INFLUENCE LINES

PRELIMINARIES

Moving loads *-- Loads applied to a structure with points of application (including their magnitude) can vary as a function of positions on the structure. Examples of moving loads include live load on buildings, traffic or vehicle loads on bridges, loads induced by wind and earthquake, etc. In the analysis, the moving loads can be modeled as varying distributed loads, a series of concentrated loads, or the combination of distributed loads and concentrated loads.*

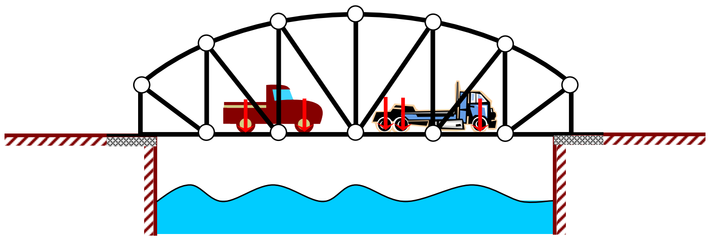


Figure 1

A moving unit load *-- a concentrated load of unit magnitude with its point of application varies as a function of position on the structure.*

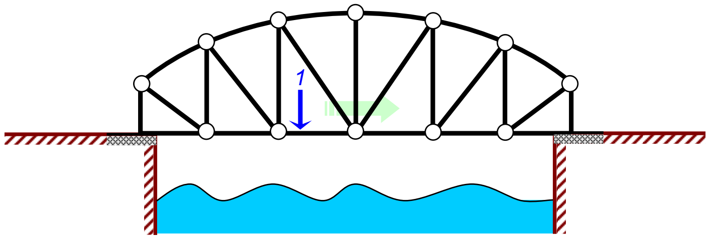


Figure 2

Responses due to moving loads *-- Quantities of interest that indicate the effect of the moving loads on a structure, e.g. internal forces, support reactions, displacements and rotations, deformations, etc.*

Responses due to a moving unit load *-- Quantities of interest at a particular point within a given structure, e.g. internal forces, support reactions, deformations, displacements and rotations, due to an applied moving unit load. The quantities are given in terms of functions of a position of a moving unit load on the structure; these response functions are termed as the* ***influence functions*** *and their graphical representations are known as the* ***influence lines****.*

Application of the influence functions (lines)

Let f_A be a quantity of interest at a point A within a given structure due to applied distributed load q and a series of concentrated loads $\{P_1, P_2, \ldots, P_N\}$ and f_{AI} denote the influence function of the corresponding quantity at point A. By a method of superposition, we obtain the relation of f_A and f_{AI} as

$$f_A = \int f_{AI} q \, dx + \sum_{i=1}^{N} f_{AI}(x_i) P_i \qquad (1)$$

where the integral is to be taken over the region on which load q is applied and x_i indicates the location on which the load P_i is applied. For instance, assume that the influence line of the support reaction at point A (R_{AI}) of the beam is given as shown in the Figure 3a. The support reaction at point A (R_A) due to applied loads as shown in Figure 3b can then be obtained using Eqn. (1) as follow:

$$R_A = \int_0^{L/2} R_{AI} q \, dx + R_{AI}(L/4)P + R_{AI}(3L/4)2P$$

$$= q\left(\int_0^{L/2} 1 - x/L \, dx\right) + (3/4)P + (1/4)(2P) = 3qL/8 + 5P/4$$

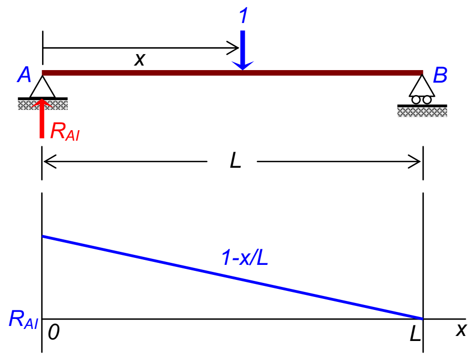


Figure 3a

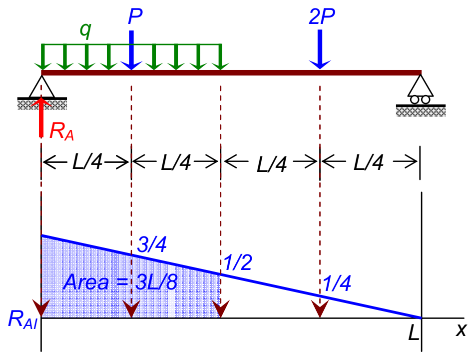
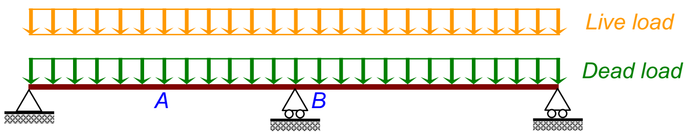
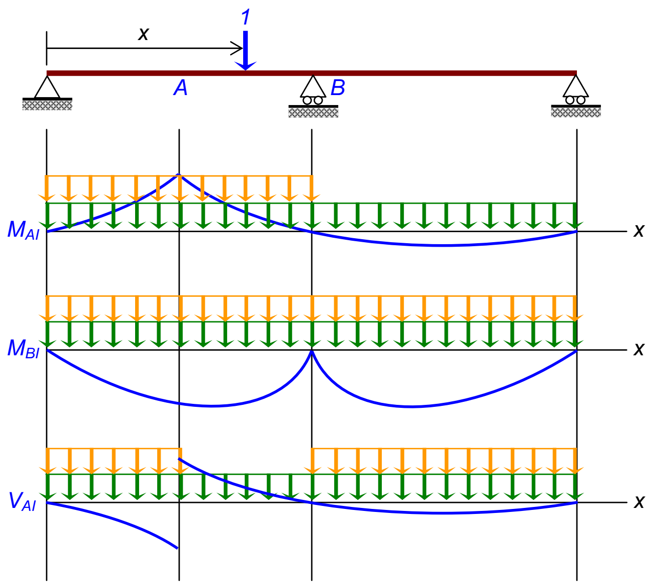


Figure 3b

In addition, the influence lines can also be used to predict the load pattern that maximizes responses at a particular point of the structure. For instance, let consider a two-span continuous beam subjected to both dead load (fixed load) and live load (varying load) as shown in the figure below.

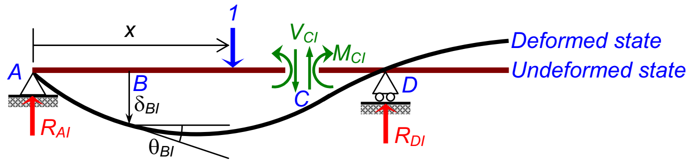


To determine the maximum positive bending moment at points A, the maximum negative moment B, and the maximum positive shear at point A due to these applied dead and live loads, we construct first the influence lines M_{AI}, M_{BI}, and V_{AI} as shown below.



It is evident from the influence lines that the maximum positive bending moment at point A occurs when the live load is placed only on the first span; the maximum negative moment at point B occurs when the live load is placed on both spans; and the maximum positive shear occurs when the live load is placed on the first half of the first span and on the second span. The maximum value of the responses can then be obtained using Eqn.(1) for each corresponding loading pattern. It is noted that the dead load is fixed and therefore it is applied to both spans of the beam for all cases.

INFLUENCE LINES FOR DETERMINATE BEAMS

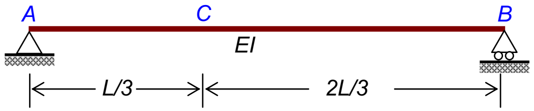


- *Support reactions (e.g. R_{AI}, R_{DI})*
- *Bending moment at a particular section (e.g. M_{CI})*
- *Shear force at a particular section (e.g. V_{CI})*
- *Deflection at a particular point (e.g. δ_{BI})*
- *Rotation at a particular point (e.g. θ_{BI})*

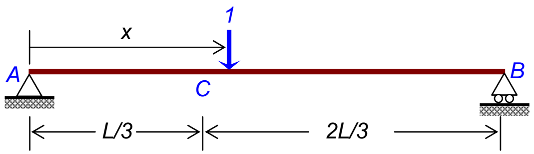
Direct Methods for Constructing Influence Lines

- *Treat a structure subjected to a moving unit load (as function of positions)*
- *Influence functions are obtained by considering all possible load locations*
- *Support reactions*
  - *-- Equilibrium equations of the entire structure*
- *Internal forces*
  - *-- Method of sections*
  - *-- Equilibrium equations of parts of the structure*
- *Displacement and rotations*
  - *-- Determining support reactions and internal forces from equilibrium*
  - *-- Displacement and rotations are obtained from*
    - ✓ *Direct integration method*
    - ✓ *Moment area and conjugate structure methods*
    - ✓ *Energy methods, etc.*

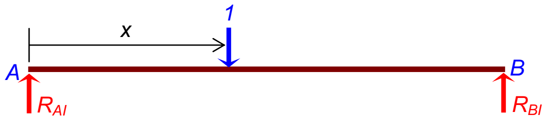
Example1: Construct influence lines R_{AI}, R_{BI}, V_{CI}, M_{CI}, δ_{CI}, θ_{CI} of a simply supported beam



Solution Consider the beam subjected to a moving unit load as shown below.



Influence lines for reactions R_{AI}, R_{BI}

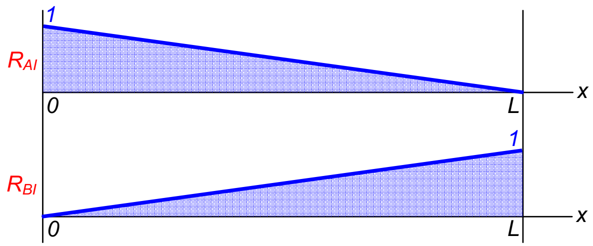


$[\Sigma M_B = 0]$ (+) $\quad -(R_{AI})(L) + (1)(L - x) = 0$

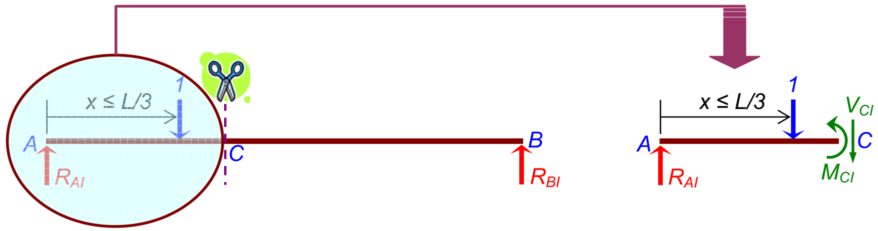
$$R_{AI} = \frac{L - x}{L}$$

$[\Sigma M_A = 0]$ (+) $\quad (R_{BI})(L) - (1)(x) = 0$

$$R_{BI} = \frac{x}{L}$$



Influence lines for shear and bending moment V_{CI}, M_{CI}

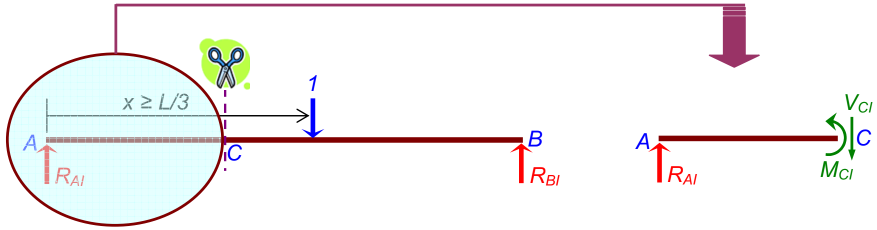


$[\Sigma F_Y = 0]$ ↑+ $\quad R_{AI} - 1 - V_{CI} = 0$

$$V_{CI} = R_{AI} - 1 = -\frac{x}{L}$$

$[\Sigma M_C = 0]$ (+) $\quad -(R_{AI})(L/3) + (1)(L/3 - x) + M_{CI} = 0$

$$M_{CI} = \frac{(R_{AI} - 1)L}{3} + x = \frac{2x}{3}$$

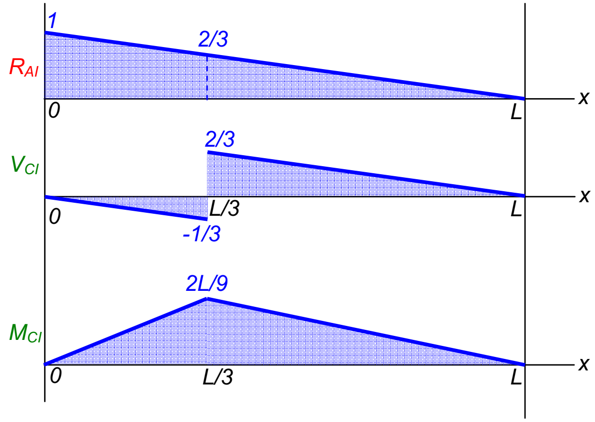


$[\Sigma F_Y = 0]$ ↑+ $\quad R_{AI} - V_{CI} = 0$

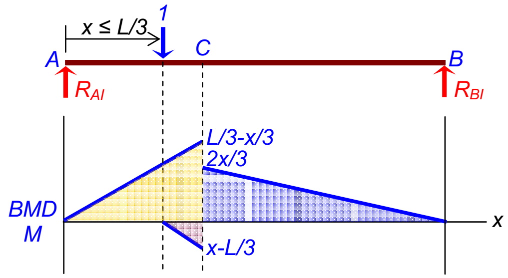
$$V_{CI} = R_{AI} = 1 - \frac{x}{L}$$

$[\Sigma M_C = 0]$ (+) $\quad -(R_{AI})(L/3) + M_{CI} = 0$

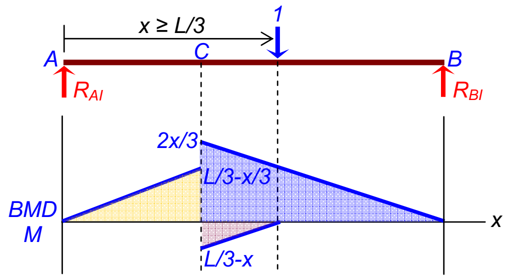
$$M_{CI} = \frac{R_{AI}L}{3} = \frac{L}{3}\left(1 - \frac{x}{L}\right)$$



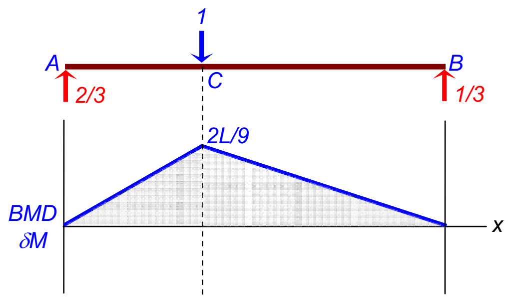
<u>Influence lines for deflection and rotation</u> δ_{CI}, θ_{CI}



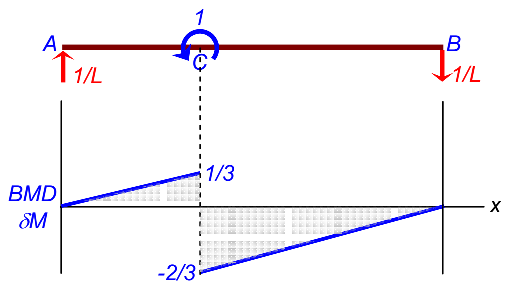
Actual System I



Actual System II



Virtual System I



Virtual System II

The deflection δ_{CI} for $x \le L/3$ can be obtained using the unit load method along with the actual system I and the virtual system I; i.e.

$$\delta_{CI} = \int_0^L \frac{M\delta M}{EI}dx$$

$$= \frac{1}{2EI}\left(\frac{L-x}{3}\right)\left(\frac{L}{3}\right)\left[\frac{2}{3}\frac{2L}{9}\right] + \frac{1}{2EI}\left(x-\frac{L}{3}\right)\left(\frac{L}{3}-x\right)\left[\left(\frac{2}{3}+\frac{x}{L}\right)\frac{2L}{9}\right]$$

$$+\frac{1}{2EI}\left(\frac{2x}{3}\right)\left(\frac{2L}{3}\right)\left[\frac{2}{3}\frac{2L}{9}\right]$$

$$= \frac{x}{81EI}\left(5L^2 - 9x^2\right)$$

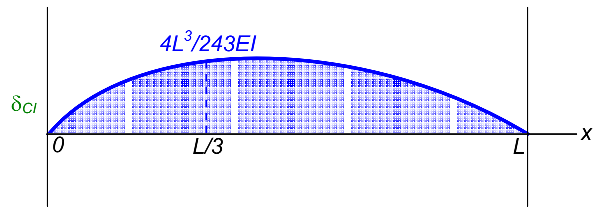
The deflection δ_{CI} for $x \ge L/3$ can be obtained using the unit load method along with the actual system II and the virtual system I; i.e.

$$\delta_{CI} = \int_0^L \frac{M\delta M}{EI}dx$$

$$= \frac{1}{2EI}\left(\frac{L-x}{3}\right)\left(\frac{L}{3}\right)\left[\frac{2}{3}\frac{2L}{9}\right] + \frac{1}{2EI}\left(\frac{L}{3}-x\right)\left(x-\frac{L}{3}\right)\left[\left(\frac{7}{6}-\frac{x}{2L}\right)\frac{2L}{9}\right]$$

$$+\frac{1}{2EI}\left(\frac{2x}{3}\right)\left(\frac{2L}{3}\right)\left[\frac{2}{3}\frac{2L}{9}\right]$$

$$= \frac{x-L}{162EI}\left(L^2 - 18Lx + 9x^2\right)$$



The rotation θ_{CI} for $x \le L/3$ can be obtained using the unit load method along with the actual system I and the virtual system II; i.e.

$$\theta_{CI} = \int_0^L \frac{M\delta M}{EI} dx$$

$$= \frac{1}{2EI}\left(\frac{L-x}{3}\right)\left(\frac{L}{3}\right)\left[\frac{2}{3}\frac{1}{3}\right] + \frac{1}{2EI}\left(x-\frac{L}{3}\right)\left(\frac{L}{3}-x\right)\left[\left(\frac{2}{3}+\frac{x}{L}\right)\frac{1}{3}\right]$$

$$+\frac{1}{2EI}\left(\frac{2x}{3}\right)\left(\frac{2L}{3}\right)\left[-\frac{2}{3}\frac{2}{3}\right]$$

$$= -\frac{x}{18EIL}\left(L^2+3x^2\right)$$

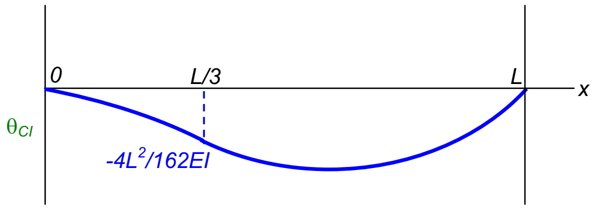
The rotation θ_{CI} for $x \ge L/3$ can be obtained using the unit load method along with the actual system II and the virtual system I; i.e.

$$\theta_{CI} = \int_0^L \frac{M\delta M}{EI} dx$$

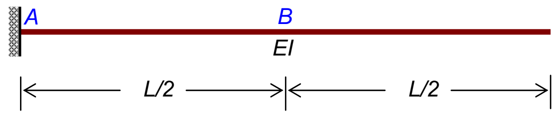
$$= \frac{1}{2EI}\left(\frac{L-x}{3}\right)\left(\frac{L}{3}\right)\left[\frac{2}{3}\frac{1}{3}\right] + \frac{1}{2EI}\left(\frac{L}{3}-x\right)\left(x-\frac{L}{3}\right)\left[-\left(\frac{7}{6}-\frac{x}{2L}\right)\frac{2}{3}\right]$$

$$+\frac{1}{2EI}\left(\frac{2x}{3}\right)\left(\frac{2L}{3}\right)\left[-\frac{2}{3}\frac{2}{3}\right]$$

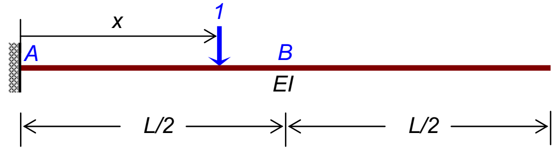
$$= \frac{L-x}{18EIL}\left(L^2-6Lx+3x^2\right)$$



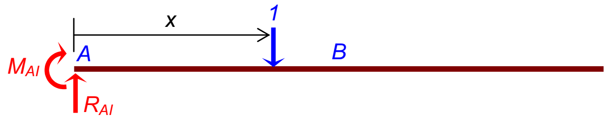
Example2: Construct influence lines R_{AI}, M_{AI}, V_{BI}, M_{BI}, δ_{BI}, θ_{BI} of a cantilever beam



Solution Consider the beam subjected to a moving unit load as shown below.



Influence lines for reactions R_{AI}, M_{AI}

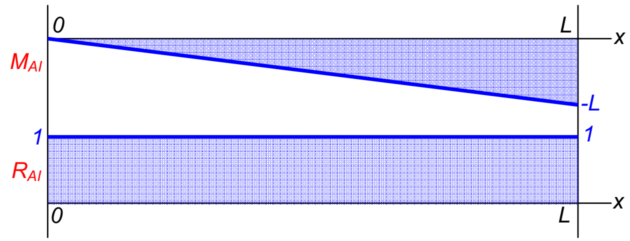


$[\Sigma M_A = 0]$ (+) $\qquad -M_{AI} - (1)(x) = 0$

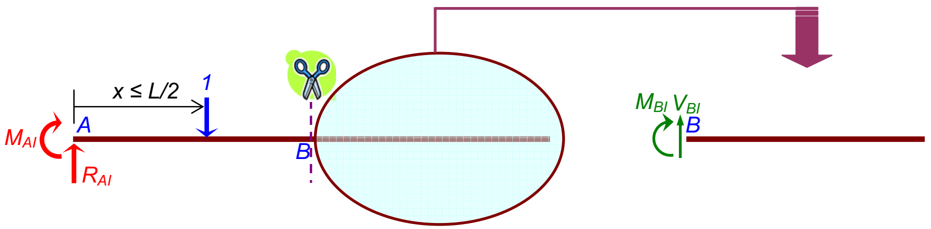
$$M_{AI} = -x$$

$[\Sigma F_Y = 0]$ ↑+ $\qquad R_{AI} - 1 = 0$

$$R_{AI} = 1$$



Influence lines for shear and bending moment V_{CI}, M_{CI}

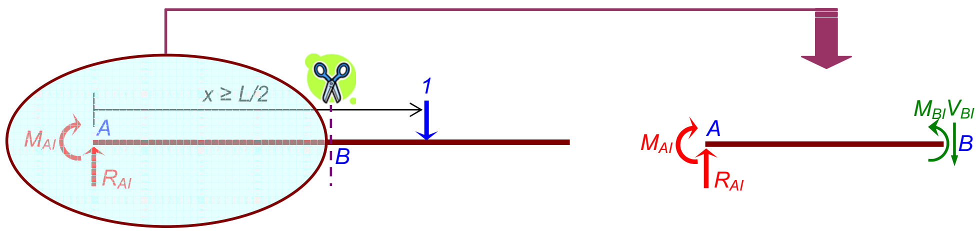


$[\Sigma F_Y = 0] \uparrow +$ $\quad V_{BI} = 0$

$$V_{BI} = 0$$

$[\Sigma M_B = 0]$ (+) $\quad -M_{BI} = 0$

$$M_{BI} = 0$$

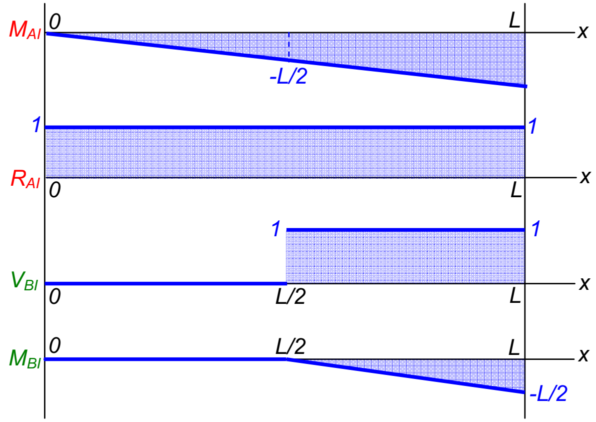


$[\Sigma F_Y = 0] \uparrow +$ $\quad R_{AI} - V_{BI} = 0$

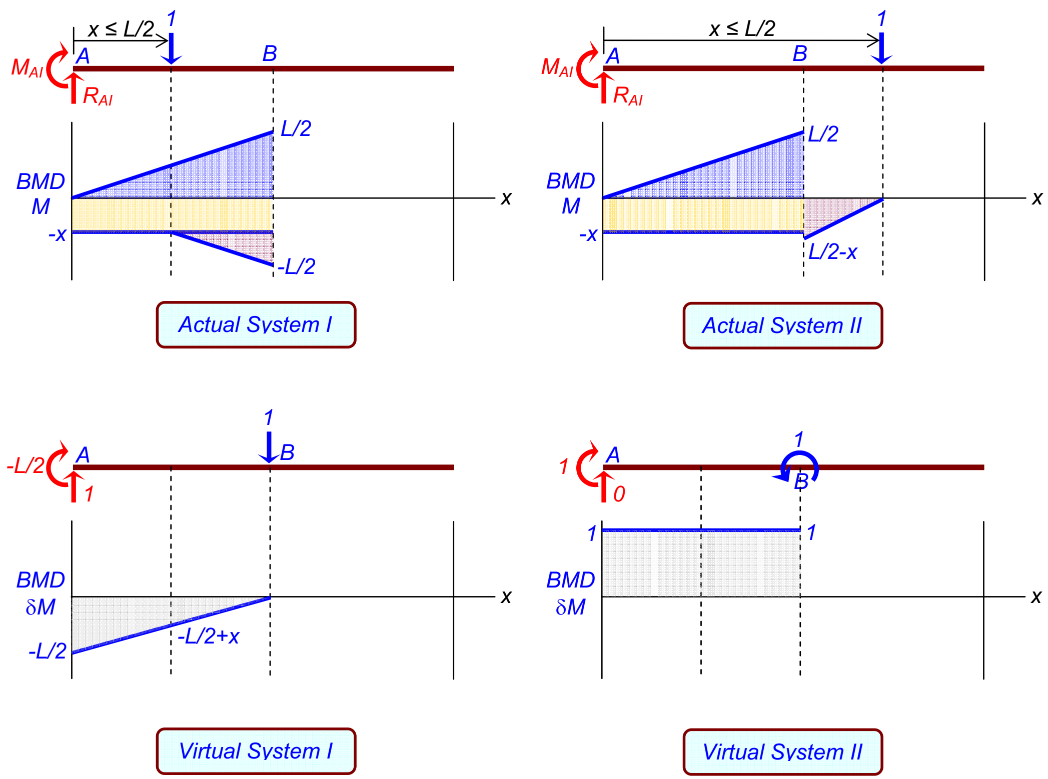
$$V_{BI} = R_{AI} = 1$$

$[\Sigma M_B = 0]$ (+) $\quad -(R_{AI})(L/2) - M_{AI} + M_{BI} = 0$

$$M_{BI} = \frac{R_{AI}L}{2} + M_{AI} = -\left(x - \frac{L}{2}\right)$$



Influence lines for deflection and rotation δ_{BI}, θ_{BI}



The deflection δ_{BI} for $x \le L/2$ can be obtained using the unit load method along with the actual system I and the virtual system I; i.e.

$$\delta_{BI} = \int_0^L \frac{M\delta M}{EI} dx$$

$$= \frac{1}{2EI}\left(\frac{L}{2}\right)\left(\frac{L}{2}\right)\left[-\frac{1}{3}\frac{L}{2}\right] + \frac{1}{EI}(-x)\left(\frac{L}{2}\right)\left[-\frac{1}{2}\frac{L}{2}\right]$$

$$+ \frac{1}{2EI}\left(-\frac{L}{2}+x\right)\left(\frac{L}{2}-x\right)\left[-\left(\frac{1}{3}-\frac{2x}{3L}\right)\frac{L}{2}\right]$$

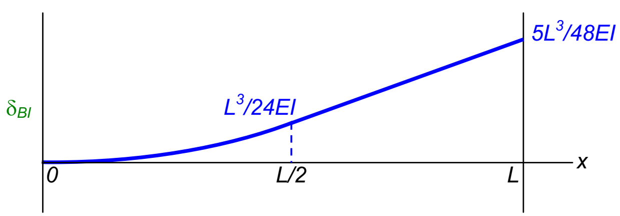
$$= \frac{x^2}{12EI}(3L-2x)$$

The deflection δ_{BI} for $x \ge L/2$ can be obtained using the unit load method along with the actual system II and the virtual system I; i.e.

$$\delta_{CI} = \int_0^L \frac{M\delta M}{EI} dx$$

$$= \frac{1}{2EI}\left(\frac{L}{2}\right)\left(\frac{L}{2}\right)\left[-\frac{1}{3}\frac{L}{2}\right] + \frac{1}{EI}(-x)\left(\frac{L}{2}\right)\left[-\frac{1}{2}\frac{L}{2}\right]$$

$$= \frac{L^2}{48EI}(6x-L)$$



The rotation θ_{BI} for $x \le L/2$ can be obtained using the unit load method along with the actual system I and the virtual system II; i.e.

$$\theta_{CI} = \int_0^L \frac{M\delta M}{EI} dx$$

$$= \frac{1}{2EI}\left(\frac{L}{2}\right)\left(\frac{L}{2}\right)[1] + \frac{1}{EI}(-x)\left(\frac{L}{2}\right)[1]$$

$$+ \frac{1}{2EI}\left(-\frac{L}{2}+x\right)\left(\frac{L}{2}-x\right)[1]$$

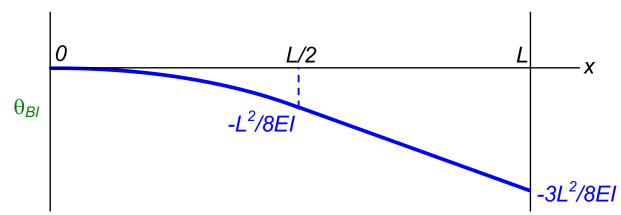
$$= -\frac{x^2}{2EI}$$

The rotation θ_{BI} for $x \ge L/2$ can be obtained using the unit load method along with the actual system II and the virtual system I; i.e.

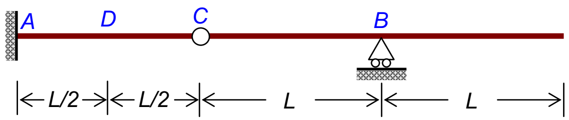
$$\theta_{CI} = \int_0^L \frac{M\delta M}{EI} dx$$

$$= \frac{1}{2EI}\left(\frac{L}{2}\right)\left(\frac{L}{2}\right)[1] + \frac{1}{EI}(-x)\left(\frac{L}{2}\right)[1]$$

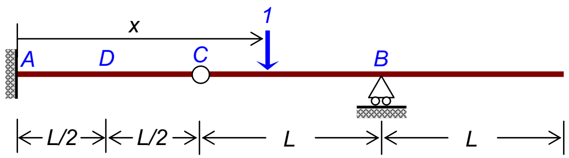
$$= \frac{L}{8EI}(L-4x)$$



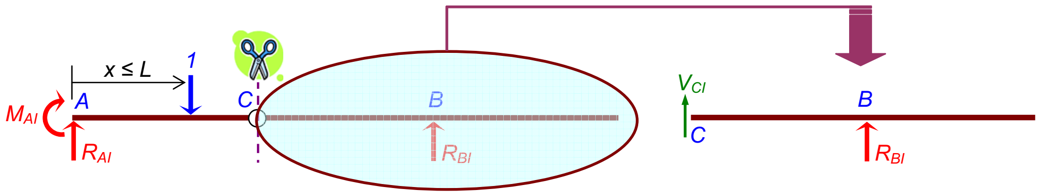
Example3: Construct influence lines R_{AI}, M_{AI}, R_{BI}, V_{CI}, V_{BLI}, M_{BLI}, V_{BRI}, M_{BRI}, V_{DI}, and M_{DI} of a beam shown below



Solution Consider the beam subjected to a moving unit load as shown below.



Influence lines for reactions R_{AI}, M_{AI}, R_{BI} *and shear force* V_{CI}



FBD I *FBD II*

From FBD II, we obtain

$$[\Sigma M_C = 0] \circlearrowleft + \quad (R_{BI})(L) = 0$$

$$\boxed{R_{BI} = 0}$$

$$[\Sigma F_Y = 0] \uparrow + \quad V_{CI} + R_{BI} = 0$$

$$\boxed{V_{CI} = -R_{BI} = 0}$$

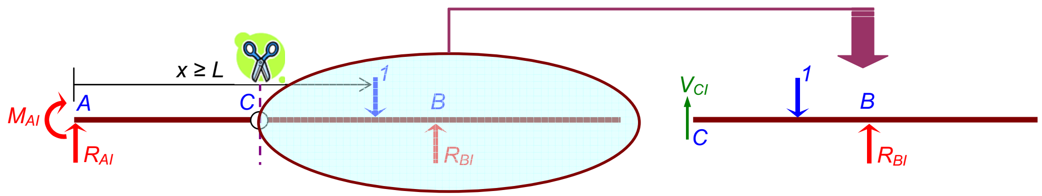
From FBD I, we obtain

$$[\Sigma F_Y = 0] \uparrow + \quad R_{AI} + R_{BI} - 1 = 0$$

$$\boxed{R_{AI} = 1 - R_{BI} = 1}$$

$$[\Sigma M_A = 0] \circlearrowleft + \quad -M_{AI} + (R_{BI})(2L) - (1)(x) = 0$$

$$\boxed{M_{AI} = 2R_{BI}L - x = -x}$$



FBD III *FBD IV*

From FBD IV, we obtain

$$[\Sigma M_C = 0] \circlearrowleft + \quad (R_{BI})(L) - (1)(x - L) = 0$$

$$\boxed{R_{BI} = \frac{x}{L} - 1}$$

$$[\Sigma F_Y = 0] \uparrow + \quad V_{CI} + R_{BI} - 1 = 0$$

$$\boxed{V_{CI} = 1 - R_{BI} = 2 - \frac{x}{L}}$$

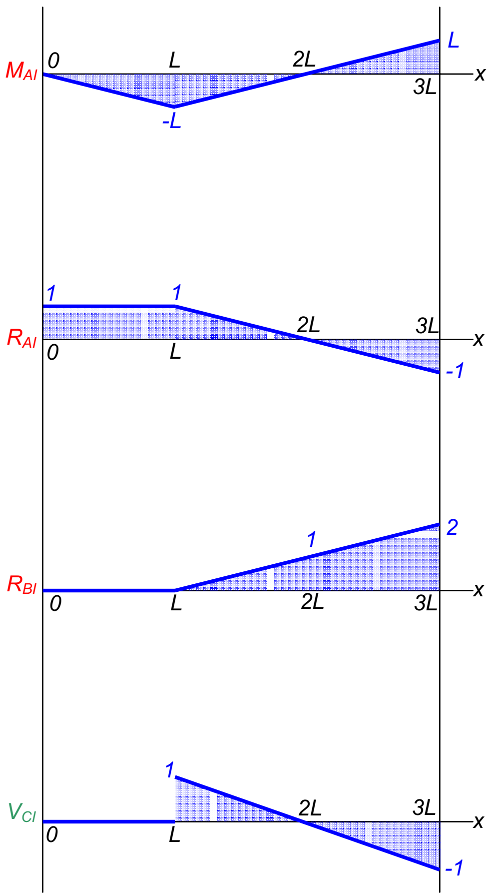
From FBD III, we obtain

$$[\Sigma F_Y = 0] \uparrow + \quad R_{AI} + R_{BI} - 1 = 0$$

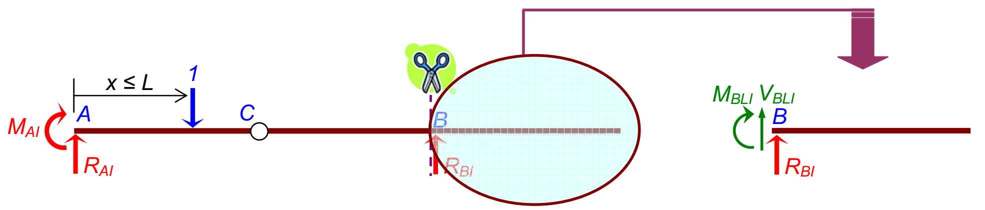
$$\boxed{R_{AI} = 1 - R_{BI} = 2 - \frac{x}{L}}$$

$$[\Sigma M_A = 0] \;\; (+) \qquad -M_{AI} + (R_{BI})(2L) - (1)(x) = 0$$

$$\boxed{M_{AI} = 2R_{BI}L - x = x - 2L}$$



<u>Influence lines for shear and bending moment</u> V_{BLI}, M_{BLI}

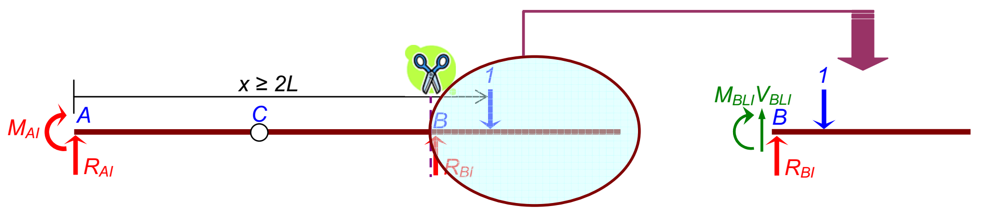


$$[\Sigma F_Y = 0] \;\; \uparrow + \qquad V_{BLI} + R_{BI} = 0$$

$$\boxed{V_{BLI} = -R_{BI}}$$

$$[\Sigma M_B = 0] \;\; (+) \qquad -M_{BLI} = 0$$

$$\boxed{M_{BLI} = 0}$$

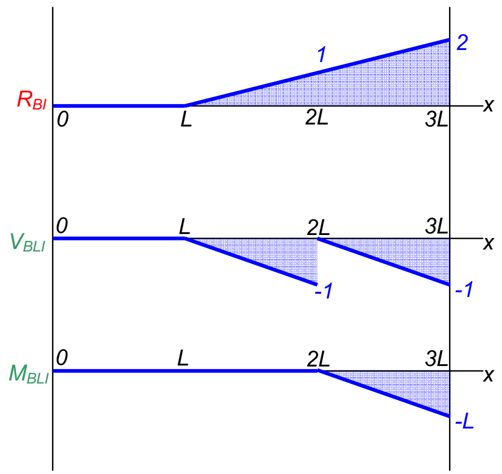


$$[\Sigma F_Y = 0] \;\; \uparrow + \qquad V_{BLI} + R_{BI} - 1 = 0$$

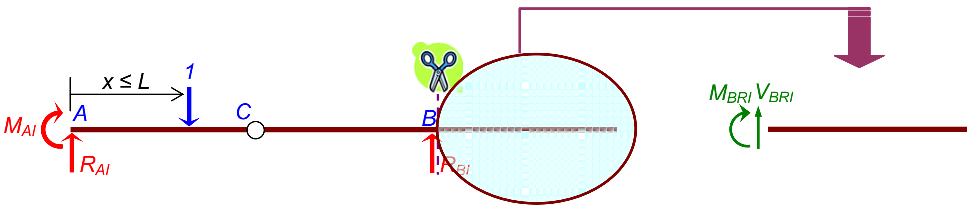
$$\boxed{V_{BLI} = 1 - R_{BI}}$$

$$[\Sigma M_B = 0] \;\; (+) \qquad -M_{BLI} - (1)(x - 2L) = 0$$

$$\boxed{M_{BLI} = 2L - x}$$



Influence lines for shear and bending moment V_{BRI}, M_{BRI}

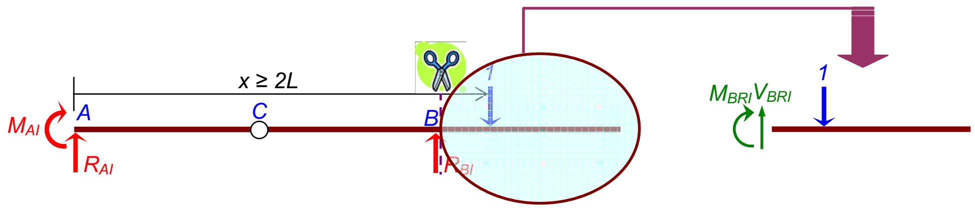


$[\Sigma F_Y = 0]$ ↑+ $\quad V_{BRI} = 0$

$$V_{BRI} = 0$$

$[\Sigma M_B = 0]$ (+) $\quad -M_{BRI} = 0$

$$M_{BRI} = 0$$

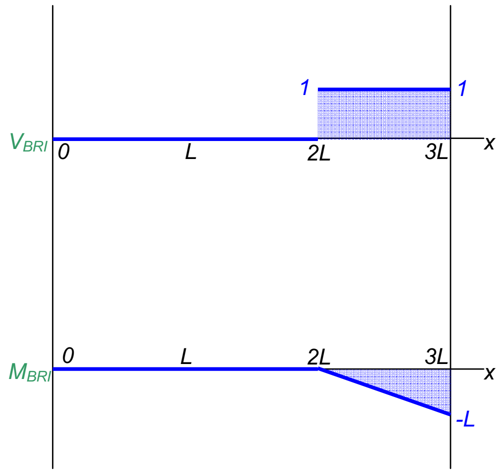


$[\Sigma F_Y = 0]$ ↑+ $\quad V_{BRI} - 1 = 0$

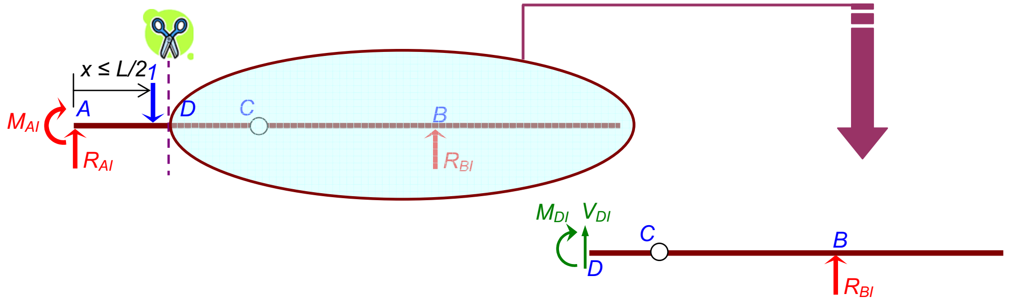
$$V_{BRI} = 1$$

$[\Sigma M_B = 0]$ (+) $\quad -M_{BRI} - (1)(x - 2L) = 0$

$$M_{BRI} = 2L - x$$



Influence lines for shear and bending moment V_{DI}, M_{DI}

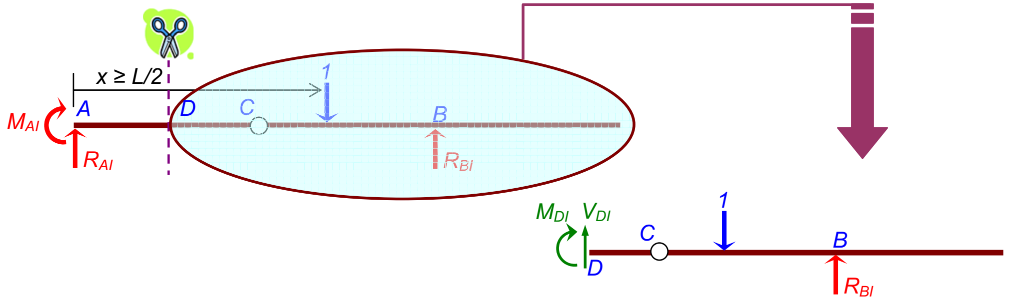


$$[\Sigma F_Y = 0] \uparrow + \qquad V_{DI} + R_{BI} = 0$$

$$\boxed{V_{DI} = -R_{BI}}$$

$$[\Sigma M_B = 0] \circlearrowleft + \qquad -M_{DI} + (R_{BI})(3L/2) = 0$$

$$\boxed{M_{DI} = 3LR_{BI}/2}$$

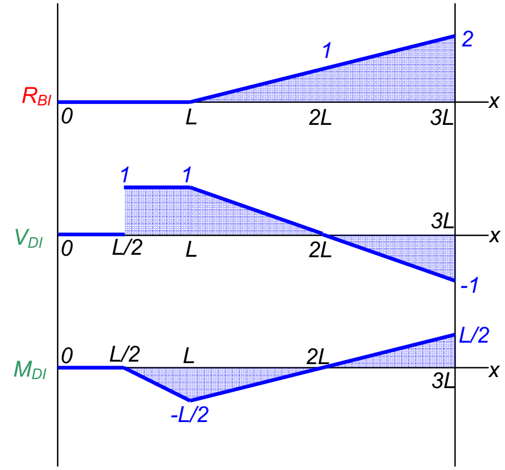


$$[\Sigma F_Y = 0] \uparrow + \qquad V_{DI} + R_{BI} - 1 = 0$$

$$\boxed{V_{BLI} = 1 - R_{BI}}$$

$$[\Sigma M_B = 0] \circlearrowleft + \qquad -M_{DI} - (1)(x - L/2) + (R_{BI})(3L/2) = 0$$

$$\boxed{M_{DI} = 3LR_{BI}/2 + L/2 - x}$$



Remarks

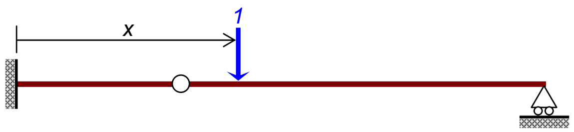
1. ***The influences lines of support reactions and internal forces (shear force and bending moment) for statically determinate beams are piecewise linear; i.e. they consists of only straight line segments.***

2. ***The influence functions of the internal forces can be obtained in terms of the influence functions of the support reactions; therefore, the influence lines of internal forces can be readily obtained from those for support reactions.***

3. ***The influence lines of the deflection and rotation at any points of the statically determinate beam generally consist of curve segments.***

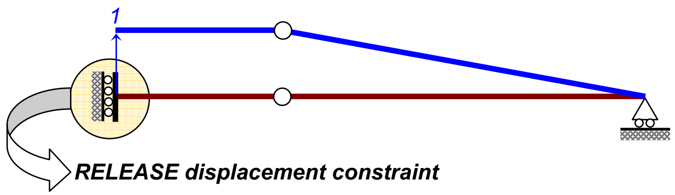
Muller-Breslau Principle

***Actual Structure**. Consider a statically determinate beam subjected to a moving unit load as shown in the figure below.*

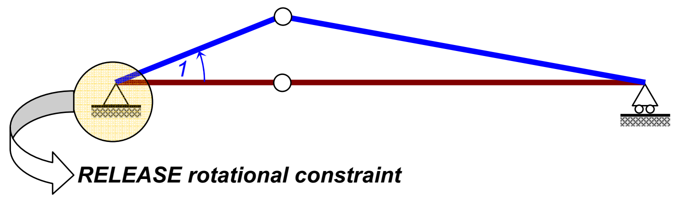


***Virtual Displacement** -- The fictitious and arbitrary displacement that is introduced to the structure. For use further below, the following three types of virtual displacement for the beam structure are considered:*

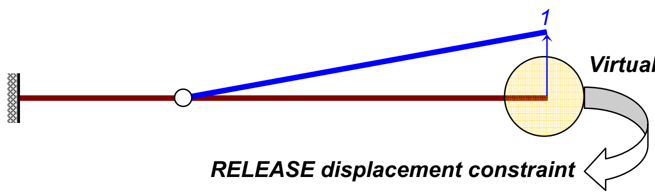
- ***Virtual displacement due to release of a support constraint**.*
  1. *Release a support constraint in the direction of interest*
  2. *The beam becomes statically unstable (partially or completely)*
  3. *Introduce **unit virtual displacement** (or **unit virtual rotation** if the rotational constraint is released) in the direction that the support constraint is released.*
  4. *The virtual displacement at all other points results from the development of the mechanism (or rigid body motion) of the entire beam*



Virtual System 1a

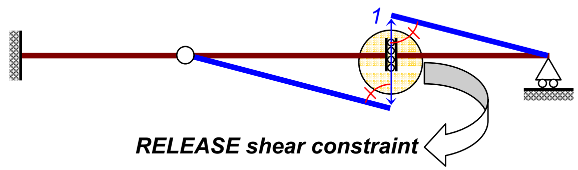


Virtual System 1b

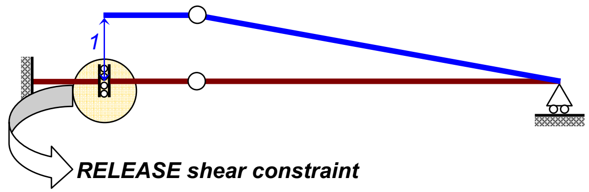


Virtual System 1c

- ***Virtual displacement due to release of shear constraint**.*
  1. *Remove the shear constraint by introducing a shear release at point of interest*
  2. *The beam becomes statically unstable (partially or completely)*
  3. *Introduce **unit relative virtual displacement** between the two ends of the shear release with their **slope remaining the same** (provided that the moment constraint exists at that point)*
  4. *The virtual displacement at all other points results from the development of the mechanism (or rigid body motion) of the entire beam.*

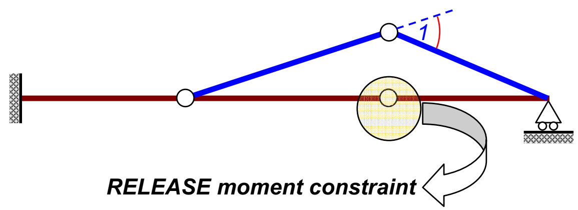


Virtual System 2a

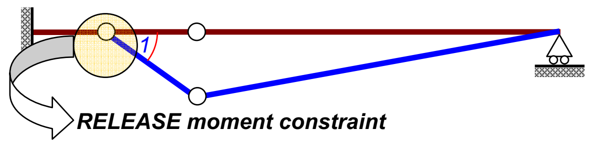


Virtual System 2b

- ***Virtual displacement due to release of bending moment constraint**.*
  1. *Remove the moment constraint by introducing a hinge at point of interest*
  2. *The beam becomes statically unstable (partially or completely)*
  3. *Introduce **unit relative virtual rotation** at the hinge without separation (provided that the shear constraint exists at that point).*
  4. *The virtual displacement at all other points results from the development of the mechanism (or rigid body motion) of the entire beam.*



Virtual System 3a



Virtual System 3b

***Principle of Virtual Work**: Consider a system or structure subjected to external applied loads. The support reactions and internal forces at any locations within the structure are in equilibrium with the applied loads if and only if the external virtual work (work done by the external applied loads) is the same as the internal virtual work (work done by the internal forces) for all admissible virtual displacements, i.e.*

$$\delta W_E = \delta W_I \tag{2}$$

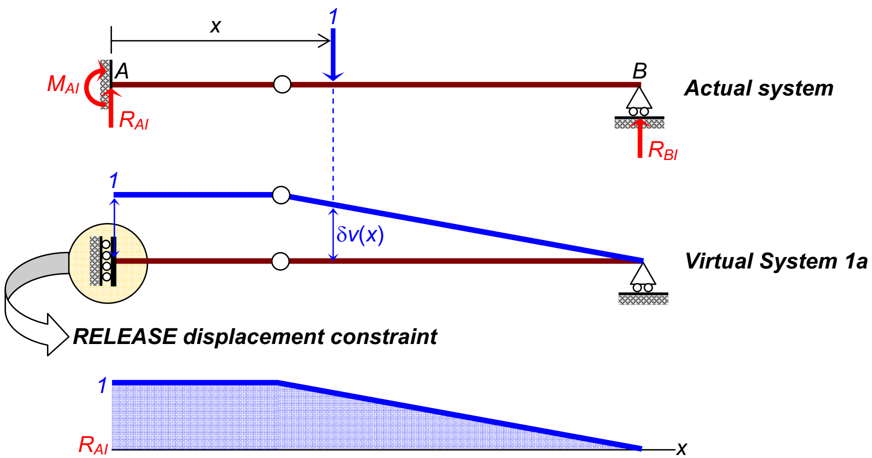
It is important to note that the portion of the structure that undergoes virtual rigid body motion (virtual displacement that produces no deformation) produces zero internal virtual work.

***Influence Line for Support Reactions**. To clearly illustrate the strategy, let assume that the influence line of the support reaction R_{AI} is to be determined. By applying the principle of virtual work to the actual system with a special choice of the virtual displacement as indicated in the virtual system 1a (the virtual displacement associated with the rigid body motion of the beam resulting from the release of the displacement constraint at A) , we obtain*

$$\delta W_E = R_{AI} \cdot 1 - 1 \cdot \delta v(x) = R_{AI} - \delta v(x) \quad ; \quad \delta W_I = 0$$

$$\delta W_E = \delta W_I$$

$$\Rightarrow \quad R_{AI} = \delta v(x) \tag{3}$$



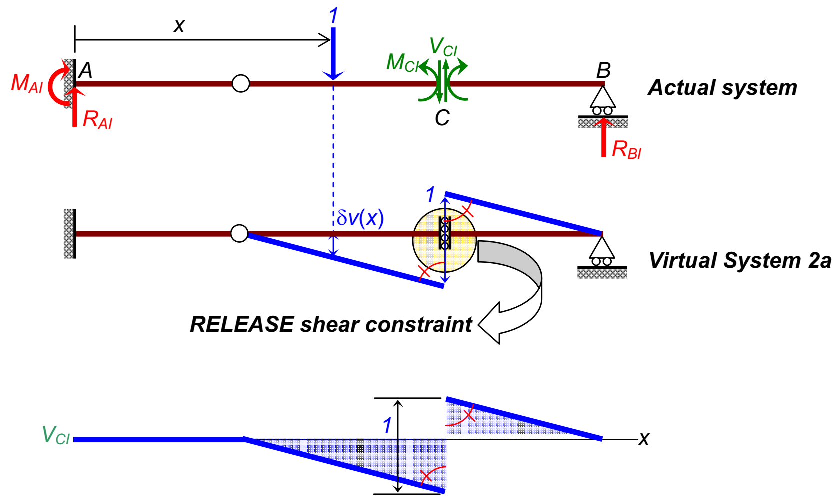
<u>Muller-Breslau Principle</u>: "The influence line of a particular support reaction has an identical shape to the virtual displacement obtained from releasing the support constraint in the direction of the support reaction (under consideration) and introducing a rigid body motion with unit displacement/unit rotation in the direction of the released constraint."

***Influence Line for Shear Force**. Let assume that the influence line of the shear force at point C, V_{CI}, is to be determined. By applying the principle of virtual work to the actual system with a special choice of the virtual displacement as indicated in the virtual system 2a (the virtual displacement associated with the rigid body motion of the beam resulting from the release of the shear constraint at C) , we obtain*

$$\delta W_E = -1 \cdot \delta v(x) = -\delta v(x) \quad ; \quad \delta W_I = -V_{CI} \cdot 1 = -V_{CI}$$

$$\delta W_E = \delta W_I$$

$$\Rightarrow \quad V_{CI} = \delta v(x) \tag{4}$$



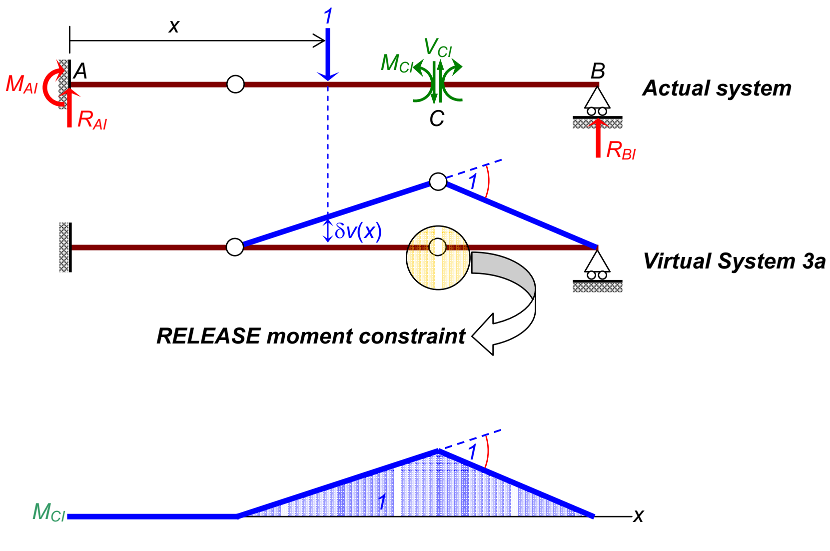
<u>Muller-Breslau Principle</u>: "The influence line of the shear force at a particular point has an identical shape to the virtual displacement obtained from releasing the shear constraint at that point and introducing a rigid body motion with unit relative virtual displacement between the two ends of the shear release with their slope remaining the same."

***Influence Line for Bending Moment**. Let assume that the influence line of the bending at point C, M_{CI}, is to be determined. By applying the principle of virtual work to the actual system with a special choice of the virtual displacement as indicated in the virtual system 3a (the virtual displacement associated with the rigid body motion of the beam resulting from the release of the bending moment constraint at C) , we obtain*

$$\delta W_E = -1 \cdot \delta v(x) = -\delta v(x) \quad ; \quad \delta W_I = -M_{CI} \cdot 1 = -M_{CI}$$

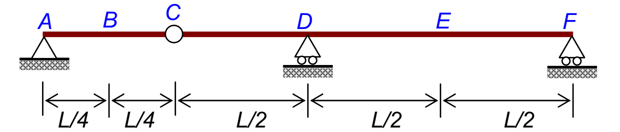
$$\delta W_E = \delta W_I$$

$$\Rightarrow \qquad \boxed{M_{CI} = \delta v(x)} \qquad (5)$$

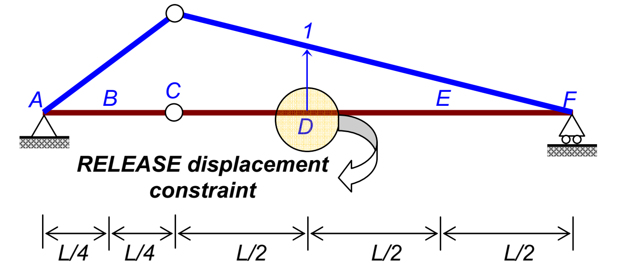


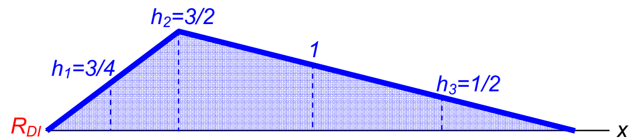
<u>Muller-Breslau Principle</u>: "The influence line of the shear force at a particular point has an identical shape to the virtual displacement obtained from releasing the shear constraint at that point and introducing a rigid body motion with unit relative virtual displacement between the two ends of the shear release with their slope remaining the same."

<u>Example4</u>: Use Muller-Breslau principle to construct influence lines R_{AI}, R_{DI}, R_{FI}, V_{BI}, V_{CLI}, V_{CRI}, V_{DLI}, V_{DRI}, V_{EI}, M_{BI}, M_{DI}, and M_{EI} of a statically determinate beam shown below



<u>Solution</u> The influence line of the support reaction R_{DI} is obtained as follow: 1) release the displacement constraint at point *D*, 2) introduce a rigid body motion, 3) impose unit displacement at point *D*, and 4) the resulting virtual displacement is the influence line of R_{DI}.





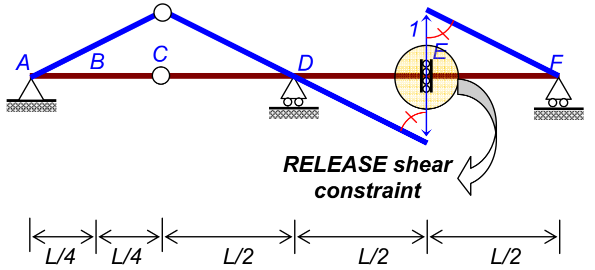
The value of the influence line at other points can be readily determined from the geometry, for instance,

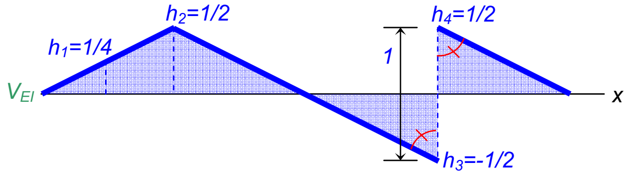
$$h_2 = (1)(3L/2)/(L) = 3/2$$

$$h_3 = (1)(L/2)/(L) = 1/2$$

$$h_1 = (3/2)(L/4)/(L/2) = 3/4$$

The influence line of the shear force V_{EI} is obtained as follow: 1) release the shear constraint at point E, 2) introduce a rigid body motion, 3) impose unit relative displacement at point E and 4) the resulting virtual displacement is the influence line of V_{EI}.





The value of the influence line at other points can be readily determined from the geometry, for instance,

$$-h_3/(L/2) = h_4/(L/2) \;\Rightarrow\; h_3 = -h_4$$

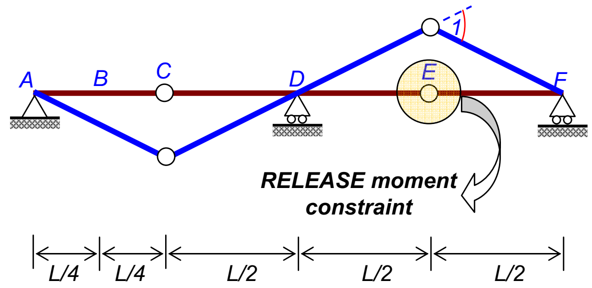
$$h_4 - h_3 = 1 \;\Rightarrow\; h_4 - (-h_4) = 2h_4 = 1 \;\Rightarrow\; h_4 = 1/2$$

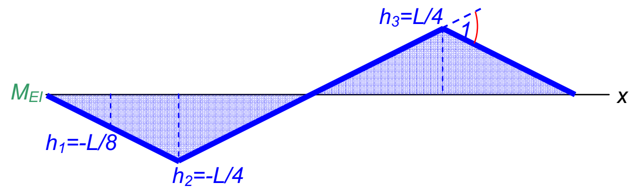
$$h_3 = -h_4 = -1/2$$

$$h_2 = (-h_3)(L/2)/(L/2) = 1/2$$

$$h_1 = (h_2)(L/4)/(L/2) = 1/4$$

The influence line of the bending moment M_{EI} is obtained as follow: 1) release the bending moment constraint at point E, 2) introduce a rigid body motion, 3) impose unit relative rotation at point E without separation and 4) the resulting virtual displacement is the influence line of M_{EI}.





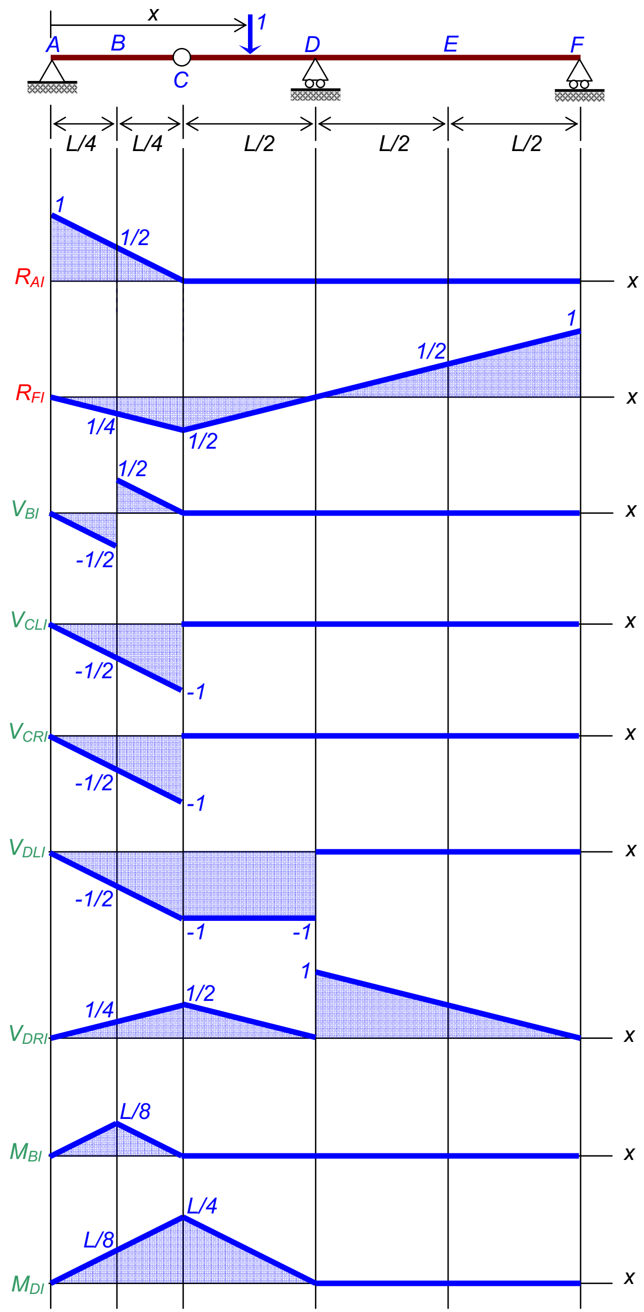
The value of the influence line at other points can be readily determined from the geometry, for instance,

$$h_3/(L/2) + h_3/(L/2) = 1 \;\Rightarrow\; h_3 = L/4$$

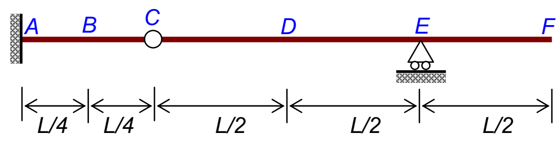
$$h_2 = (-h_3)(L/2)/(L/2) = -L/4$$

$$h_1 = (h_2)(L/4)/(L/2) = -L/8$$

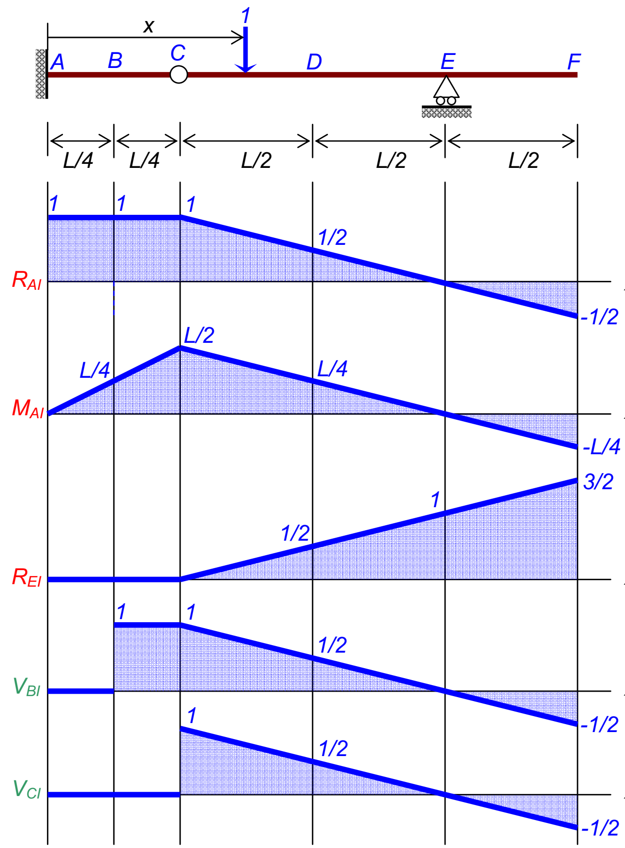
The rest of the influence lines can be determined in the same manner and results are given below.

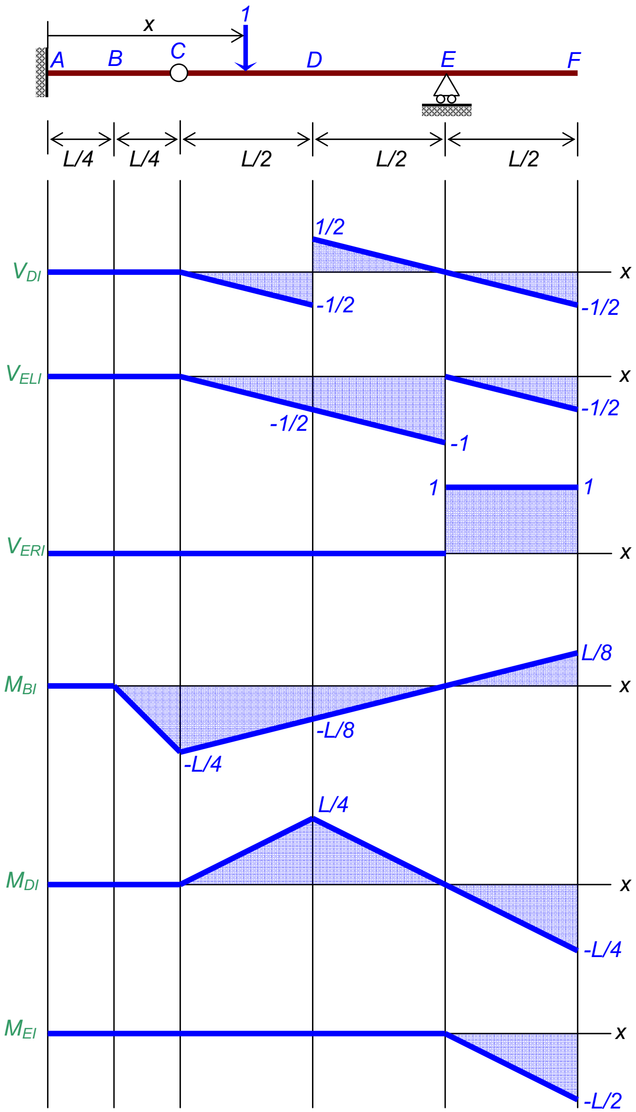


Example5: Use Muller-Breslau principle to construct influence lines R_{AI}, M_{AI}, R_{DI}, V_{BI}, V_{CI}, V_{DI}, V_{ELI}, V_{ERI}, M_{BI}, M_{DI}, and M_{EI} of a statically determinate beam shown below.

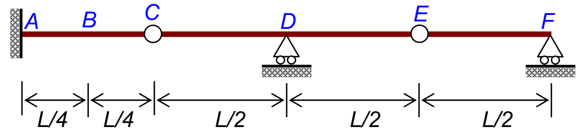


Solution By Muller-Breslau principle, we obtain the influence lines as follow: 1) release the constraint associated with the quantity of interest, 2) introduce a rigid body motion, 3) impose unit virtual displacement/rotation in the direction of released constraint, and 4) the resulting virtual displacement is the influence line to be determined. It is noted that values at points on the influence line can be readily determined from the geometry.

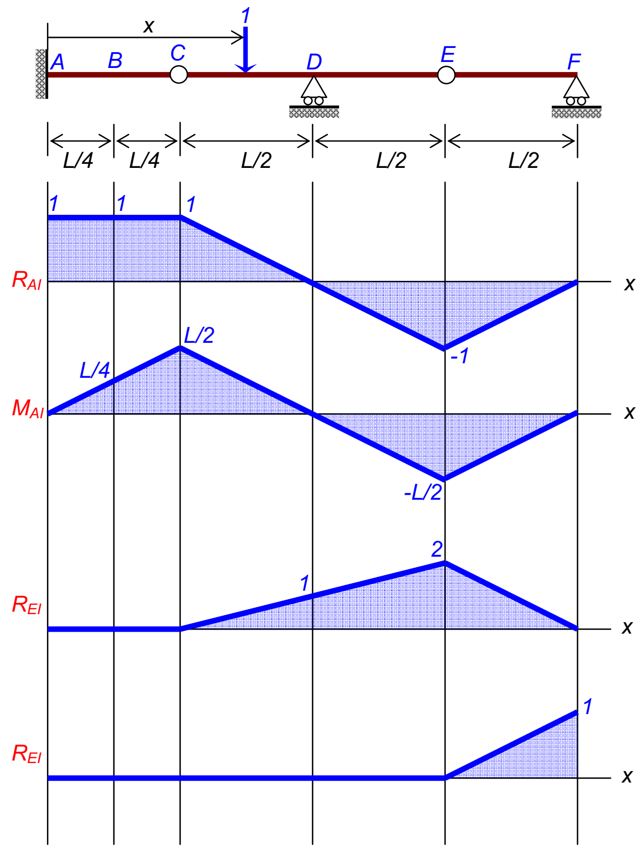


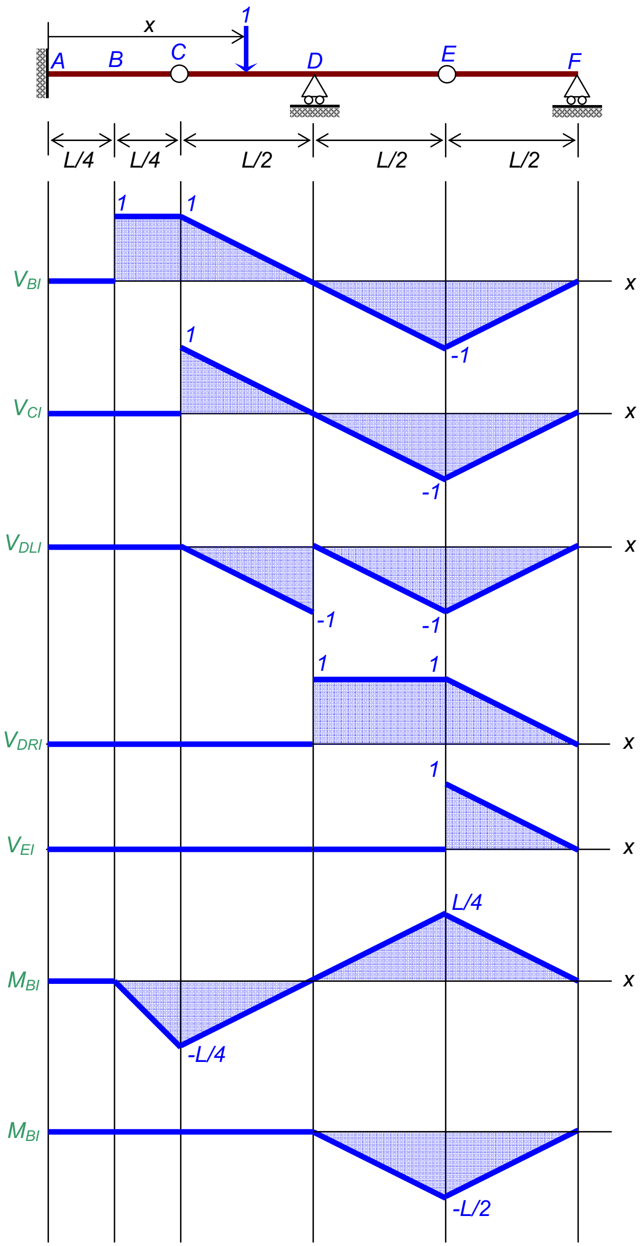


Example6: Use Muller-Breslau principle to construct influence lines R_{AI}, M_{AI}, R_{DI}, R_{FI}, V_{BI}, V_{CI}, V_{DLI}, V_{DRI}, V_{EI}, M_{BI}, and M_{DI} of a statically determinate beam shown below.



Solution By Muller-Breslau principle, we obtain the influence lines as follow: 1) release the constraint associated with the quantity of interest, 2) introduce a rigid body motion, 3) impose unit virtual displacement/rotation in the direction of released constraint, and 4) the resulting virtual displacement is the influence line to be determined. It is noted that values at points on the influence line can be readily determined from the geometry.




x
1
A
B
C
D
E
F
L/4
L/4
L/2
L/2
L/2
V_BI
V_CI
V_DLI
V_DRI
V_EI
M_BI
M_BI
1
-1
-1
-1
L/4
-L/4
-L/2
x