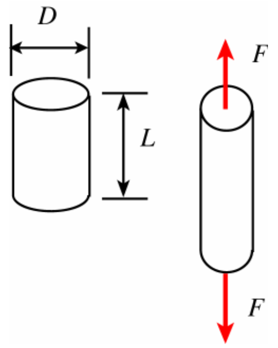
Strain Measurements

2103-602

Measurement and Instrumentations

Review of Stress and Strain

Axial stress σ_a, axial strain ε_a, transverse strain ε_t, Poisson's ratio ν, and Young modulus E are



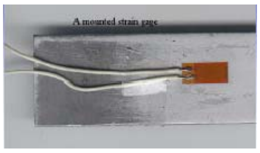
$$\sigma_a = \frac{F}{A} \qquad \varepsilon_a = \frac{dL}{L} \qquad \varepsilon_t = \frac{dD}{D}$$

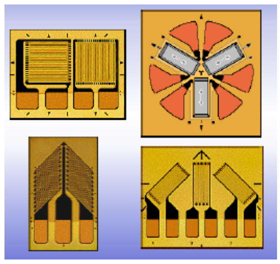
$$\upsilon = \frac{-\varepsilon_t}{\varepsilon_a} = \frac{-dD/D}{dL/L} \approx 0.3$$

$$E = \frac{\sigma}{\varepsilon}$$

Electrical-Resistance Strain Gages

- Widely used for strain measurement
- Resistance of gage material changes with deformation
- Use to measure local strain in axial direction of the gage
- Can measure various components of strain using rosette gages





Principle of Operation

Electrical resistance of any gage wire is determined by

$$\boxed{R = \frac{\rho L}{A}}$$

where L is the gage length

A is area of the wire

ρ is resistivity in Ω-m

Note: - The strain increased, R increased

- R linearly increases with strain, provided a constant temperature

Relation of R and ε

From $R = \dfrac{\rho L}{A}$ where $A = cD^2$

c = 1 (square) and $c = \pi/4$ (circular).

The change of R is then

$$dR = d\left(\frac{\rho L}{A}\right)$$

$$= \frac{L}{A}d\rho + \frac{\rho}{A}dL - \frac{\rho L}{A^2}dA$$

$$= \frac{L}{A}d\rho + \frac{\rho}{A}dL - \frac{2\rho L}{A}\frac{dD}{D}$$

or $$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{2dD}{D}$$

$$\frac{dR/R}{dL/L} = \frac{dR/R}{\varepsilon_a} = \frac{d\rho/\rho}{\varepsilon_a} + 1 + 2\upsilon$$

If ρ is constant, i.e. resistivity does not change with strain, thus

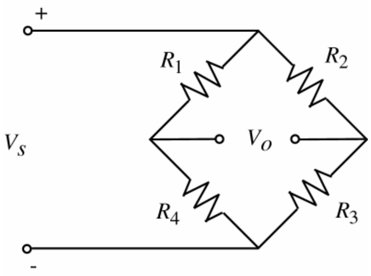
$$\boxed{\frac{dR/R}{\varepsilon_a} = 1 + 2\upsilon \equiv \text{Gage Factor (GF)}}$$

Gage Factor

$$\boxed{\frac{dR/R}{\varepsilon_a} = 1 + 2\upsilon \equiv \text{Gage Factor (GF)}}$$

- GF or the sensitivity typically varies from 1.6 to 4
- Large GF (a large change in resistance for a given strain) is desirable
- R is normally 120 Ω or 350 Ω.
- Axial strain ε_a is in the range of 10^{-6} to 10^{-3}, resulting in dR = 0.00024-0.24 Ω.
- Note that how small dR is, so we cannot simply stick an ohm meter across the strain gages.
- Wheatstone bridges circuit are commonly used to amplify such low resistance values.

Wheatstone Bridge

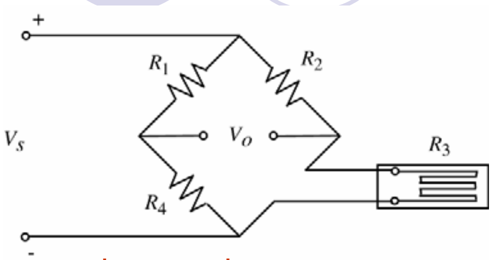


Output voltage V_o is

$$\boxed{V_o = V_s \frac{R_3 R_1 - R_4 R_2}{(R_2 + R_3)(R_1 + R_4)}}$$

If all four resistors are identical or if $R_3 R_1 = R_4 R_2$, the bridge is balanced, i.e. $V_o = 0$.

Quarter Bridge Circuit (one active gage)



R_3 is an active gage

The bridge is initially balanced, when $R_{3i}R_1 = R_4R_2$. Thus

$$V_{oi} = 0$$

If $R_1 = R_2 = R_{3i} = R_4$, then the strain causes a change in R_3 as

$$R_{3i} + dR_3$$

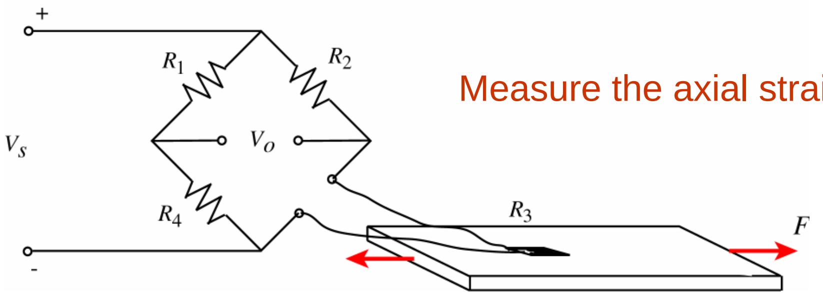
The output voltage per input voltage is

$$\frac{V_o}{V_s} = \frac{(R_{3i} + dR_3)R_1 - R_4R_2}{(R_2 + R_{3i} + dR_3)(R_1 + R_4)} \cong \frac{R_2 R_{3i}}{(R_2 + R_{3i})^2}\left(\frac{dR_3}{R_{3i}}\right)$$

Since $\frac{dR_3 / R_3}{\varepsilon_a} = GF$ and $R_2 = R_{3i}$, therefore $\frac{V_o}{V_s} = \frac{1}{4}\varepsilon_a GF$

Or the sensitivity $\boxed{\frac{V_o}{\varepsilon_a} = \frac{1}{4} V_s GF}$ (in unit of V/ppm)

Installation of One Active Gage



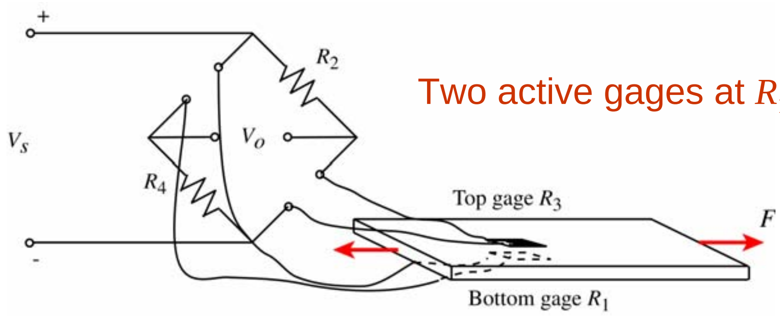
Measure the axial strain of a bar

Ex: If under tension the output voltage is V_o = 2.1 mV. Given V_s = 10 V and GF = 1.8, determine the measured strain ε_a.

From the sensitivity, we obtain

$$\varepsilon_a = \frac{4V_o}{V_s \mathrm{GF}} = \frac{4 \times 2.1 \times 10^{-3}}{10 \times 1.8} = 0.000933$$, or 933 ppm

Half Bridge Circuit (two active gages)



Two active gages at R_1 and R_3

Initially balance the gage, hence $R_{3i}R_{1i} = R_4R_2$.

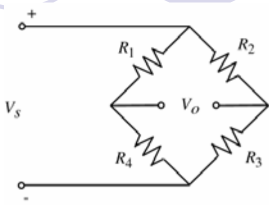
When loaded, we can get the output voltage per input voltage as

$$\frac{V_o}{V_s} \approx \frac{R_2 R_{3i}}{(R_2 + R_{3i})^2}\left(\frac{dR_1}{R_{1i}} + \frac{dR_3}{R_{3i}}\right)$$ where $$\frac{dR_1}{R_{1i}} = \frac{dR_3}{R_{3i}} = \varepsilon_a \mathrm{GF}$$

Therefore the sensitivity for this case is $$\boxed{\frac{V_o}{\varepsilon_a} = \frac{2}{4} V_s GF}$$

Change of R in Bridge Circuit

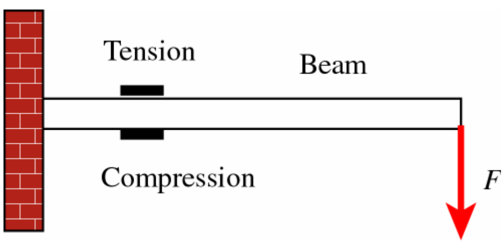
If all four resistors R in bridge circuit have a change,

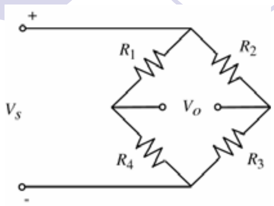


$$\boxed{\frac{V_o}{V_s} \approx \frac{R_2 R_{3i}}{(R_2 + R_{3i})^2}\left(\frac{dR_1}{R_{1i}} - \frac{dR_2}{R_{2i}} + \frac{dR_3}{R_{3i}} - \frac{dR_4}{R_{4i}}\right)}$$

- As strain increased, dR_1 and dR_3 contribute to + V_o whereas dR_2 and dR_4 contribute to - V_o.
- According to the ± signs, it is important where you place strain gages in the circuit.
- For the previous ex., if we put the first gage at R_1, it does not make sense to put the second gage at R_2 and R_4, because dR could cancel and V_o is zero.

Strain In a Transverse Beam





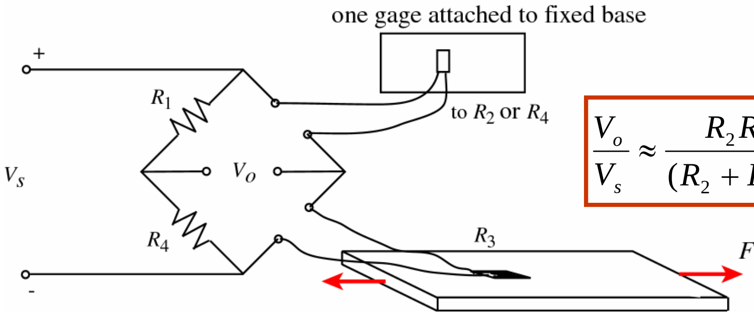
$$\boxed{\frac{V_o}{V_s} \approx \frac{R_2 R_{3i}}{(R_2 + R_{3i})^2}\left(\frac{dR_1}{R_{1i}} - \frac{dR_2}{R_{2i}} + \frac{dR_3}{R_{3i}} - \frac{dR_4}{R_{4i}}\right)}$$

For a little exercise, if we have two active gages on a beam as shown, where to connect these two gages in the Wheatstone bridge?

Note that the resistance change due to compression is minus of that in tension.

Answer: At 1 and 2, 3 and 4, 2 and 3, or 1 and 4, else result in zero output voltage.

Temperature Compensation



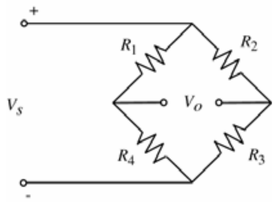
$$\boxed{\frac{V_o}{V_s} \approx \frac{R_2 R_{3i}}{(R_2 + R_{3i})^2}\left(\frac{dR_1}{R_{1i}} - \frac{dR_2}{R_{2i}} + \frac{dR_3}{R_{3i}} - \frac{dR_4}{R_{4i}}\right)}$$

To cancel the resistance change of the strain gages due to thermal effect, we can setup the gages as shown.

With R_3 as an active gage, we can use R_2 or R_4 to compensate for the temperature.

Full Bridge Circuit (four active gages)

$$\boxed{\frac{V_o}{V_s} \approx \frac{R_2 R_{3i}}{(R_2 + R_{3i})^2}\left(\frac{dR_1}{R_{1i}} - \frac{dR_2}{R_{2i}} + \frac{dR_3}{R_{3i}} - \frac{dR_4}{R_{4i}}\right)}$$



When all R on the bridge are replaced by strain gages:

"R_1 and R_3 must have positive strain (tension), while R_2 and R_4 have negative strain (compression)".

We can get the sensitivity ***four*** times of the quarter bridge and ***two*** times of the half bridge.

Bridge Constant (K)

Defined as output of the bridge per output of the primary (one active) gage.

For one active gage ($K=1$), the sensitivity is

$$\frac{V_0}{\varepsilon_a} = \frac{1}{4} V_s \mathrm{GF}$$

For two active gages ($K=2$), the sensitivity is

$$\frac{V_0}{\varepsilon_a} = \frac{2}{4} V_s \mathrm{GF}$$

For four active gages ($K=4$), the sensitivity is

$$\frac{V_0}{\varepsilon_a} = \frac{4}{4} V_s \mathrm{GF}$$

$$\boxed{\frac{V_0}{\varepsilon_a} = \frac{K}{4} V_s \mathrm{GF}}$$