2103-602 Measurement and Instrumentation

Lecture Note: Strain Measurement

Review of Stress and Strain

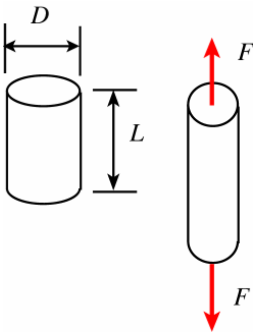


Figure 1: Structure under tension

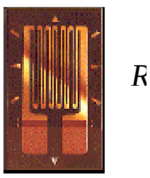
From Fig. 1, axial stress σ_a, axial strain ε_a, transverse strain ε_t, Poisson's ratio ν, and Young modulus E are

$$\sigma_a = \frac{F}{A}, \qquad \varepsilon_a = \frac{dL}{L}, \quad \varepsilon_t = \frac{dD}{D}$$

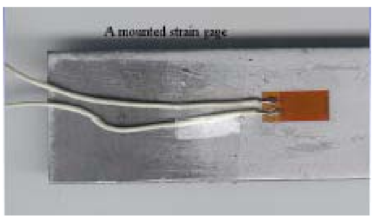
$$\upsilon = \frac{-\varepsilon_t}{\varepsilon_a} = \frac{-dD/D}{dL/L} \approx 0.3, \quad E = \frac{\sigma}{\varepsilon},$$

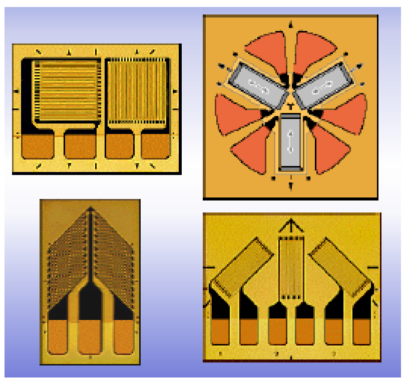
where A is the cross-sectional area.

Electrical-Resistance Strain Gages



1-D strain gage





Rosette gage

Figure 2: Different types of strain gages

- Widely used for strain measurement
- Resistance of gage material changes with deformation
- Use to measure local strain in a lateral direction of the gage (see Figure 2)
- Can measure multi-directional strain using rosette gages (see Figure 2)

Principle of Operation

Electrical resistance of any gage wire is determined from

$$R = \frac{\rho L}{A},$$

where L is the gage length, A is cross-sectional area of the wire and ρ is the resistivity in Ω-m.

Electrical resistance of the wire changes with strain, i.e. as strain ε increased, L decreased, A decreased and ρ increased. Therefore with the strain increased, the resistance R increased. For the strain gages, the electrical wires are designed such that their resistance R linearly increases with strain, provided constant temperature.

Relationship of R and ε

From $R = \frac{\rho L}{A}$ where $A = cD^2$, c = 1 (square) and c = π/4 (circular). The change of R is then

$$\begin{aligned} dR &= d\left(\frac{\rho L}{A}\right) \\ &= \frac{L}{A}d\rho + \frac{\rho}{A}dL - \frac{\rho L}{A^2}dA \\ &= \frac{L}{A}d\rho + \frac{\rho}{A}dL - \frac{2\rho L}{A}\frac{dD}{D} \\ \text{or} \quad \frac{dR}{R} &= \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{2dD}{D} \end{aligned}$$

Divide the whole equation with *dL/L*

$$\frac{dR/R}{dL/L} = \frac{dR/R}{\varepsilon_a} = \frac{d\rho/\rho}{\varepsilon_a} + 1 + 2\upsilon$$

If ρ is constant, i.e. resistivity does not change with strain, thus

$$\frac{dR/R}{\varepsilon_a} = 1 + 2\upsilon \equiv \text{Gage Factor (GF)}$$

Remarks:

- GF typically varies from 1.6 to 4.
- High GF (high sensitivity) is desirable because a large change in resistance can be produced for a given strain.
- *R* is normally 120 Ω or 350 Ω.
- Axial strain ε_a is in the range of 10^{-6} to 10^{-3}, resulting in dR = 0.00024-0.24 Ω. Note that how small *dR* is, therefore we cannot simply stick an ohm meter across the strain gages.
- Wheatstone bridges are commonly circuit used to amplify such low resistance values in strain measurement.

Wheatstone Bridge

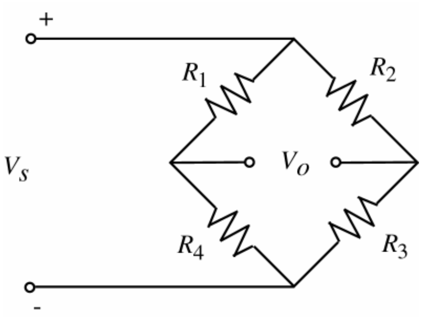


Figure 3: Wheatstone bridge

From the circuit in Fig. 3, the output voltage V_o is

$$V_o = V_s \frac{R_3 R_1 - R_4 R_2}{(R_2 + R_3)(R_1 + R_4)}$$

If all four resistors are identical or if $R_3R_1 = R_4R_2$, the bridge is balanced or $V_o = 0$.

1) Quarter Bridge Circuit (one active gage)

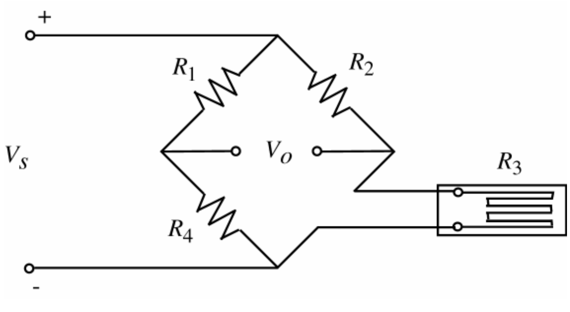


Figure 4: Quarter bridge-one active gage

Figure 4 shows the quarter bridge circuit where the strain gage is set at R_3 in the bridge, or R_3 is an active gage. Before the measurement, the bridge is initially balanced, i.e. $V_{oi} = 0$ or $R_{3i}R_1 = R_4R_2$. If $R_1 = R_2 = R_{3i} = R_4 = 120\ \Omega$, then the strain causes a change in R_3 as $R_{3i} + dR_3$. The output voltage per input voltage is therefore

$$\frac{V_o}{V_s} = \frac{(R_{3i} + dR_3)R_1 - R_4R_2}{(R_2 + R_{3i} + dR_3)(R_1 + R_4)} \cong \frac{R_2R_{3i}}{(R_2 + R_{3i})^2}\left(\frac{dR_3}{R_{3i}}\right)$$

Since $\dfrac{dR_3 / R_3}{\varepsilon_a} = GF$ and $R_2 = R_{3i}$, hence

$$\frac{V_o}{V_s} = \frac{1}{4}\varepsilon_a GF$$

Or the sensitivity (in unit of V/ppm) of one active gage (quarter bridge) is

$$\frac{V_o}{\varepsilon_a} = \frac{1}{4}V_s GF$$

Figure 5 shows one active gage installation to measure the axial strain on a bar.

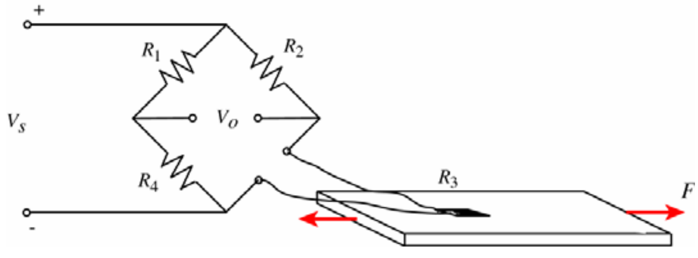


Figure 5: Quarter bridge installation

Example: For the setup in Fig. 5, if under tension the output voltage is $V_o = 2.1$ mV. Given $V_s = 10$ V and GF = 1.8, determine the measured strain ε_a.

From the sensitivity relation, we obtain

$$\varepsilon_a = \frac{4V_o}{V_s\mathrm{GF}} = \frac{4 \times 2.1 \times 10^{-3}}{10 \times 1.8} = 0.000933, \text{ or 933 ppm}$$

2) Half Bridge Circuit (two active gages)

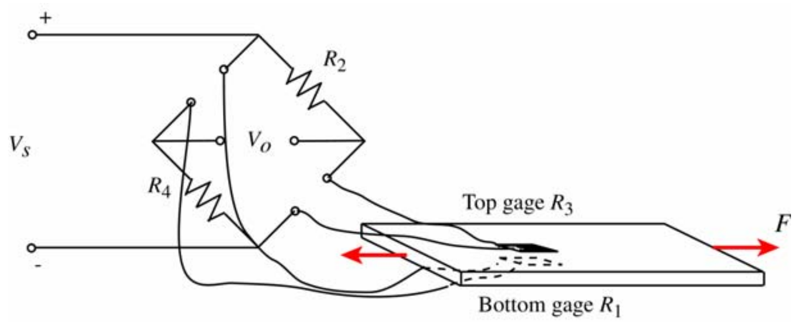


Figure 6: Half bridge installation

We can use two active gages at R_1 and R_3 as shown in Figure 6. Initially balance the gage, hence $R_{3i}R_{1i} = R_4R_2$. When loaded, we can get the output voltage per input voltage as

$$\frac{V_o}{V_s} \approx \frac{R_2 R_{3i}}{(R_2 + R_{3i})^2}\left(\frac{dR_1}{R_{1i}} + \frac{dR_3}{R_{3i}}\right)$$

where $\frac{dR_1}{R_{1i}} = \frac{dR_3}{R_{3i}} = \varepsilon_a \mathrm{GF}$. Therefore the sensitivity, for this case, is

$$\frac{V_o}{\varepsilon_a} = \frac{2}{4} V_s GF$$

In general, if we use all four resistors as active gages the output voltage per input voltage is

$$\frac{V_o}{V_s} \approx \frac{R_2 R_{3i}}{(R_2 + R_{3i})^2}\left(\frac{dR_1}{R_{1i}} - \frac{dR_2}{R_{2i}} + \frac{dR_3}{R_{3i}} - \frac{dR_4}{R_{4i}}\right)$$

Note that:

- dR_1 and dR_3 both contribute to positive output voltage whereas dR_2 and dR_4 contribute to negative voltage as strain is increased.
- According to positive and negative signs in bridge circuit, it is important where you place strain gages in the circuit. For the previous example, if we put the first gage at R_1, it does not make sense to put the second gage at R_2 and R_4, because resistance changes would cancel and output voltage would be zero.

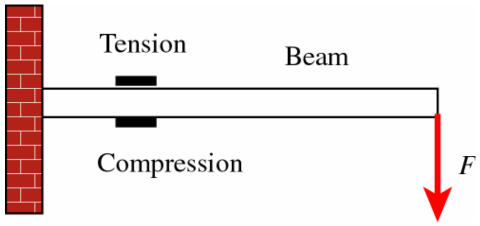


Figure 7: Strain measurement of beam

For a little exercise, if we have two active gages, one is in tension and the other is in compression as shown in Fig. 7, where can we place these two active gages in the Wheatstone bridge? Note that the resistance change due to compression is minus of that in tension. *(Answer: 1 and 2, 3 and 4, 2 and 3, or 1 and 4, else result in zero output voltage.)*

3) Temperature Compensation

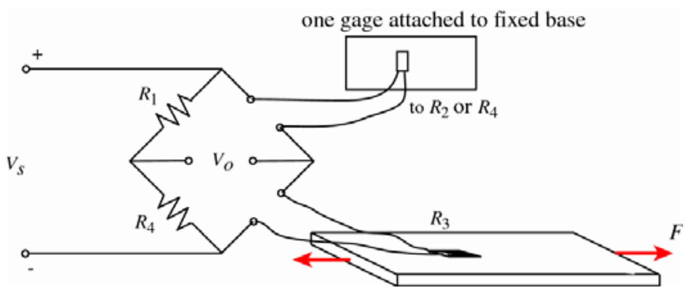


Figure 8: Temperature compensation

To cancel the resistance change of the strain gages due to thermal effects, we can setup the gages as shown in Fig. 8. With R_3 as an active gage, we can use R_2 or R_4 to compensate for the temperature.

4) Full Bridge Circuit (four active gages)

When all four resistors on the bridge are replaced by the strain gages, we can make a careful choice as follows:

"R_1 and R_3 must have positive strain (tension) while R_2 and R_4 have negative strain (compression)".

In this case, we get the sensitivity of full bridge four times of that for the quarter bridge and two times of that for the half bridge.

5) Bridge Constant

Bridge constant K is defined as the ratio of the output from an active bridge to the output of the bridge with primary (one active) gage.

For one active gage (K=1), the sensitivity is $\frac{V_0}{\varepsilon_a} = \frac{1}{4} V_s \mathrm{GF}$, for two active gages or half bridge (K=2), $\frac{V_0}{\varepsilon_a} = \frac{2}{4} V_s \mathrm{GF}$, and for four active gages or full bridge (K=4), $\frac{V_0}{\varepsilon_a} = \frac{4}{4} V_s \mathrm{GF}$. In general, $\frac{V_0}{\varepsilon_a} = \frac{K}{4} V_s \mathrm{GF}$, where K is the bridge constant.

Note that both gage factor GF and bridge constant K determine overall sensitivity of the strain gage system.