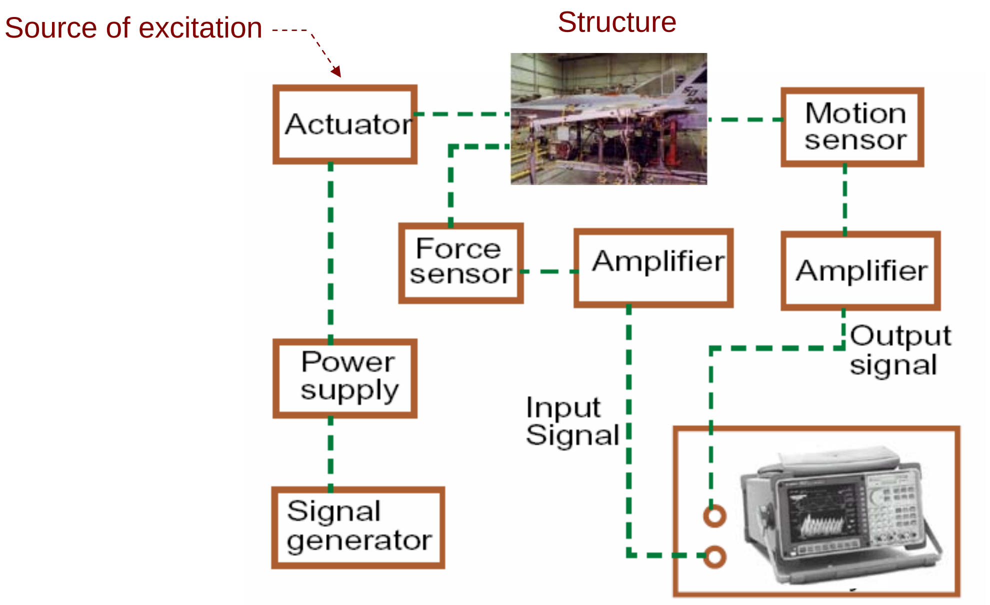
Vibration Testing

2103-602: Measurement and Instrumentations

Why vibration testing needed?

- ***Modal testing*** to determine vibration parameters such as natural frequencies, modal damping and mode shapes
- Verification of analytical model
- Product reliability test, e.g., shock and vibration
- Machine condition monitoring

Modal testing diagram



Dynamic signal analyzer: display both time and frequency responses and real time calculation of Fourier transform.

Vibration sensor

- Contact sensors:

Strain Gauge

Accelerometers

- Non-contact sensors

Displacement sensor: capacitance type

Laser Dlopper Vibrometer

Sensor senses and converts the motion into electrical signal.

Actuator

Need excitation with wide bandwidth of frequency covering all interested modes and operating frequencies.

Impulse hammer

provides impulsive force to the tested structure

Vibration shaker

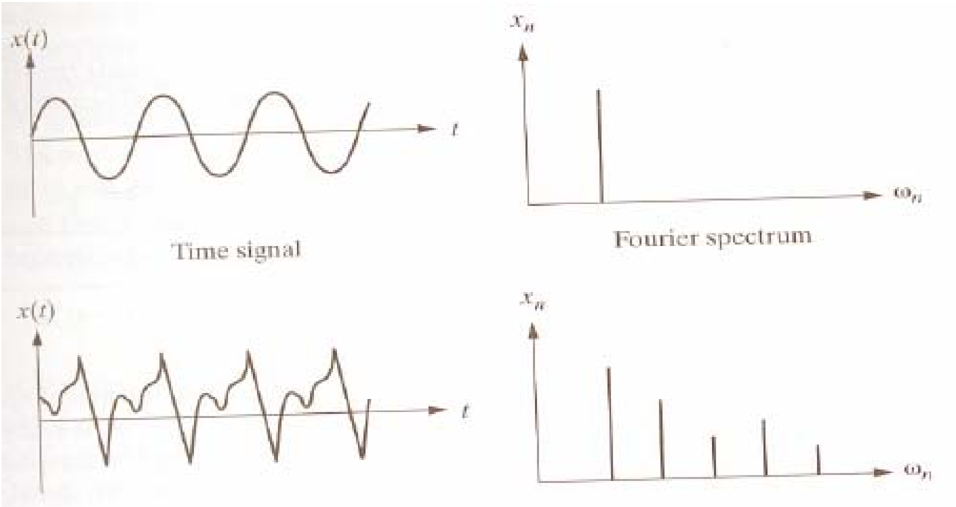
provide harmonic, swept-sine, or random excitation to the tested structure

Signal Analysis

Basic concept:

- Fourier transform
- Correlation
- Power Spectral Density (PSD)
- FRF determined from input- and output-PSD
- Digital processing

Fourier series of periodic signals



Coefficients of Fourier series indicate the density of signal at various discrete harmonic frequencies.

Fourier Transform

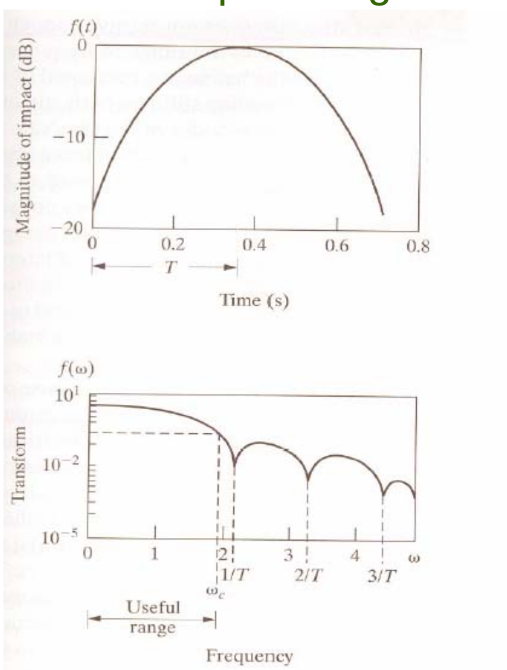
For any non-periodic signal, Fourier transform indicates density of the signal at various continuous frequencies.

Fourier transform of $x(t)$ is

$$X(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

"*Coefficients of continuous series of harmonics*"

Real impulse signal

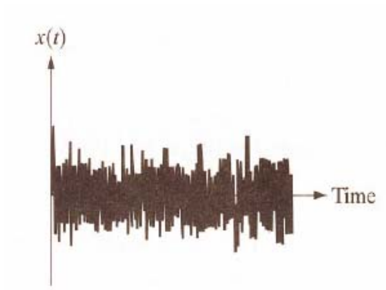


Correlation

Correlation indicates how fast the signal is changing compared to itself (auto) or other signal (cross).

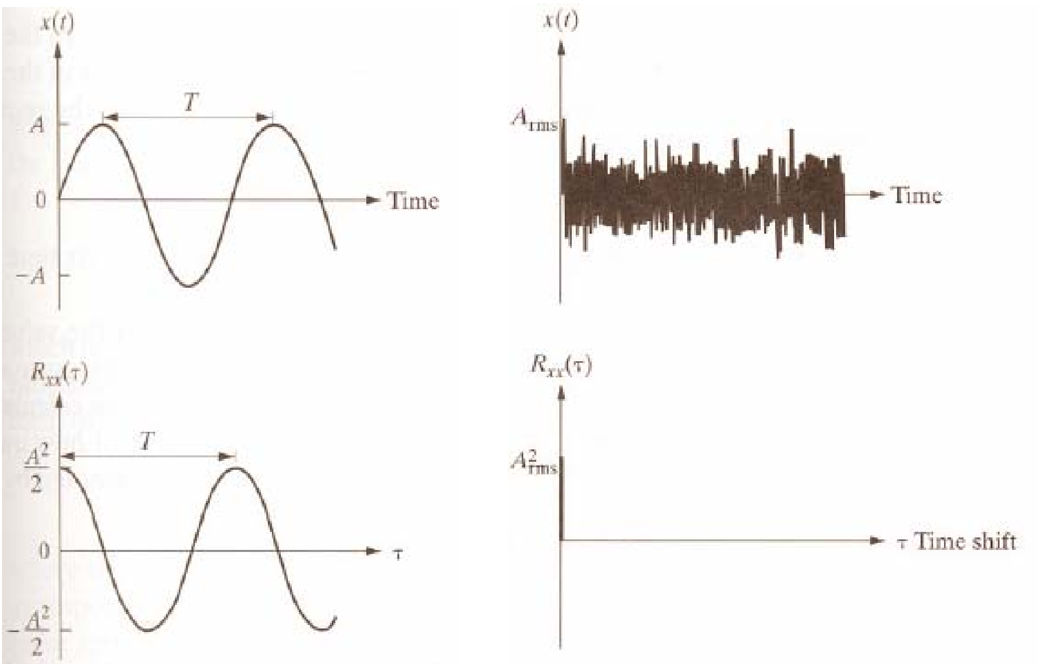
Auto-correlation

$$R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)x(t+\tau)dt$$



Cross-correlation

$$R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)y(t+\tau)dt$$



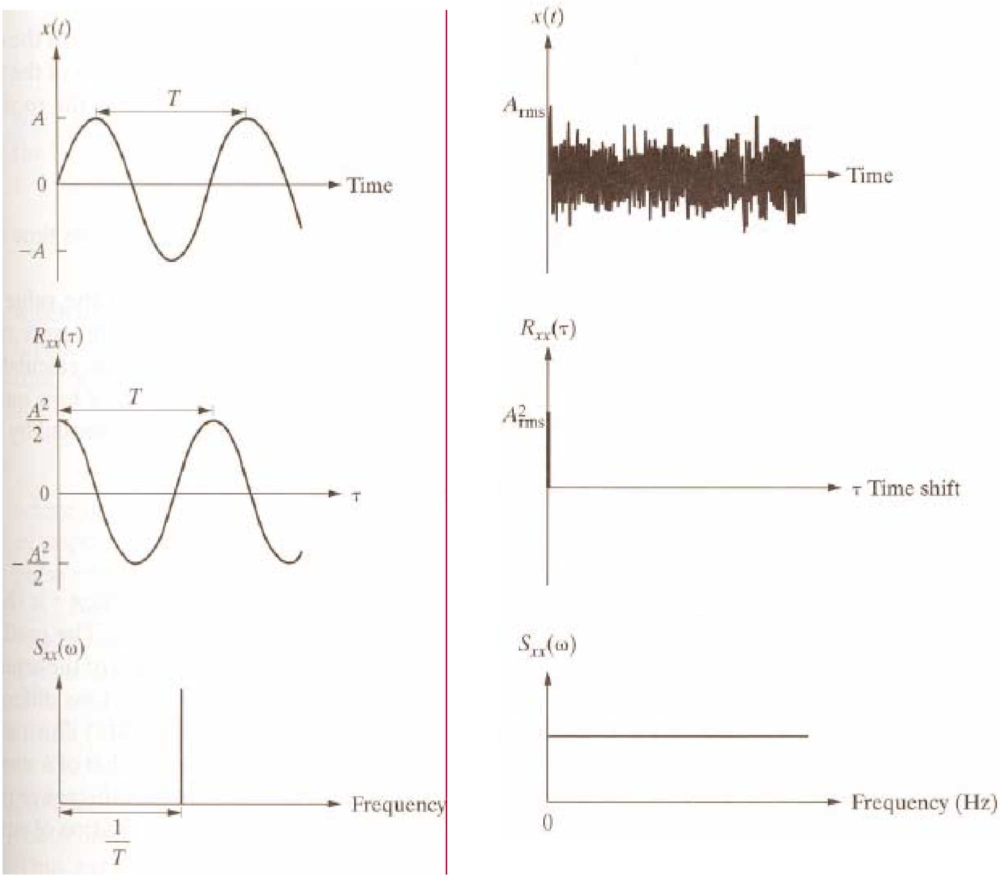
Power Spectral Density (PSD)

PSD is Fourier transform of the correlation.

Auto-PSD: $$S_{xx}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{xx}(\tau)e^{-j\omega\tau}d\tau$$

Cross-PSD: $$S_{xy}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{xy}(\tau)e^{-j\omega\tau}d\tau$$

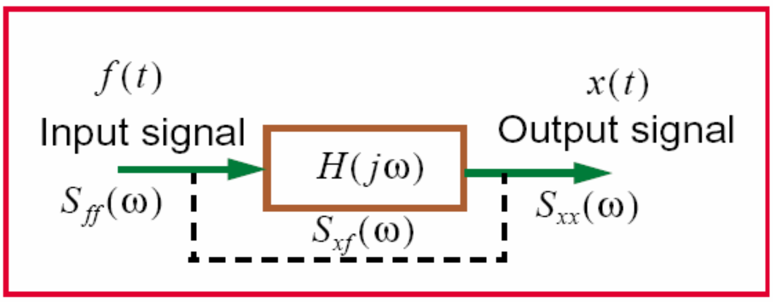
PSD indicates the energy density of signal at various frequencies and has a unit of power.



When subject to the uniform input at various frequencies, what does the PSD of vibration response, tell us about?

PSD or vibration energy at resonance frequencies will be maximum.

FRF determined from PSD

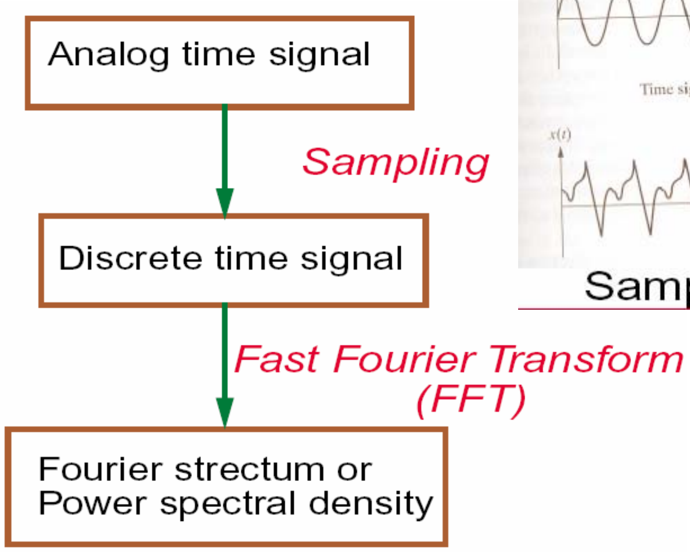


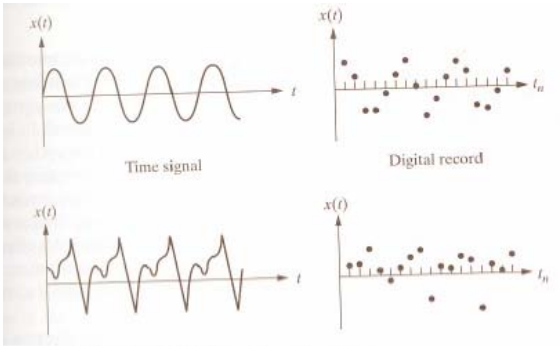
$$S_{xx}(\omega) = |H(j\omega)|^2 S_{ff}(\omega)$$

and

$$S_{xx}(\omega) = H(j\omega) S_{xf}(\omega)$$

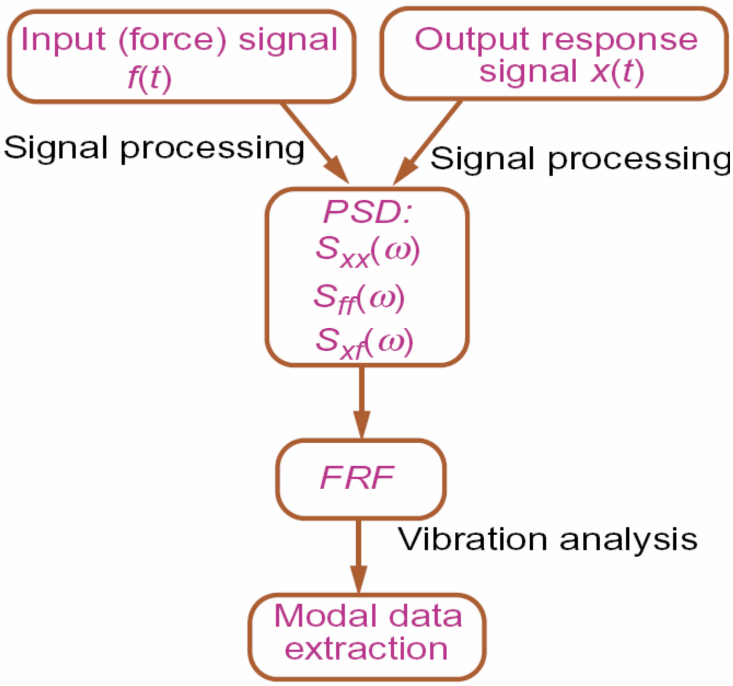
Digital Signal Processing





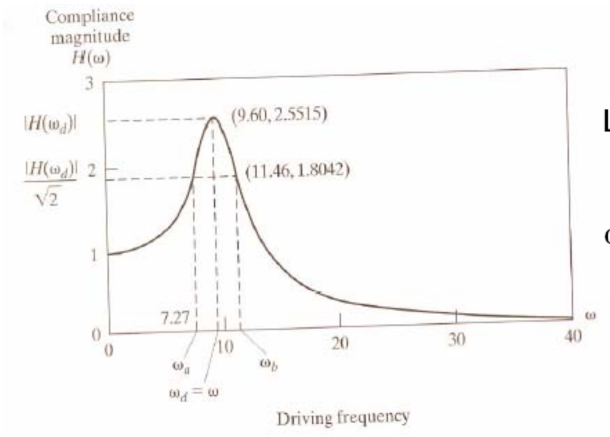
Sampling data

Modal testing



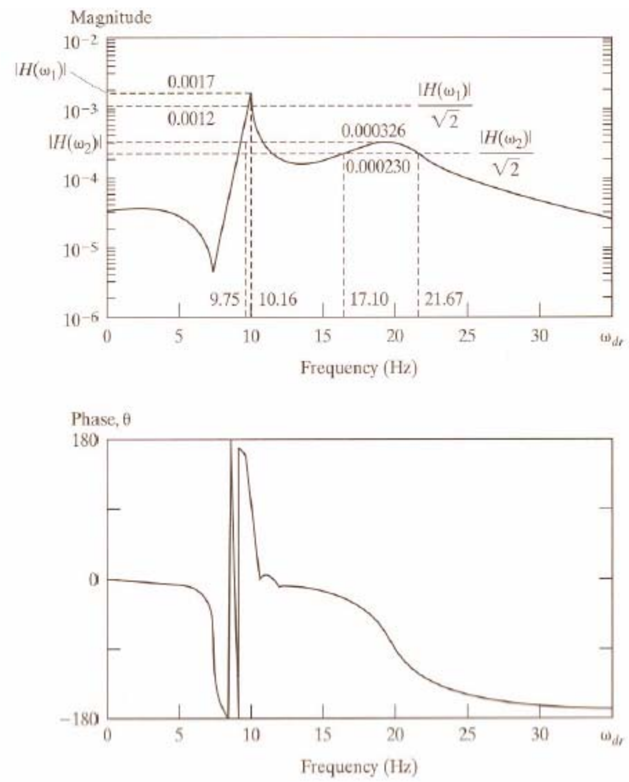
Modal data extraction

- Natural frequencies: frequencies at peaks
- Modal damping: half-power method for 1-DOF lightly damped system



Let $|H(\omega_a)| = |H(\omega_b)| = \dfrac{|H(\omega_d)|}{\sqrt{2}}$, then

$\omega_b - \omega_a = 2\zeta\omega_d$ or $\zeta = \dfrac{\omega_b - \omega_a}{2\omega_d}$



Half power method can be applied to M-DOF lightly damped system whose resonance frequencies are far apart.

Mode Shapes

From FRF

$$H_{ij} = \sum_{m=1}^{k} \frac{(u_m^{(i)} u_m^{(j)})/\omega_{nm}^2}{\left[1 - \left(\frac{\omega}{\omega_{nm}}\right)^2\right] + j\left(2\zeta_m \frac{\omega}{\omega_{nm}}\right)}$$

Consider the *q*-th mode and evaluate H_{ij} at $\omega = \omega_q$:

$$H_{ij}(\omega_q) = h^2 + \frac{(u_q^{(i)} u_q^{(j)})/\omega_{nq}^2}{\left[1 - \left(\frac{\omega_q}{\omega_{nq}}\right)^2\right] + j\left(2\zeta_q \frac{\omega_q}{\omega_{nq}}\right)} + h^2$$

the q-th term

where h^2 are the neglegible higher order terms.

With small damping and all natural frequencies are far apart, hence:

$$|H_{ij}(\omega_q)| \cong \left|\frac{u_q^{(i)} u_q^{(j)}}{j2\zeta_q \omega_{nq}^2}\right|$$

E.g. consider the 1-st mode of a 3-DOF system:

$$|H_{11}(\omega_1)| = \left|\frac{u_1^{(1)} u_1^{(1)}}{j2\zeta_1 \omega_{n1}^2}\right| = c_1|u_1^{(1)}|$$

$$|H_{12}(\omega_1)| = \frac{u_1^{(1)} u_1^{(2)}}{j2\zeta_1 \omega_{n1}^2} = c_1|u_1^{(2)}|$$

$$|H_{13}(\omega_1)| = \frac{u_1^{(1)} u_1^{(3)}}{j2\zeta_1 \omega_{n1}^2} = c_1|u_1^{(3)}|$$

or

$$\mathbf{u}_1 = \begin{pmatrix} u_1^{(1)} \\ u_1^{(2)} \\ u_1^{(3)} \end{pmatrix} = c_1 \begin{pmatrix} |H_{11}(\omega_1)| \\ |H_{12}(\omega_1)| \\ |H_{13}(\omega_1)| \end{pmatrix}$$

Consider phase plot for plus or minus sign

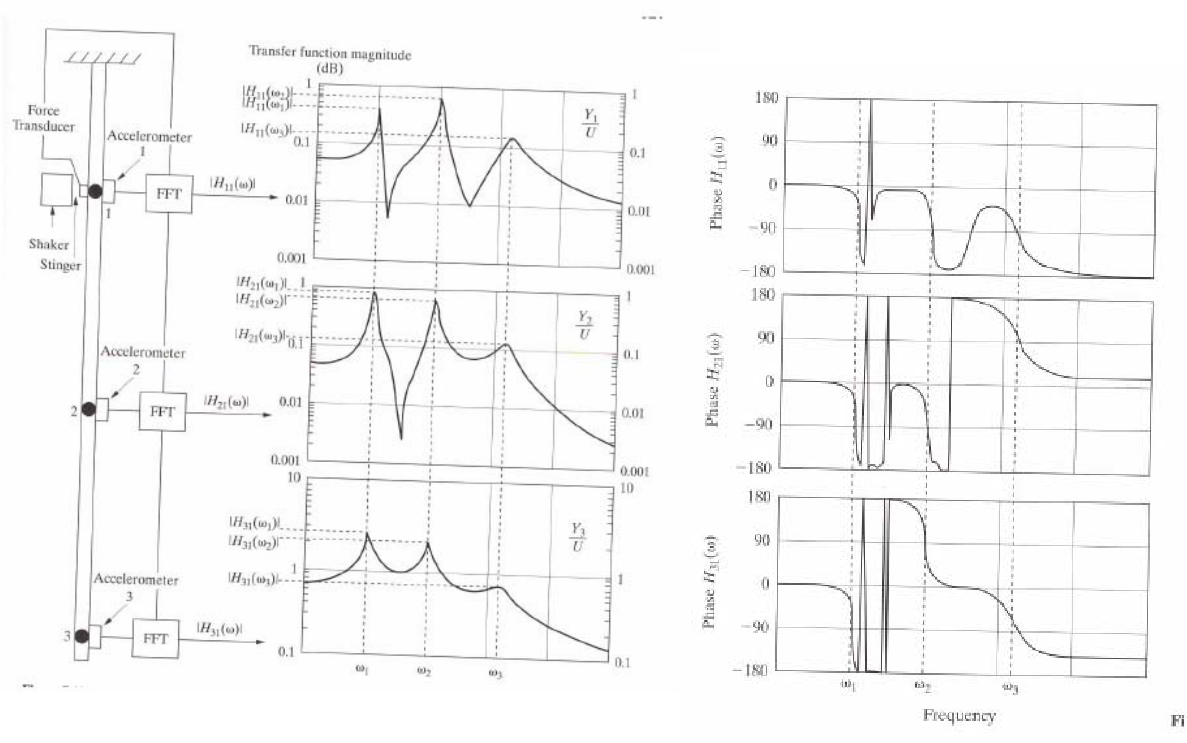
Similarly:

$$\mathbf{u}_2 = \begin{pmatrix} u_2^{(1)} \\ u_2^{(2)} \\ u_2^{(3)} \end{pmatrix} = c_2 \begin{pmatrix} |H_{11}(\omega_2)| \\ |H_{12}(\omega_2)| \\ |H_{13}(\omega_2)| \end{pmatrix}$$

and

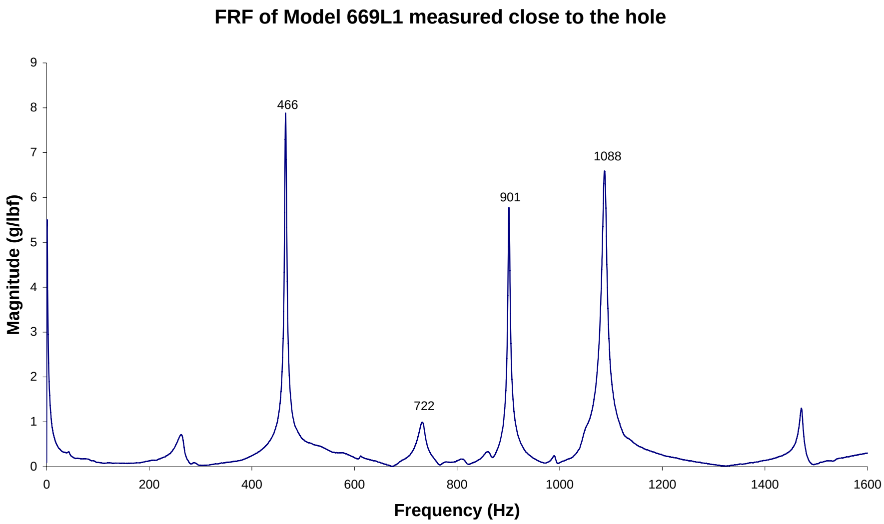
$$\mathbf{u}_3 = \begin{pmatrix} u_3^{(1)} \\ u_3^{(2)} \\ u_3^{(3)} \end{pmatrix} = c_3 \begin{pmatrix} |H_{11}(\omega_3)| \\ |H_{12}(\omega_3)| \\ |H_{13}(\omega_3)| \end{pmatrix}$$

Example



Modal Testing of Alloy Wheel

Test Result



FRF of Model 669L1 measured close to the hole
Magnitude (g/lbf)
Frequency (Hz)
466
722
901
1088
0
1
2
3
4
5
6
7
8
9
0
200
400
600
800
1000
1200
1400
1600