9-4 Method of virtual work: Trusses

- External loading
  - Consider the vertical disp Δ of joint B in Fig 9.7
  - If the applied loadings P_1 & P_2 cause a linear elastic material response, the element will deform $\Delta L = NL / AE$

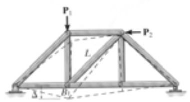


Fig. 9-7

9-4 Method of virtual work: Trusses

- External loading (cont'd)
  - Applying eqn 9.13, the virtual work eqn of the truss is:

$$1.\Delta = \sum \frac{nNL}{AE} \qquad \text{eqn 9.15}$$

1 = ext virtual unit load acting on the truss joint in the stated direction of Δ
n = int virtual normal force in a truss member caused by the ext virtual unit load
Δ = ext joint disp caused by the real loads on the truss
N = internal normal force in a truss member caused by the real load
L = length of the member
A = cross-sectional area of member
E = modulus elasticity of a member

9-4 Method of virtual work: Trusses

- External loading (cont'd)
  - The external virtual load creates internal virtual forces n in each of the members
  - The real loads caused the truss joints to be displaced in the same direction as the virtual unit load
  - Each member is disp NL/AE in the same direction as its respective n force
  - Hence, ext virtual work = sum of int. (virtual) strain energy stored in truss members

9-4 Method of virtual work: Trusses

- Temperature
  - In some cases, truss members may change their length due to temperature

$$\Delta L = \alpha \Delta T L$$

  - The disp of a selected truss joint may be written as

$$1.\Delta = \sum n \alpha \Delta T L \qquad \text{eqn 9.16}$$

Δ = ext joint disp caused by temperature change
α = coefficient of thermal expansion of member
ΔT = change in temperature of a member

9-4 Method of virtual work: Trusses

- Fabrication errors & camber
  - Errors in fabricating the lengths of the members of a truss may occur
  - Truss members may also be made slightly longer or shorter in order to give the truss a camber
  - Camber is often built into bridge truss so that the bottom cord will curve upward by the same amount equivalent to the downward deflection when subjected to the bridge's full dead weight

9-4 Method of virtual work: Trusses

- Fabrication errors & camber (cont'd)
  - The disp of a truss joint from its expected position can be written as:

$$1.\Delta = \sum n\Delta L \qquad \text{eqn 9.17}$$

Δ = ext joint disp caused by fabrication errors
ΔL = difference in length of the member from its intended size as caused by fabrication error

  - A combination of right sides of eqn 9.15 to 9.17 may be necessary if both external loads, thermal change & fabrication errors are taking place

Example 9.1

- Determine the vertical disp of joint C of the steel truss shown in Fig 9.8(a)
- The cross-sectional area of each member = $300mm^2$
- E = 200GPa

Example 9.1

- Fig 9.8

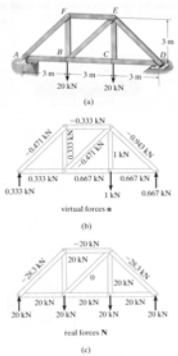


Fig. 9-8

Example 9.1 - solution

- Virtual force
  - Only a vertical 1kN load is placed at joint C
  - The force in each member is calculated using method of joints
  - Results are shown in Fig 9.8(b)
- Real forces
  - The real forces are calculated using method of joints
  - Results are shown in Fig 9.8(c)

Example 9.1 - solution

- Virtual work eqn
  - Arranging the data in tabular form, we have
  - Table

$$1kN.\Delta_{c_v} = \sum \frac{nNL}{AE} = \frac{369.6kN^2 m}{AE}$$

$$1kN.\Delta_{c_v} = \frac{369.6kN^2 m}{(300(10^{-6})m^2)[200(10^6)kN/m^2]}$$

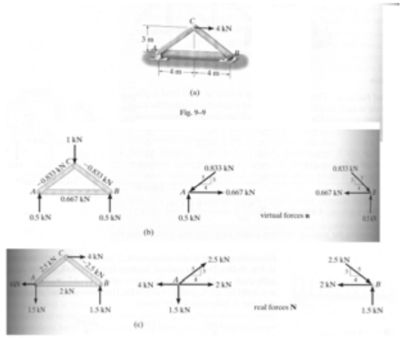
$$\Delta_{c_v} = 0.00616m = 6.16mm$$

Example 9.2

- The cross-sectional area of each member shown in Fig 9.9(a)
  - A = 400mm^2
  - E = 200GPa
- Determine the vertical disp of joint C if no loads act on the truss, what would be the vertical disp of joint C if member AB is 5mm too short

Example 9.2

- Fig 9.9



Example 9.2 - solution

- Virtual forces
  - The support reactions at A & B are calculated
  - The n force in each member is determined using method of joints as shown in Fig 9.9(b)
- Applying eqn 9.17

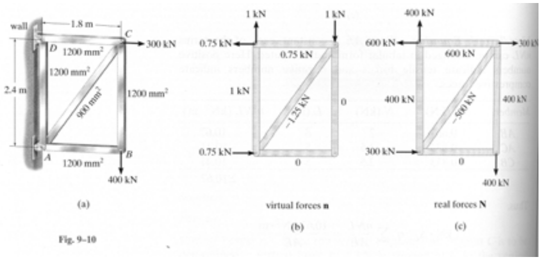
$$1.\Delta = \sum n\Delta L$$
$$1kN.\Delta_{c_v} = (0.667kN)(-0.005m)$$
$$\Delta_{c_v} = -0.00333m = -3.33mm$$

Example 9.3

- Determine the vertical disp of joint C of the steel truss shown in Fig 9.10(a)
- Due to radiant heating from the wall, member AD is subjected to increase in temp = +60°C
- Take $\alpha = 1.08(10^{-5})/^{\circ}C$ and E = 200GPa
- The cross-sectional area of each member is indicated in the figure

Example 9.3

- Fig 9.10



Example 9.3 - solution

- Virtual forces n
  - The forces in members are computed, Fig 9.10(b)
- Real forces N
  - Since n forces in AB & BC are zero, N forces need not be computed
- Virtual work eqn
  - Both loads & temp affect the deformation
  - Eqn 9.15 & 9.16 are combined

Example 9.3 - solution

- Virtual work eqn (cont'd)

$$1.\Delta = \sum \frac{nNL}{AE} + \sum n\alpha\Delta TL$$

$$1.\Delta_{c_v} = \frac{0.75(600)(1.8)}{1200(10^{-6})[200(10^{6})]} + \frac{1(400)(2.4)}{1200(10^{-6})[200(10^{6})]} + \frac{(-1.25)(-500)(3)}{900(10^{-6})[200(10^{6})]} + (1)[1.08(10^{-5})](60)(2.4)$$

$$\Delta_{c_v} = 0.00193m = 1.93mm$$

9-5 Method of virtual work: Beams & Frames

- The Principle of virtual work may be formulated for beam & frame deflections by considering the beam shown in Fig 9.11(a)
- To compute Δ a virtual unit load acting in the direction of Δ is placed on the beam at A
- The internal virtual moment m is determined by the method of sections at an arbitrary location x from the left support, Fig 9.11(b)
- When point A is displaced Δ, the element dx deforms or rotates dθ = (M/EI)dx

9-5 Method of virtual work: Beams & Frames

- Fig 9.11

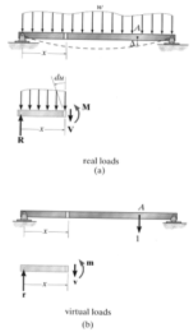


Fig. 9–11

9-5 Method of virtual work: Beams & Frames

$$1.\Delta = \int_0^L \frac{mM}{EI} dx \quad \text{eqn 9.18}$$

= external virtual unit load acting on the beam or frame in the direction of Δ
= internal virtual moment in the beam or frame, expressed as a function of x & caused by the ext virtual unit load
= ext disp of the point caused by real loads acting on the beam or frame
= int moment in the beam or frame, expressed as a function of x & caused by the real loads
= modulus of elasticity of the material
= moment of inertia of cross - sectional area, computed about the neutral axis

9-5 Method of virtual work: Beams & Frames

- If the tangent rotation or slope angle θ at a point on the beam's elastic curve is to be determined, a unit couple moment is applied at the point
- The corresponding int moment m_θ have to be determined

$$1.\theta = \int_0^L \frac{m_\theta M}{EI} dx \quad \text{eqn 9.19}$$

9-5 Method of virtual work: Beams & Frames

- If concentrated forces or couple moments act on the beam or the distributed load is discontinuous, separate x coordinates will have to chosen within regions that have no discontinuity of loading

Example 9.4

- Determine the disp of point B of the steel beam shown in Fig 9.13(a)
- Take
  - E = 200GPa
  - I = 500(10^6) mm^4

Example 9.4

- Fig 9.13

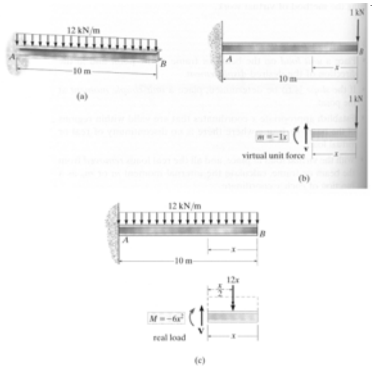


Fig. 9-13

Example 9.4 - solution

- Virtual moment m
  - The vertical disp of point B is obtained by placing a virtual unit load of 1kN at B, Fig 9.13(b)
  - Using method of sections, the internal moment m is formulated as shown in Fig 9.13(b)
- Real moment M
  - Using the same x coordinate, M is formulated as shown in Fig 9.13(c)

Example 9.4 - solution

- Virtual work eqn

$$1kN.\Delta_B = \int_0^L \frac{mM}{EI} dx = \int_0^{10} \frac{(-1x)(-6x^2)}{EI} dx$$

$$1kN.\Delta_B = \frac{15(10^3)kN^2m^3}{EI}$$

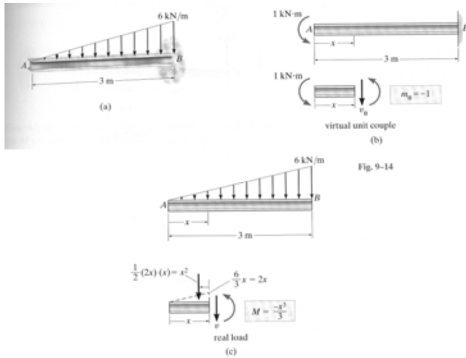
$$\Delta_B = 0.150m = 150mm$$

Example 9.5

- Determine the tangential rotation at point A of the steel beam shown in Fig 9.14(a)
- Take
  - $E = 200GPa$
  - $I = 60(10^6)\ mm^4$

Example 9.5

- Fig 9.14



Example 9.5 - solution

- Virtual moment m_θ
  - The tangential rotation of point A is obtained by placing a virtual unit couple of 1kNm at A, Fig 9.14(b)
  - Using method of sections, the internal moment m_θ is formulated as shown in Fig 9.14(b)
- Real moment, M
  - The internal moment is formulated as shown in Fig 9.14(c)

Example 9.5 - solution

- Virtual work eqn

$$1kN.m.\theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$= \int_0^3 \frac{(-1)\left(\frac{-x^3}{3}\right)}{EI} dx = \frac{1}{3EI}\int_0^3 x^3 dx$$

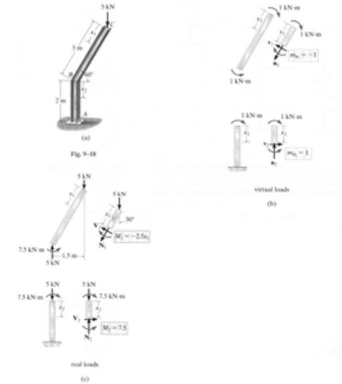
$$\theta_A = 0.000563 rad$$

Example 9.9

- Determine the tangential rotation at point C of the frame shown in Fig 9.18(a)
- Take
  - E = 200GPa
  - I =15(10^6) mm^4

Example 9.9

- Fig 9.18



Example 9.9 - solution

- Virtual moments m_θ
  - A unit moment is applied at C and the int moments m_θ are calculated, Fig 9.18(b)
- Real moments M
  - In a similar manner, the real moments are are calculated as shown in Fig 9.18(c)

Example 9.9 - solution

- Virtual work eqn
  - Using the data in Fig 9.18(b) & 9.18(c), we have:

$$1.\theta_C = \int_0^L \frac{m_\theta M}{EI} dx$$

$$= \int_0^3 \frac{(-1)(-2.5x_1)}{EI} dx_1 + \int_0^2 \frac{(1)(7.5)}{EI} dx_2$$

$$\theta_C = \frac{11.25}{EI} + \frac{15}{EI} = \frac{26.25kNm^2}{EI} = 0.00875rad$$