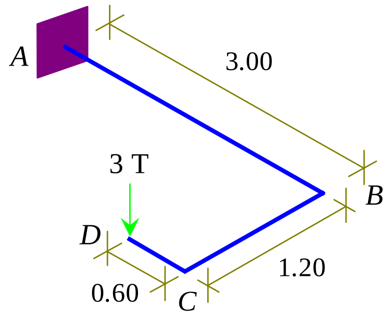
$$R_y^A - 10 + 29 - 6 \times 2 - 6 = 0$$

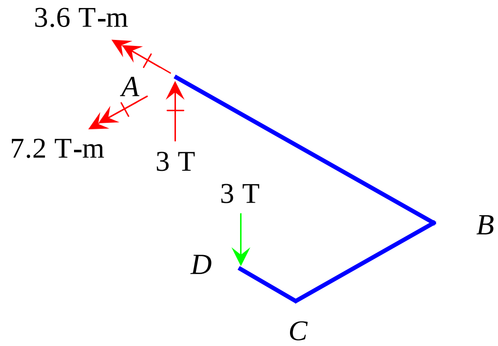
$$R_y^A = 10 - 29 + 12 + 6 = -1 \text{ T} \downarrow$$

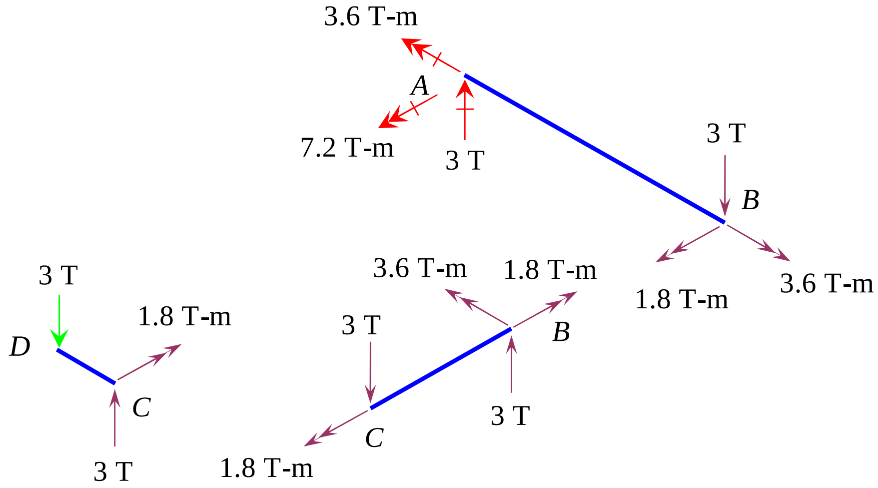
Check: $\sum M_E = 0$

$$-R_y^A \times 6 + 10 \times 4 - R_y^B \times 2 + 6 \times 2 \times 1 = 6 + 40 - 58 + 12 = 0$$ **O.K.**

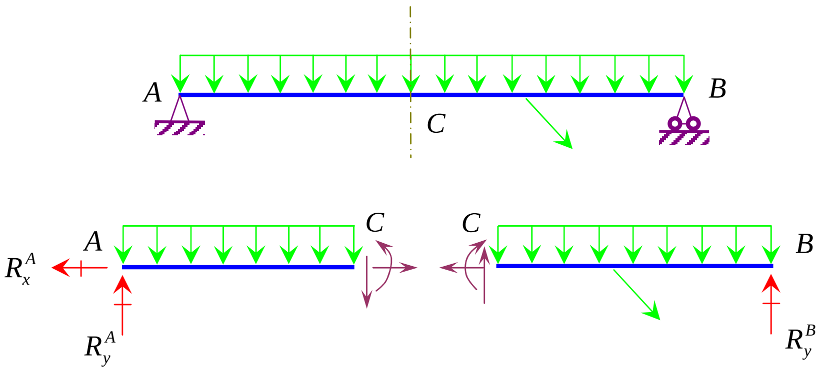
Example 3



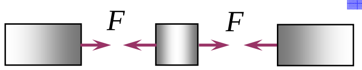




Beams and Plane Frames

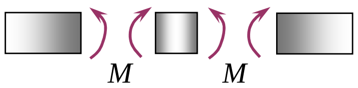


Axial Force F → **Internal force *in the direction of* the centroidal axis of the element**



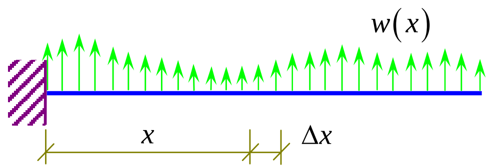
Shear Force V → **Internal force in the direction *perpendicular* to the centroidal axis**

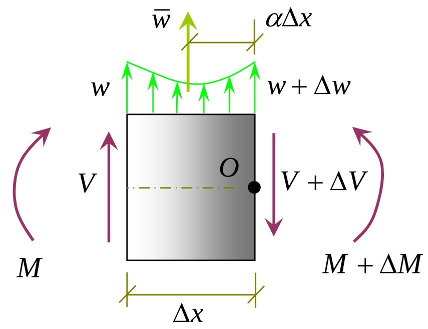
Bending Moment M → **Internal *couple* acting on the cross section of the element**



The values of these internal forces can vary along the length of each structural member depending on the loading and boundary (or support) conditions

Relationships between Loads: V & M





$\sum F_y = 0$:

$$V + \bar{w}\Delta x - (V + \Delta V) = 0$$

$$\frac{\Delta V}{\Delta x} = \bar{w}$$

As $\Delta x \to 0$:

$$\frac{dV}{dx} = w \qquad \textbf{(1)}$$

Slope of SFD at any point = load intensity at that point

$\sum M_O = 0$:

$$M + V\Delta x + (\bar{w}\Delta x)(\alpha\Delta x) - (M + \Delta M) = 0$$

$$\frac{\Delta M}{\Delta x} = V + \cancel{\bar{w}\alpha\Delta x} \quad 0 \text{ as } \Delta x \to 0$$

$$\frac{dM}{dx} = V \qquad \textbf{(2)}$$

Slope of BMD at any point = shear at that point

Equations (1) & (2) are D.E. of equilibrium. Substitution of (1) into (2) gives

$$\frac{d^2M}{dx^2} = w \qquad \textbf{(3)}$$

This equation relates M to w at a section.
Equation (1) can be written

$$dV = wdx$$

$$V = \int wdx \qquad \textbf{(4)}$$

$$V_B - V_A = \int_A^B wdx \qquad \textbf{(5)}$$

Change in shear between sections *A* and *B*
= area under load intensity diagram between *A* and *B*

Similarly, from Eq. (2)

$$M = \int Vdx \qquad \textbf{(6)}$$

$$M_B - M_A = \int_A^B Vdx \qquad \textbf{(7)}$$

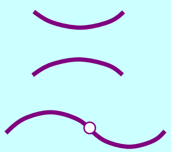
Change in moment between sections *A* and *B*
= area under shear force diagram (SFD) between *A* and *B*

Remarks

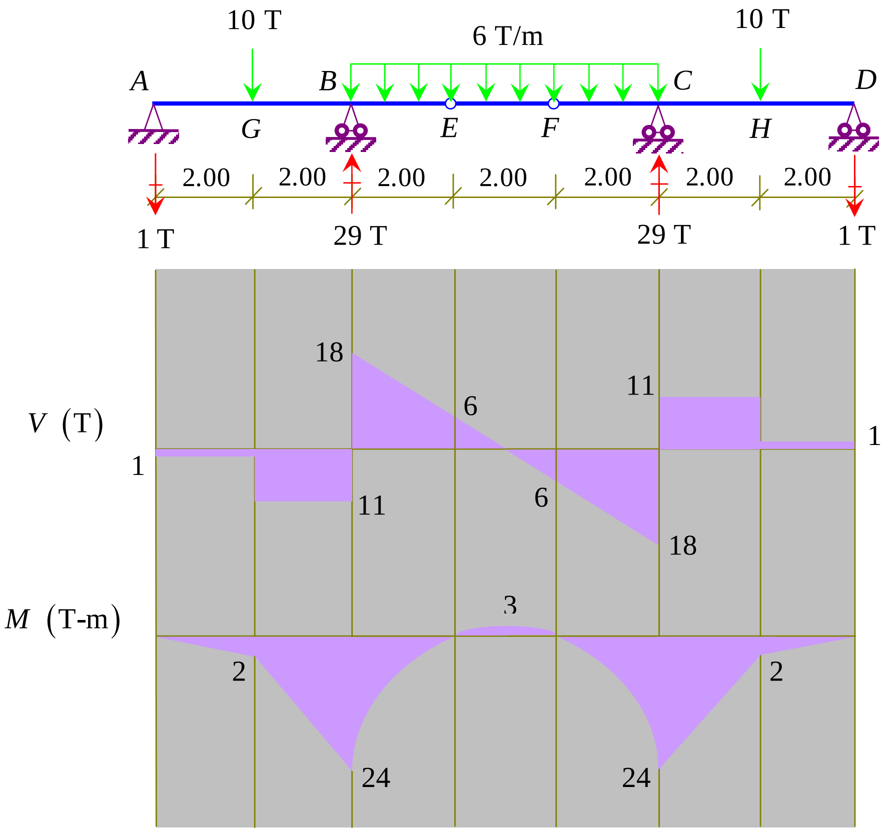
- For distributed load, slope of SFD at any point = load intensity at that point
- For point load, slope of SFD ➔ ∞
- For distributed load, change in shear over a segment = area under load intensity diagram over the segment
- For point load, change in shear = load magnitude
- Slope of BMD at any point = shear at that point and is constant for constant shear
- Slope of BMD = zero at point of zero shear and BM is max or min
- Slope of BMD changes abruptly at point load
- Change in moment over a segment = area under SFD over the segment
- \+ M ➔ concave member

  \- M ➔ convex member

  zero M ➔ change in curvature at point of inflection



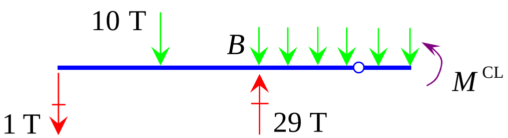
Example 4



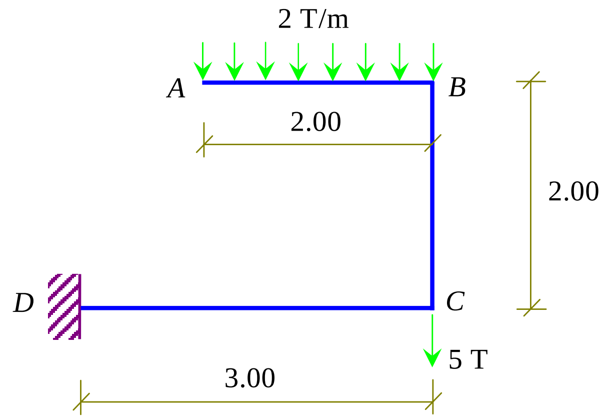
$M^G = -1\times 2 = -2$ T-m

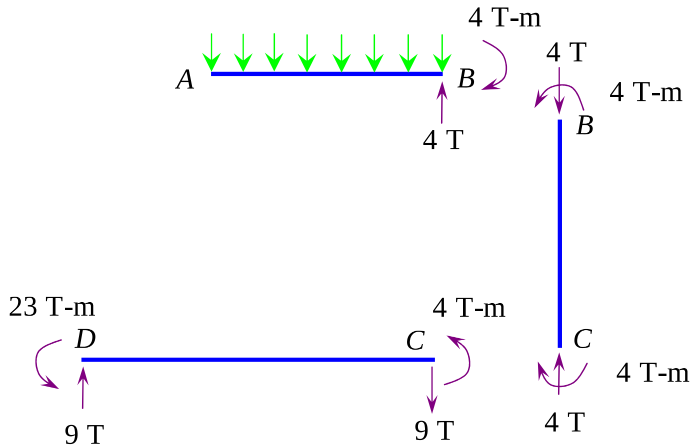
$M^B = -1\times 4 - 10\times 2 = -24$ T-m

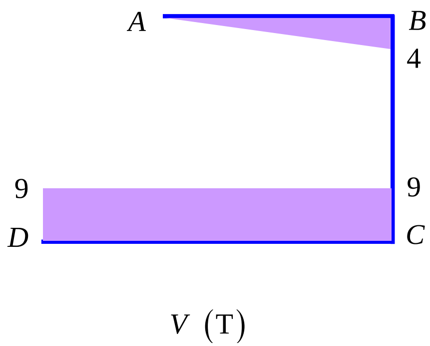
$M^{CL} = -1\times 7 - 10\times 5 + 29\times 3 - 6\times 3\times 1.5 = 3$ T-m

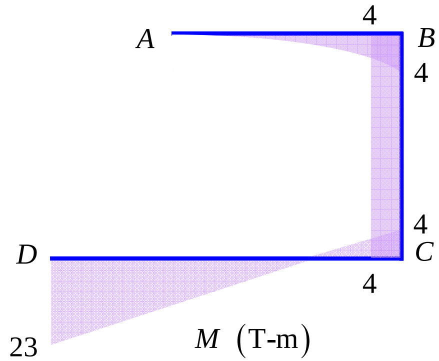


Example 5

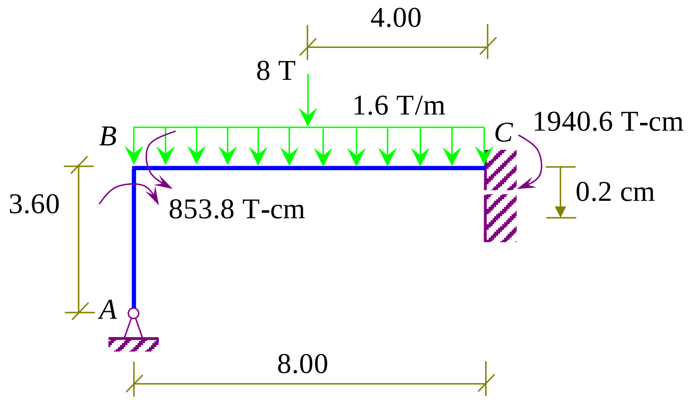




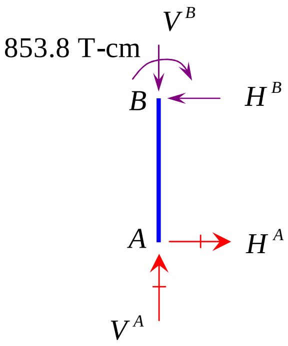




Example 6

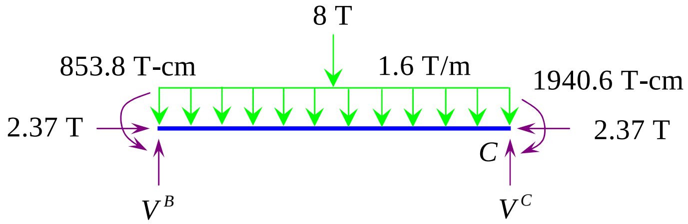


Degree of indeterminacy: $(3\times 2+5)-(3\times 3+0)=2$



$\sum M_B = 0$:

$$H^A = -H^B = \frac{853.8}{360} = 2.372 \text{ T} \rightarrow$$

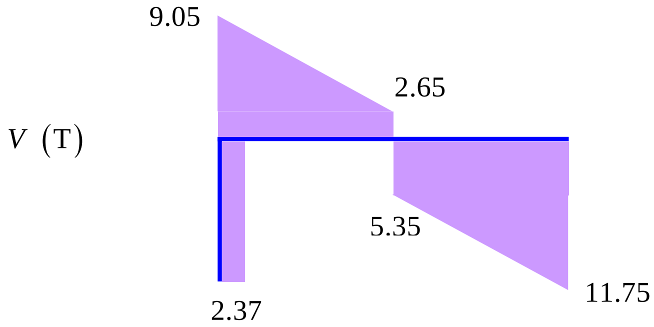


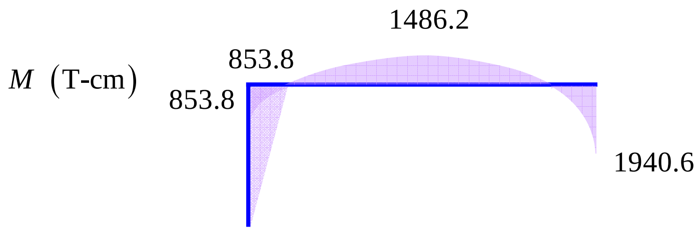
$\sum M_C = 0$:

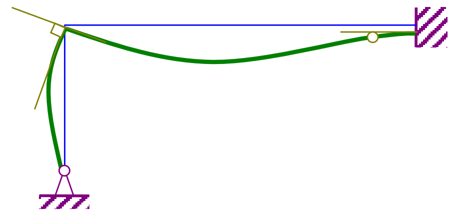
$$V^B = \frac{1}{8}\left(\frac{853.8}{100} + 8\times\frac{8}{2} + 1.6\times 8\times\frac{8}{2} - \frac{1940.6}{100}\right) = 9.05 \text{ T} \uparrow$$

$\sum F_y = 0$:

$$V^C = 8 + 1.6\times 8 - 9.05 = 11.75 \text{ T} \uparrow$$

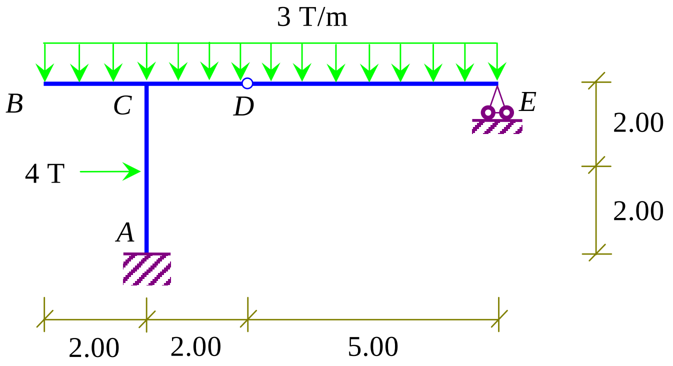


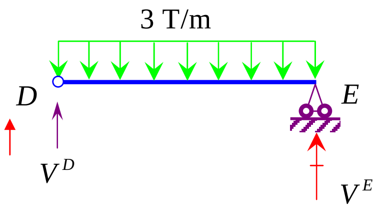




Example 7

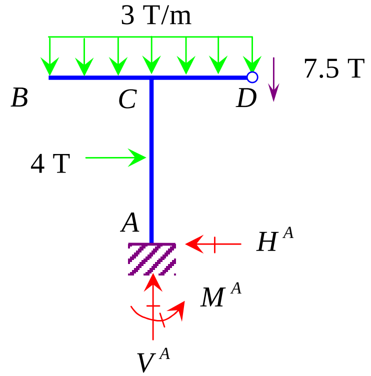
Draw the BMD and sketch the elastic curve for the frame shown.





$\sum V = 0:$

$$V^D = V^E = 3 \times \frac{5}{2} = 7.5 \text{ T} \uparrow$$



$\sum M_A = 0:$

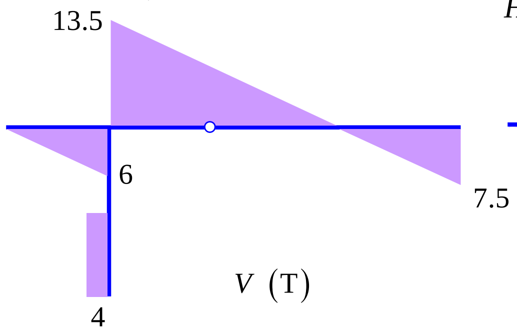
$$M^A = 7.5 \times 2 + 3 \times 2 \times 1 - 3 \times 2 \times 1 + 4 \times 2 = 23 \text{ T-m}$$

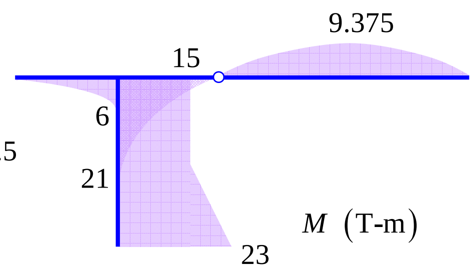
$\sum V = 0:$

$$V^A = 3 \times 4 + 7.5 = 19.5 \text{ T} \uparrow$$

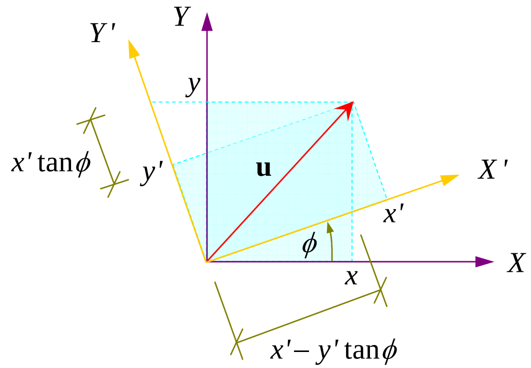
$\sum H = 0:$

$$H^A = 4 \text{ T} \leftarrow$$





Transformation of Coordinates



Consider the transformation of the coordinates of a vector $\mathbf{u}$ rotated through an angle ϕ in a plane

$$x = (x' - y' \tan\phi)\cos\phi = x'\cos\phi - y'\sin\phi$$
$$y = (x'\tan\phi + y')\cos\phi = x'\sin\phi + y'\cos\phi$$

$$\mathbf{u} = \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{Bmatrix} x' \\ y' \end{Bmatrix} = \mathbf{R}^T \mathbf{u}'$$

Hence the rotation matrix $\mathbf{R}$ in a 2-D problem is defined as

$$\mathbf{R} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$

Any vector $\mathbf{u}$ in global coordinates can therefore be obtained from its components in local coordinates by the relationship

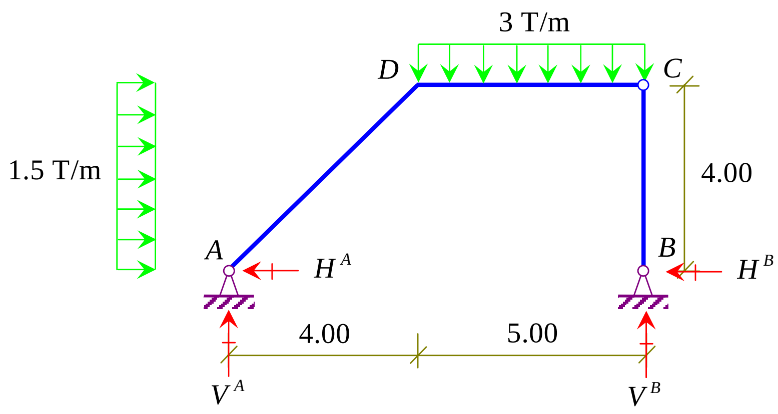
$$\mathbf{u} = \mathbf{R}^T \mathbf{u}'$$

Conversely, for the transformation to global coordinates

$$\mathbf{u}' = \mathbf{R}\mathbf{u}$$

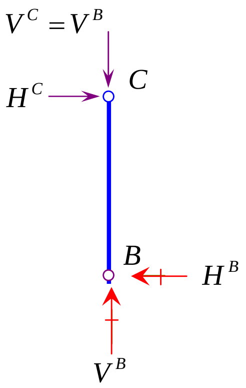
Note: $\mathbf{R}^T = \mathbf{R}^{-1}$
$\mathbf{R} \rightarrow$ **orthogonal matrix**
$\mathbf{R}^T\mathbf{R} = \mathbf{I}$

Example 8 Analyze the frame shown and sketch the BMD and the elastic curve.



$$\sum M_B = 0:$$
$$9V^A + 1.5\times 4\times 2 - 3\times 5\times 2.5 = 0$$
$$V^A = 2.83 \text{ T} \uparrow$$

$$\sum M_A = 0:$$
$$9V^B - 1.5\times 4\times 2 - 3\times 5\times 6.5 = 0$$
$$V^B = 12.17 \text{ T} \uparrow$$

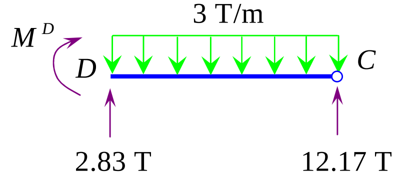


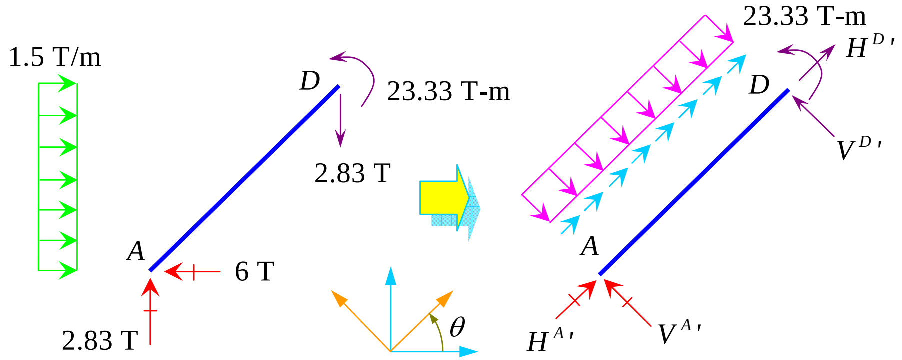
$\sum M_C = 0$: **(FBD** BC**)**
$$H^B = 0 = H^C$$

$\sum H = 0$: **(FBD Structure)**
$$H^A = 6 \text{ T}$$

Check: $\sum M_C = 0$ **(FBD** ADC**)**
$2.83\times 9 + 6\times 4 - 1.5\times 4\times 2 - 3\times 5\times 2.5 = 0$ **O.K.**

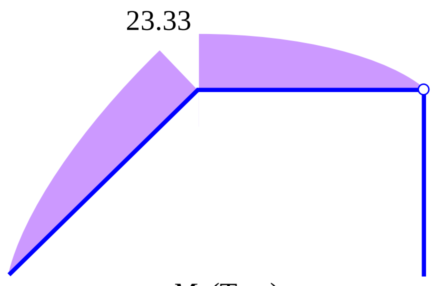
$\sum M_C = 0$: **(FBD** DC**)**
$$M^D = 3\times 5\times 2.5 - 2.83\times 5 = 23.33 \text{ T-m}$$





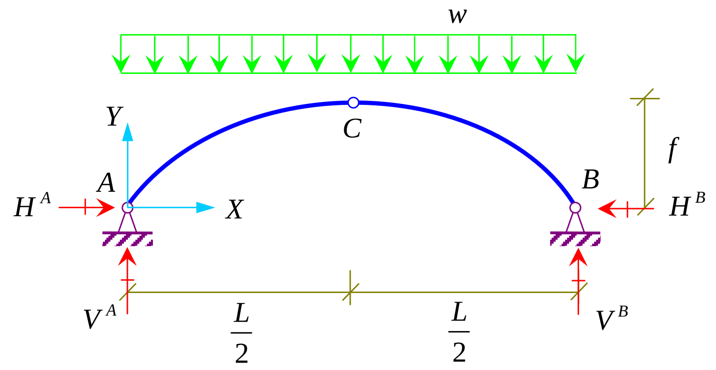
$$\begin{Bmatrix} H^{A\prime} \\ V^{A\prime} \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} H^A \\ V^A \end{Bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{Bmatrix} -6 \text{ T} \\ 2.83 \text{ T} \end{Bmatrix} \begin{matrix} \leftarrow \\ \uparrow \end{matrix} = \begin{Bmatrix} -2.24 \text{ T} \\ 6.65 \text{ T} \end{Bmatrix} \begin{matrix} \swarrow \\ \nwarrow \end{matrix}$$

$$\begin{Bmatrix} H^{D\prime} \\ V^{D\prime} \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} H^D \\ V^D \end{Bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{Bmatrix} 0 \text{ T} \\ -2.83 \text{ T} \end{Bmatrix} \begin{matrix} \rightarrow \\ \downarrow \end{matrix} = \begin{Bmatrix} -2 \text{ T} \\ -2 \text{ T} \end{Bmatrix} \begin{matrix} \swarrow \\ \searrow \end{matrix}$$



M (T-m)

Three-hinged Arches



Parabolic Arch:

The equation of the arch takes the form

$$y = a_0 + a_1 x + a_2 x^2$$

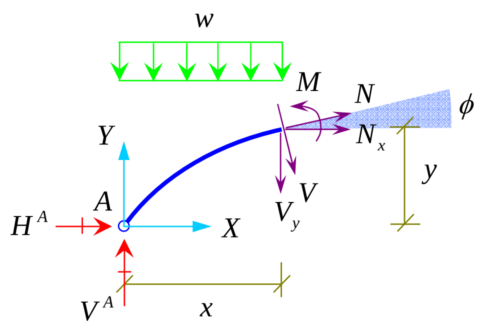
with $y = 0$ **at** $x = 0$ **and** $x = L$

and $y = f$ **at** $x = \frac{L}{2}$

$$\therefore \quad a_0 = 0,\ a_1 = \frac{4f}{L},\ a_2 = -\frac{4f}{L^2}$$

$$y = \frac{4fx}{L}\left(1 - \frac{x}{L}\right)$$

The four reaction components can be found from four equations of equilibrium



$$\tan\phi = y' = \frac{dy}{dx}$$

$$\sin\phi = \frac{\tan\phi}{\sqrt{1+\tan^2\phi}}$$

$$\cos\phi = \frac{1}{\sqrt{1+\tan^2\phi}}$$

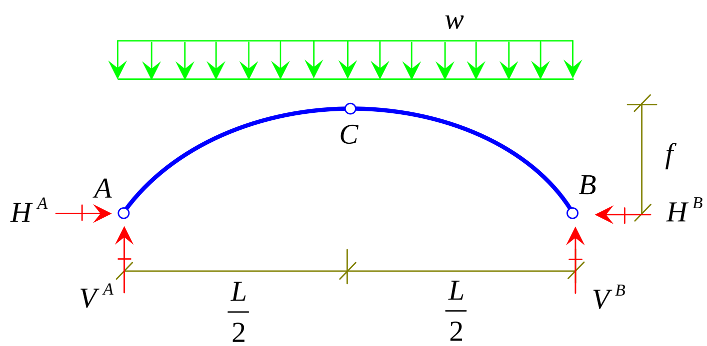
$$\begin{Bmatrix} N \\ -V \end{Bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{Bmatrix} N_x \\ -V_y \end{Bmatrix}$$

$$N_x = H_A$$

$$V_y = V_A - wx$$

$$N = N_x \cos\phi - V_y \sin\phi$$
$$= -H^A \cos\phi - \left(V^A - wx\right)\sin\phi$$

$$V = N_x \sin\phi + V_y \cos\phi$$
$$= -H^A \sin\phi + \left(V^A - wx\right)\cos\phi$$



$$\sum V = 0:$$
$$V^A = V^B = \frac{wL}{2}$$

$$\sum M_c = 0:$$
$$= H^A f - V^A \frac{L}{2} + \frac{wL}{2} \cdot \frac{L}{4}$$
$$\Rightarrow H^A = \frac{wL^2}{8f} = H^B$$

$$\tan\phi = \frac{dy}{dx} = \frac{4f}{L^2}\left(L-2x\right)$$

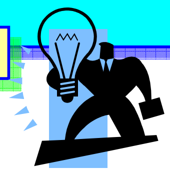
$$N = -\frac{wL^2}{8f}\frac{1}{\sqrt{1+\left[\frac{4f}{L^2}\left(L-2x\right)\right]^2}} - w\left(\frac{L}{2}-x\right)\frac{\frac{4f}{L^2}\left(L-2x\right)}{\sqrt{1+\left[\frac{4f}{L^2}\left(L-2x\right)\right]^2}}$$

$$= -\frac{w}{8f}\sqrt{L^4+16f^2\left(L-2x\right)^2}$$

$$V = -\frac{wL^2}{8f}\frac{\frac{4f}{L^2}\left(L-2x\right)}{\sqrt{1+\left[\frac{4f}{L^2}\left(L-2x\right)\right]^2}} + w\left(\frac{L}{2}-x\right)\frac{1}{\sqrt{1+\left[\frac{4f}{L^2}\left(L-2x\right)\right]^2}}$$

$$= 0$$

Magic !!



$$M = V^A x - H^A y - \frac{wx^2}{2}$$

$$= \frac{wLx}{2} - \frac{wL^2}{8f}\frac{4fx}{L}\left(1-\frac{x}{L}\right) - \frac{wx^2}{2}$$

$$= 0$$

Magic !!

Therefore the load is carried *entirely* by *axial compression* which *varies parabolically* along the span.

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