

Competitive Markets (chapter 13)

(Partial Equilibrium Analysis)

So far: prices were taken as given. Here we study how market prices are determined by action of individual agents.

The Competitive Firm

Assumption/Definition: A competitive firm takes the price as given.

Let p^* be the market price, the competitive firms assumes that demand is given by

$$D(p) = \begin{cases} 0 & \text{if } p > p^* \\ \text{any amount} & \text{if } p = p^* \\ +\infty & \text{if } p < p^* \end{cases}$$

In theory firm can set price and quantity. However, *given above belief* the “best” price is $p = p^*$.

Profit maximization problem

No link between output (y) and price.

$$\max_y py - c(y),$$

where $c(y)$ is a cost of producing y units, with $c'(\cdot) > 0$ and $c''(y) > 0$.

FOC

$$p - c'(y) = 0$$

defines implicit supply function:

$$\Rightarrow y(p) = (c'^{-1}(p))$$

E.g. $c(y) = \frac{1}{2}y^2 - 2y \Rightarrow$

$$p - y + 2 = 0 \Rightarrow y(p) = p + 2.$$

If there are (sunked) fixed costs, the firm will produce iff

$$py(p) - C_v(y(p)) - F \geq -F$$

$$\Leftrightarrow p \geq \frac{C_v(y(p))}{y(p)} = AVC.$$

Industry Supply

If there are m firms. Total supply:

$$Y(p) = \sum_{i=1}^m y_i(p)$$

Eg 2 firms. $c_1(y) = \frac{1}{2}y^2 - 2y$, $c_2(y) = y^2$

$$y_1(p) = p + 2$$

$$y_2(p) = \frac{p}{2}$$

$$Y(p) = \frac{3}{2}p + 2.$$

Market Equilibrium Price

Definition: the market equilibrium price is the price at which demand equals supply.

If there are n consumers, each with Walrasian demand $x_i(p)$ and m firms, the market equilibrium price is p^* such that

$$\sum_{i=1, \dots, n} x_i(p) = \sum_{j=1, \dots, m} y_j(p).$$

Welfare Economics

Simple model: representative consumer, representative firm and 2 goods x (*output*, price p) and y (*input*, price $p_y = 1$).

1-Let the markets work...

Consumer: quasi-linear utility function

$$U(x, y) = u(x_d) + y$$

FOC $u'(x_d) = p$.

Firm: Assume $c(x_s)$ units of y are needed to supply x_s units.

$$\Pi = px_s - c(x_s)$$

FOC $p = c'(x_s)$.

Equilibrium price solves $x_d(p^*) = x_s(p^*)$.

Equilibrium allocation is such that $u'(x_d^*) = c'(x_s^*)$: marginal willingness to pay = marginal cost.

2-Instead of markets, let a benevolent 3rd party allocate the goods so as to maximize the consumer's utility.

Assume that there exists w units of input to be given to consumer and to the firm.

$$\max_{x,y} u(x_d) + y$$

$$\text{s.t. } w = y + c(x_s)$$

$$\text{and } x_s = x_d$$

what is produced is consumed.

$$\Leftrightarrow \max_{x,y} u(x_d) + w - c(x_d)$$

FOC $u'(x_d) = c'(x_d)$.

Conclusion: market works well.

Note: Surplus is $u(x) - c(x)$:

$$CS = u(x) - px$$

$$PS = px - c(x)$$

$$TS = u(x) - c(x)$$

Competitive market maximizes surplus.

Several consumers and firms

Consumer i ($i = 1, \dots, n$) has utility $u_i(x_i) + y_i$.

Firm j ($j = 1, \dots, m$) has technology $c_j(x_j)$.

1-Let the markets work

For each consumer $p = u'_i(x_i^d)$.

For each firm $p = c'_j(x_j^s)$.

In equilibrium $u'_i(x_i^d) = c'_j(x_j^s)$ for all i, j .

2-Benevolent 3rd party

$$\max_{x_i^d, x_j^s, y} \sum_{i=1, \dots, n} u_i(x_i^d) + \sum_{i=1, \dots, n} y_i$$

$$\text{s.t. } w = \sum_{i=1, \dots, n} y_i + \sum_{j=1, \dots, m} c_j(x_j^s)$$

$$\text{and } \sum_{j=1, \dots, m} x_j^s = \sum_{i=1, \dots, n} x_i^d$$

Example with 2 consumers 1 firm

$$\max_{x_1^d, x_2^d, x^s, y} u_1(x_1^d) + y_1 + u_2(x_2^d) + y_2$$

$$\text{s.t. } w = y_1 + y_2 + c(x^s)$$

$$x^s = x_1^d + x_2^d$$

$$\Leftrightarrow \max_{x_1^d, x_2^d} u_1(x_1^d) + u_2(x_2^d) + w - c(x_1^d + x_2^d)$$

FOC

$$u'_i(x_i^d) = c'(x^s) \text{ for } i = 1, 2.$$

Same conclusion: a competitive equilibrium maximizes the sum of utilities.

Thus, a *competitive market* achieves the same allocation as a 3rd party benevolent who *maximizes the sum of consumers' utilities*.

Pareto efficiency and the link with previous measure of welfare.

Definition: an allocation is Pareto efficient (P.E.) if one cannot re-allocate the goods so as to increase the utility of 1 agent without decreasing the utility of another.

Consider a pure exchange economy with a fixed amount of goods (\bar{x} and \bar{y}) and 2 consumers, each with utility

$$u_i(x_i) + y_i.$$

Pareto Efficiency

$$\max_{x_1, x_2, y_1, y_2} u_1(x_1) + y_1$$

$$\text{s.t. } u_2(x_2) + y_2 = \bar{u}$$

$$\text{and } x_1 + x_2 = \bar{x}$$

$$y_1 + y_2 = \bar{y}$$

$$\max_{x_1} u_1(x_1) + u_2(\bar{x} - x_1) + \bar{y} - \bar{u}$$

Pareto efficient allocation solves

$$\begin{aligned}
 u'_1(x_1) &= u'_2(x_2), \\
 y_2 &= \bar{u} - u_2(x_2) \\
 y_1 &= \bar{y} - y_2
 \end{aligned}$$

Note 1: Markets would achieve an allocation such that $p = u'_1(x_1) = u'_2(x_2)$. Thus the allocation of good x satisfies a necessary condition for P.E..

In “nice” settings, there are many P.E. allocation with only some of these being the issue of a competitive market. It all depends on the initial allocation.

An allocation maximizing the sum of utilities solves

$$\max_{x_1, y_1} u_1(x_1) + y_1 + u_2(\bar{x} - x_1) + \bar{y} - y_1$$

FOC $u'_1(x_1) = u'_2(x_2)$ but the allocation of \bar{y} does not matter here.

Thus, any allocation that is P.E. also maximizes the sum of utilities. However, an allocation that maximizes the sum of utilities is not necessarily P.E..

MSU= set of allocations that maximize the sum of utilities

SPE= set of PE allocations

SCA= set of competitive allocations

$$SCA \subseteq SPE \subseteq MSU$$