A. Electrical and Thermal Properties of Materials

• Conduction Theory: Drude model

• $\rho(T)$
  – pure metals
  – non-pure metals (Matthiessen’s rule)

• $\rho$ for alloys
  – 1-phase: Nordheim’s rule
  – 2-phase: mixture rules

• $\rho$ for non-metals
  – semiconductors, glasses, polymers (plastic)

• special cases for EE
  – $\rho$ of amorphous / thin films
  – $\rho$ at high frequencies (skin effect)
  – electromigration

• thermal conduction $\kappa(T)$
  – metals, non-metals

References:
- S.O. Kasap, 3rd edition, Chapter 2
- R.J.D. Tilley, 2nd ed, Chapter 13
2.1* Classical Theory: Drude Model

- Key EE materials: metals, semiconductors
  - metals (conductors): transport (drift) of very large number of charge carriers, sea of electrons (at energy $\sim E_F$), under electric field $E$
  - semiconductors: transport (drift, diffusion) of large number of electrons ($\sim E_C$) and/or holes ($\sim E_V$), under electric field $E$

- $E \rightarrow J$: when electric field $E$ (V/cm) is applied, conduction electrons respond, causing current density $J$ (A/cm$^2$)

- Three important questions
  Q1) source of conduction electrons
  Q2) their location
  Q3) their response under electric field
Q1) Where do (conduction) electrons in metals come from?

Reminders (band theory):
- electron energy levels in atoms split and form bands in crystals due to Pauli exclusion principle, at equilibrium distance, some bands overlap
- most important bands: top-most occupied band (▲) and bottom-most empty (▼). Electrons in these bands are valence (VB) or conduction (CB) electrons, the latter respond to electric fields
- examples: metallic $^{11}\text{Na}$, $^{12}\text{Mg}$, $^{13}\text{Al}$
Q1) Where do *(conduction)* electrons in metals come from?

- Materials can be classified according to the type of band diagrams into three main groups: insulators, semiconductors, metals.

- The conduction electrons in metals arise from the partially filled band (\( \phi \)).

- \(^{29}\text{Cu}\): electrons in the half-filled 4s band are responsible for conduction (electrons in the filled 3d band is shielded by 4s band, does not involve in bonding, thus localized around each Cu atom).

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**Insulators**  **Semiconductors**  **Metals**  **Semi-metals**

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**Copper**

- **Valence electrons in the form of individual negative charges**
- **Valence electrons in the form of electron charge clouds**

---

**S\(_{1/2}\)**

- **Cu**
- **Copper**
- **63.546**
- **[Ar]3d\(^{10}\)4s\(^{1}\)**
- **7.7264**
Q2) Where are the conduction electrons?

A) “throughout” metal, moving (a) without- or (b) with preferred direction depending on the presence of external electric fields. The conduction electrons (4s\(^1\)) are said to be delocalized. In contrast, the core electrons ([Ar] 3d\(^{10}\)) are localized.

Q3) How do they respond to electric field \((E)\)?

The short answer (from 2102385): they move, resulting in current \((J)\)

\[ J (E) \]

- Drude: \( J = \sigma E = n e \mu E \)
- \( \sigma \): conductivity \((\Omega \cdot \text{cm})^{-1}\)
- \( n \): electron concentration \((/\text{cm}^3, \text{cm}^{-3})\)
- \( \mu \): drift mobility \((\text{cm}^2/\text{V} \cdot \text{s})\), measures the ease with which charge carriers drift
Derivation (the long answer)

Fig. 2.1: Drift of electrons in a conductor in the presence of an applied electric field. Electrons drift with an average velocity \( v_{dx} \) in the \( x \)-direction. \( (E_x \) is the electric field.)

How is \( v_{dx} \) related to \( E \)?

\[ J \equiv \frac{\Delta q}{A \cdot \Delta t} \]

\[ \Delta q = ne(A \cdot \Delta x) \]

\[ v_{dx} = \frac{\Delta x}{\Delta t} \]

\[ i/p \quad E_x \quad \Delta x \quad o/p: \quad J_x = nev_{dx} \]

\( v_{dx} \): drift velocity

random motion
\[ v = u + at \]

\[ v_d x = \frac{eE_x}{m} \tau \]

Classical mechanics
\[ F = ma \]

Mean free time \( \tau \) (or relaxation time)
- Time between successive collisions (\( \bullet \) \( \bullet \))
- With each collision (\( \bullet \)), the electron is assumed to lose all its velocity (\( \bullet \))
- Energy transferred to lattice (heat: \( P = I^2R \))
- Average free time depends on lattice vibration, defects, impurities, etc

<table>
<thead>
<tr>
<th>Material</th>
<th>Device</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J ): current density (A/cm(^2))</td>
<td>( I ): current (A)</td>
</tr>
<tr>
<td>( \sigma ): conductivity (/( \Omega ).cm)</td>
<td>( G ): conductance (/( \Omega ))</td>
</tr>
<tr>
<td>( E ): field strength (V/cm)</td>
<td>( V ): voltage (V)</td>
</tr>
<tr>
<td>( \rho ): resistivity (( \Omega ).cm)</td>
<td>( R ): resistance (( \Omega ))</td>
</tr>
<tr>
<td>( \mu ): mobility (cm(^2)/V.s)</td>
<td></td>
</tr>
</tbody>
</table>

Ex: \textbf{Cu}
FCC, \( a = 3.6 \) Å
\( N = 4/a^3 = 8.6 \times 10^{22} / \text{cm}^3 \)

Fig. 2.3: Velocity gained in the x-direction at time \( t \) from the electric field (\( E_x \)) for three electrons. There will be \( N \) electrons to consider in the metal.
Ex. 2.2

Calculate drift mobility and mean scattering time of conduction electrons in Cu at room temperature, given that $\sigma(Cu) = 5.9 \times 10^5 \ \Omega^{-1} \ \text{cm}^{-1}$. The density of Cu is 8.96 g/cm$^3$ and atomic mass is 63.5 (g/mol).

\[
\mu = 43.3 \ \text{cm}^2/\text{Vs} \\
\tau = 2.5 \times 10^{-14} \ \text{s}
\]
given $v = 1.5 \times 10^6 \ \text{m/s}$ (Ex. 4.4 p. 299 Fermi velocity)

\[
\text{m.f.p.} = 37 \ \text{nm}
\]
cf: Cu FCC lattice constant 3.610 Å

Note: conduction electrons in metals travel at Fermi velocity: $\frac{1}{2} m v_F^2 = E_{FO}$ (temperature insensitive)
2.2 $\rho(T)$ for ideal pure metals

- Drift of conduction electrons limited mainly by lattice scattering (phonon scattering).
- Increasing temperature $T$ results in increased scattering cross-section (yellow shaded area $a$), decreased mfp mft, mobility, and conductivity, summarized by

\[ T \uparrow \rightarrow a \uparrow \rightarrow \tau \downarrow \rightarrow \mu \downarrow \rightarrow \sigma \downarrow \rightarrow \rho \uparrow \]

\[ \rho \equiv \frac{1}{\sigma} = \frac{1}{ne\mu} \propto \frac{1}{\tau} \propto T \]

**Pure metal wire:** (99.99\% Cu)

\[
\frac{R_{\text{summer}} - R_{\text{winter}}}{R_{\text{summer}}} = \frac{(40 + 273) - (0 + 273)}{(40 + 273)} \approx 13\%
\]

---

**Lattice scattering**

scattering centers (host atoms, impurity, defects,...)

Electron

\[ S = \pi a^2 \]

\[ \ell := \frac{v}{\tau} \]

l: mean free path
u: mean speed
\( \tau \): mean free time

Fig. 2.4: Scattering of an electron from the thermal vibrations of the atoms. The electron travels a mean distance $\ell = u \tau$ between collisions. Since the scattering cross-sectional area is $S$, in the volume $S\ell$ there must be at least one scatterer, $N_S(S\ell \tau) = 1$. 
2.3 $\rho (T)$ for non-pure metals

- “non-pure” means metals with impurities. When the impurities are unintentional, they are called contamination. If they are intentional (controlled type and concentration) they are called dopants (process called doping)

- drift of conduction electrons in non-pure metals are limited by lattice scattering (as in pure metals) and impurity scattering

- experiments show that scattering rates combine linearly, this is known as Matthiessen’s rule (next slide)

Fig. 2.5: Two different types of scattering processes involving scattering from impurities alone and thermal vibrations alone.
**Matthiessen’s Rule**

Overall frequency of independent scattering events:  \( \frac{1}{\tau} = \frac{1}{\tau_T} + \frac{1}{\tau_I} \)

- Due to lattice vibration, \( \rho \propto \frac{1}{\tau} \)  
- Due to impurity scattering, \( \rho \propto \frac{1}{\tau} \)

In general:  \( \rho(T) = \rho_T(T) + \rho_R \)

**Microstructure sensitive**

Residual resistivity  
- [impurities, dislocations, vacancies, interstitial atoms, grain boundaries…]

**Temperature sensitive**

\( \therefore \rho \approx AT + B \)  
[\( A, B \) temperature independent]

Let’s check the validity of Matthiessen’s Rule:  
1. overall trends, 2. boundary conditions (0 K), 3. many metals
Q1) How good is M’s rule? \[ \rho(T) = \rho^*(T) + \rho_R \approx AT + B \]

A) Quite good! Measured results (Fig. 2.8) match well with schematic graphs showing increased resistivities with (Fig. 13.4) temperature (☉), presence of impurity (☉), and (Fig. 13.5) linear dependence with impurity level.

**Figure 13.4** The variation of the resistivity of a metal with temperature (schematic).

**Figure 13.5** The variation of the resistivity of alloys with concentration of the alloying elements (schematic).

Fig. 2.8: Typical temperature dependence of the resistivity of annealed and cold worked (deformed) copper containing various amount of Ni in atomic percentage (data adapted from J.O. Linde, *Ann. Physik*, 5, 219 (1932)).

Q2) ... for all temperatures?
A) No, it fails at low T.

\[ \rho = A T + \rho_R \]

\( \rho = DT^5 + \rho_R \) experiments

\( \rho \sim T \)

\( \rho \sim T^5 \)

\( \rho = \rho_R \)

# atoms that vibrate with sufficient energy reduced rapidly

\( \rho_R = 0 \)
for superconductors

predicted
Q3) ... for all metals?

A) No, it depends on the nature of material

**IDEAL**
\[ \rho \propto T \]
\[ \rho \approx AT + B \]

**REAL**
\[ \rho \propto T^n \]

log:
\[ \frac{\rho_T}{\rho_o} = \left( \frac{T}{T_o} \right)^n \]

linear:
\[ \rho_T = \rho_o [1 + \alpha (T - T_o)] \]

temperature coefficient of resistivity (TCR):
\[ \alpha = \left. \frac{1}{\rho_o} \frac{d\rho}{dT} \right|_{T_o} \]

<table>
<thead>
<tr>
<th>material</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>pure metal</td>
<td>1</td>
</tr>
<tr>
<td>magnetic</td>
<td>~2</td>
</tr>
<tr>
<td>non-magnetic</td>
<td>~1</td>
</tr>
<tr>
<td>alloy (NiCr)</td>
<td>&lt;&lt; 1</td>
</tr>
</tbody>
</table>

\( \rho \propto T \)

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Fig. 2.6: The resistivity of various metals as a function of temperature above 0 °C. Tin melts at 505 K whereas nickel and iron go through a magnetic to non-magnetic (Curie) transformations at about 627 K and 1043 K respectively. The theoretical behavior (\( \rho \sim T \)) is shown for reference.


<table>
<thead>
<tr>
<th>Metal</th>
<th>( \rho_0 ) (N(\Omega) m)</th>
<th>( \alpha_0 \left( \frac{1}{K} \right) )</th>
<th>( n )</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum, Al</td>
<td>25.0</td>
<td>( \frac{1}{233} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Antimony, Sb</td>
<td>38</td>
<td>( \frac{1}{196} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Copper, Cu</td>
<td>15.7</td>
<td>( \frac{1}{232} )</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>Gold, Au</td>
<td>22.8</td>
<td>( \frac{1}{251} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indium, In</td>
<td>78.0</td>
<td>( \frac{1}{196} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Platinum, Pt</td>
<td>98</td>
<td>( \frac{1}{255} )</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>Silver, Ag</td>
<td>14.6</td>
<td>( \frac{1}{244} )</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>Tantalum, Ta</td>
<td>117</td>
<td>( \frac{1}{294} )</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>Tin, Sn</td>
<td>110</td>
<td>( \frac{1}{217} )</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>Tungsten, W</td>
<td>50</td>
<td>( \frac{1}{202} )</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>Iron, Fe</td>
<td>84.0</td>
<td>( \frac{1}{152} )</td>
<td>1.80</td>
<td>Magnetic metal; 273 &lt; ( T ) &lt; 1043 K</td>
</tr>
<tr>
<td>Nickel, Ni</td>
<td>59.0</td>
<td>( \frac{1}{125} )</td>
<td>1.72</td>
<td>Magnetic metal; 273 &lt; ( T ) &lt; 627 K</td>
</tr>
</tbody>
</table>

Example 1:

\[
\rho_{\text{In} @100^\circ C} = 78 \left(1 + \frac{1}{196}\right) = 117.8 \text{ n}\Omega \text{.m}
\]

Example 2:

\[
\rho_{\text{Cu} @100 \text{ K}} = ?
\]

\[
\log : \quad 15.7 \left(\frac{100}{273}\right)^{1.15} = 4.95 \text{ n}\Omega \text{.m}
\]

linear : \[15.7 \left[1 + \frac{1}{232} (100 - 273)\right] \approx 4\]

Which is more accurate? See curve p.13

**SOURCE:** Data were extracted and combined from several sources.
Defects interrupt periodic arrangement of atoms (scatter electrons, reduce mobility, increase resistivity). Defects have many origins; can be grouped according to geometry/dimension (0D, 1D, 2D, 3D)

**0D: point defects**

- Vacancies are equilibrium defects (always exist), concentration in metals is < 1 in 10,000 atoms
- (a) A vacancy in the crystal.
- (b) A substitutional impurity in the crystal. The impurity atom is larger than the host atom.
- (c) A substitutional impurity in the crystal. The impurity atom is smaller than the host atom.
- (d) An interstitial impurity in the crystal. It occupies an empty space between host atoms.

**Fig. 1.44:** Point defects in the crystal structure. The regions around the point defect become distorted; the lattice becomes strained.
**1D: line defects**

Dislocations are non-equilibrium defects (removable with careful process control); When present, energies are stored in the distorted regions.

Dislocation in active area of optical emitter causes dark-line defects, dark pixels

Dislocation in active area of optical detector causes dead pixels (always black, white, or whatever color the pixel is associated with)

(a) Dislocation is a line defect. The dislocation shown runs into the paper.

(b) Around the dislocation there is a strain field as the atomic bonds have been compressed above and stretched below the islocation line
2D: area defects

2D defects include surfaces, grain boundaries (in polycrystalline materials), twins (twin boundary), twists, stacking faults (pile-up faults)

3D: volume defects

3D defects include voids (think air bubbles), inclusions, precipitates

Grain Boundaries
width ~2-5 atoms

Fig. 1.51: The grain boundaries have broken bonds, voids, vacancies, strained bonds and "interstitial" type atoms. The structure of the grain boundary is disordered and the atoms in the grain boundaries have higher energies than those within the grains.

Ex. 2.10 (a) Light bulb rated 40W, 120V. Has W filament 0.381 m long with diameter of 33 μm. Room temperature resistivity of W is 5.51×10⁻⁸ Ωm. Given that resistivity of W varies at $T^{1.2}$, estimate operating temperature of the filament.

Ans: 2,812 K
2.3.2 Nordheim’s Rule (for solid solutions)

- ‘Pure’ metals rarely used in practice, except pure Cu (99.99%) for low $\rho$
- Metals are often mixed (alloyed) to achieve functionality, properties, cost advantages over pure metals
- Alloys: the simple (brass: Cu70 Zn30 wt%), the complex (Inconel 718: 10 elements)
- Simplest type of alloys: solid solution or single-phase alloys
- **Single-phase alloys**: atoms mixed at atomic level throughout solid. This is possible only if the constituent atoms are completely miscible (similar in size and crystalline structure)
- What determine single-phase alloy resistivities?

$$\rho = \rho_T + \rho_I$$ still applies

Alloying $\rightarrow$ effect of $\rho_I \uparrow$, $\rho_T \downarrow$

$\therefore \alpha \downarrow$

**Example 1: NiCr (heater wire)**

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity at 20 °C</th>
<th>$\alpha_0$ at 20 °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nickel</td>
<td>69</td>
<td>0.006</td>
</tr>
<tr>
<td>Chrome</td>
<td>129</td>
<td>0.003</td>
</tr>
<tr>
<td>Nichrome (80%Ni-20%Cr)</td>
<td>1120</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
Example 2: Cu-Ni (thermocouple wire)

\[ \alpha \sim \text{ppm/K} \]

\[ \rho = \rho_T + \rho_I \]

\( C \): Nordheim’s coefficient, effectiveness of impurity atoms in increasing resistivity

\( X \): atomic % of solute in solvent

\[ \rho_I = CX(1 - X) \]

\[ \therefore \rho = \rho\text{matrix} + CX(1 - X) \]

\[ \rho\text{matrix} = \rho_T + \rho_R \]

Cu, Ni are both FCC.

Fig. 2.10(a) Phase diagram of the Cu-Ni alloy system. Above the liquidus line only the liquid phase exists. In the \( L + S \) region, the liquid (\( L \)) and solid (\( S \)) phases coexist whereas below the solidus line, only the solid phase (a solid solution) exists. (b) The resistivity of the Cu-Ni alloy as a function of Ni content (at.%) at room temperature. [Data extracted from

Nordheim (alloys) + Matthiesen (metals)
Semi-empirical equation:

\[ \rho_I = CX \left( 1 - X \right) \]

for small \( X \): \( \rho_I \approx CX \)

Figure 18.8 The electrical resistivity versus temperature for copper and three copper–nickel alloys, one of which has been deformed. Thermal, impurity, and deformation contributions to the resistivity are indicated at \(-100^\circ C\). [Adapted from J. O. Linde, Ann. Physik, 5, 219 (1932); and C. A. Wert and R. M. Thomson, Physics of Solids, 2nd edition, McGraw-Hill Book Company, New York, 1970.]
Q) How good is Nordheim’s rule?
A1) Good, if solutions are randomly mixed
A2) Bad (over-estimate), if solutions are ordered

* orderly structure, viewed as pure compound

Self-check: \( \rho \) of Cu25\% - Au75\%

\[
22.8 + 450(0.25)(0.75) = 107.2 \text{ n}\Omega\text{.m}
\]

Fig. 2.11: Electrical resistivity vs. composition at room temperature in Cu-Au alloys. The quenched sample (dashed curve) is obtained by quenching the liquid and has the Cu and Au atoms randomly mixed. The resistivity obeys the Nordheim rule. On the other hand, when the quenched sample is annealed or the liquid slowly cooled (solid curve), certain compositions (Cu\(_3\)Au and CuAu) result in an ordered crystalline structure in which Cu and Au atoms are positioned in an ordered fashion in the crystal and the scattering effect is reduced.

note: wt.\% ≠ at.\%
<table>
<thead>
<tr>
<th>Solute in Solvent (Element in matrix)</th>
<th>Nordheim coefficient C (nΩ m)</th>
<th>Maximum Solubility at 25°C at.%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Au in Cu matrix</td>
<td>500</td>
<td>100</td>
</tr>
<tr>
<td>Mn in Cu matrix</td>
<td>2900</td>
<td>24</td>
</tr>
<tr>
<td>Ni in Cu matrix</td>
<td>1570</td>
<td>100</td>
</tr>
<tr>
<td>Sn in Cu matrix</td>
<td>2900</td>
<td>0.6</td>
</tr>
<tr>
<td>Zn in Cu matrix</td>
<td>300</td>
<td>30</td>
</tr>
<tr>
<td>Cu in Au matrix</td>
<td>450</td>
<td>100</td>
</tr>
<tr>
<td>Mn in Au matrix</td>
<td>2410</td>
<td>25</td>
</tr>
<tr>
<td>Ni in Au matrix</td>
<td>790</td>
<td>100</td>
</tr>
<tr>
<td>Sn in Au matrix</td>
<td>3360</td>
<td>5</td>
</tr>
<tr>
<td>Zn in Au matrix</td>
<td>950</td>
<td>15</td>
</tr>
</tbody>
</table>

Brass: 70%Cu – 30%Zn

miscible  "solubility limit"  immiscible
ผสมเข้ากันได้ จึงจำกัดความละลายได้ ผสมเข้ากันไม่ได้
- **Single-phase alloys**: atoms mixed at atomic level throughout solid. This is possible only if the constituent atoms are completely miscible (similar in size and crystalline structure).
- **Two-phase alloys**: constituent atoms are very different in size and/or have different crystalline structure. They prefer to phase separate.

**Phases:**
physically and chemically distinct material regions ($\alpha$ and $\beta$).

**Aluminum-Copper Alloy**

solid solubility limit of Al in Cu = $\%$

Adapted from Fig. 9.0, *Callister 3e.*

What determine two-phase alloy resistivities?
2.4 Mixture Rules (for 2-phase alloy)

- The three scenarios of two-phase alloys: a) ordered mixture in series, use Rule 1, b) ordered mixture in parallel, use Rule 2, c) disordered mixture apply linear combination (Rules 3, 4…).
- Nature = c (not a, b), hence 2-phase region has a linear change in resistivity between two end points, (see Cu-Ag, next slide)

\[ R_{\text{eff}} = \frac{L_\alpha \rho_\alpha + L_\beta \rho_\beta}{A} \]
\[ \rho_{\text{eff}} = \chi_\alpha \rho_\alpha + \chi_\beta \rho_\beta \]

\[ \sigma_{\text{eff}} = \chi_\alpha \sigma_\alpha + \chi_\beta \sigma_\beta \]

\( \chi_\alpha \) volume fraction of phase \( \alpha \)
\( \chi_\beta \) volume fraction of phase \( \beta \)
2-phase alloys A-B

Fig. 2.14 (a) The phase diagram for a binary, eutectic forming alloy. (b) The resistivity vs composition for the binary alloy.

Electrical & Thermal Properties
Electrical materials in power distribution

Criteria:
- electrical (low loss)
- mechanical (high strength)
- chemical (long-term stability)
- environmental

Wires: Cu, Al
Plugs: brass (Cu-Zn alloy)
Switches: (Ag-Ni alloy)

Ag has high electrical and thermal conductivities → best for electrical contacts/switches (but expensive). Usually use Ag alloy (Ag-Ni preferred over Ag-Pd despite similar strength because Ag-Ni forms a 2-phase alloy whereas Ag-Pd is a solid solution)

... in signal transmission
Criteria: + noise, cross-talk… hence, twisted pair, shields

<table>
<thead>
<tr>
<th></th>
<th>Ag-Pd</th>
<th>Ag-Ni</th>
</tr>
</thead>
<tbody>
<tr>
<td>solid solution</td>
<td></td>
<td>two-phase alloy</td>
</tr>
<tr>
<td>Nordheim’s rule</td>
<td>Mixture rule</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
</tr>
</tbody>
</table>
2.7 Electrical Conductivity of Non-metals

- previous sections concerned with metals and alloys
- EEE applications also rely heavily on non-metals
- non-metals conducts poorly (semiconductors) or very poorly (insulators), but they conduct
- this section: electrical properties of semiconductors (next 3 slides) and insulators (11 slides)

> temperature dependency
> conductivity
> transport mechanisms: drift-diffusion
> material

**Superinsulators**

\[ \sigma = 0 \]

Many ceramics
- Alumina
- Diamond
- Inorganic Glasses
- Mica
- Polypropylene
- PVDF
- PET
- SiO
- Soda silica glass
- Borosilicate
- Amorphous
- As\textsubscript{2}Se\textsubscript{3}
- Pure SnO\textsubscript{2}
- Intrinsic Si
- Intrinsic GaAs
- Te
- Graphite
- NiCr
- Ag

\[ \rho = AT \]

\[ ne\mu \]

Conductivity (\(\Omega\text{m}\))\(^{-1}\)

Note the range: 10\(^{27}\). No other physical property has this range. Age of universe: 10\(^{10}\) years or 10\(^{17}\) s.
Semiconductors: key material for electrical/electronic engineer (EEE)

Reminder (from course 2102385)
- semiconductors: intrinsic vs extrinsic
- carriers origin: intrinsic (thermal) vs extrinsic (doping)
- carriers concentration: equilibrium vs excess (optical, electrical injection)

Intrinsic semiconductors
- pure, undoped crystals of some elemental (IV), III-V and II-VI compounds
- the greater the size of the component atoms, the smaller the bandgap (see table).
  This is due to the orbitals involved are larger, overlap more, easier to break
- the higher the temperature, the more bonds break, the greater number of carriers,
  the increased conductivity (see graph)

\[
\sigma(T) = ne\mu_e + pe\mu_h
\]

Figure 13.7 The variation of resistivity versus reciprocal temperature for an intrinsic semiconductor.
**Extrinsic semiconductors**
- crystals intentionally doped with small amount of atoms having different valence
dopant atoms in crystal ionize to donate mobile electron ($n$-type) or accept valence electron or equivalently to create mobile hole ($p$-type)
- though conductivity is lower than metals (conductors), the carriers are more mobile due to less scattering (see graph and Cu vs Si below)
- temperature-dependent conductivity is a bit more complicated (see next slide), but can be briefly stated as “it shows the opposite trend to metals”

\[
\begin{align*}
\text{Cu} & \quad n = 8.5 \times 10^{22} \text{ cm}^{-3} \\
\mu &= 43 \text{ cm}^2/\text{Vs}
\end{align*}
\]

**Fig. 5.19:** The variation of the drift mobility with dopant concentration in Si for electrons and holes


\( \sigma(T) \) in extrinsic semiconductors

\[
\mu (\text{cm}^2/\text{V-s}) \quad (\log \text{scale})
\]

\( T^{3/2} \quad T^{-3/2} \quad \text{Impurity scattering} \quad \text{Lattice scattering} \)

\[
\log(n)
\]

\[ \log(\sigma) \quad \log(\mu) \]

\( T (\text{K}) \quad (\log \text{scale}) \)

\[
10^{17} \quad 10^{15} \quad 10^{13} \quad 10^{11}
\]

\[ n_0, \quad n_i, \quad \text{Ionization} \]

\[ 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12 \quad 1000/T (\text{K}^{-1}) \]

Temperature dependence of electrical conductivity for a doped (n-type) semiconductor.
**Insulators?** Which materials exactly? Why we (as an EE) care?

Two examples: plastics, glasses

- **Plastics** are by-products of petroleum industry (think C,H—light atoms). Plastics are **flexible**, when conduct bring unique applications

- **Glasses** are (nominally) non-conducting ionic solids. Glasses are **transparent**, when doped they conduct and bring unique applications

keywords for the (near) future

- plastic electronics
- flexible electronics
… glass & ceramics (ionic conduction)

- glass: SiO$_2$-framework, depending on purity/manufacturing can be crystalline (quartz) or amorphous (glass)
- example: soda-lime glass 73% SiO$_2$ – 15% Na$_2$O – 7% CaO – 4% MgO – 1% Al$_2$O$_3$
- glass (all solids) framework contains vacancies and interstitials (often ionised or charged)
- V$_o$ (oxygen vacancy) introduces an energy level close to the conduction band, thus acts as donor
- glass often contains mobile ions (Na$^+$, for example) from raw materials
- glass $E_G$ large (9 eV) hence transparent, for appreciable conduction glass has to be heated up (impractical)

![Diagram](image)

(E = voltage)

 Vacancy aids the diffusion of positive ion

 Anion vacancy acts as a donor

 Interstitial cation diffuses

窗口玻璃在 > 300°C时导电

$\sigma = \sum Zn_i q_i \mu_i$

$i$: anion, cation

$Z$: valency

Fig. 2.27: Possible contributions to the conductivity of ceramic and glass insulators

(a) Possible mobile charges in a ceramic

(b) A Na$^+$ ion in the glass structure diffuses and therefore drifts in the direction of the field. ($E$ is the electric field.)
... transparent conducting oxides (TCO), or oxide semiconductors

- portable devices demand transparency + electrical conductivity (e.g. touch screen)
  - typical metals (Cu), semiconductors (Si) are opaque (visible light cannot go through)
  - typical glasses are transparent, but electrical conduction at room temperature is negligible
- TCO: combines electrical conduction (metal-like) & transparency (glass like)
- three important TCO materials: ZnO (Wurtzite), In$_2$O$_3$ (cubic), SnO$_2$ (see table below)
- ionic bonding: anion (O) yields localized 2p band (VB), cation (Zn, In) yields delocalized ns band (CB), see next page, orbitals Fig. 2.2, electronic band model Fig. 3.4a.
- carriers come from defects: for examples
  - V$_O$ (oxygen vacancy, anion cavancy) in ZnO acts as donor, $E_d$ close to CB, hence n-type ZnO
  - V$_{Cu}$ (copper vacancy, cation cavancy) in Cu$_2$O acts as acceptor, $E_a$ close to VB, hence p-type Cu$_2$O
- ideal: n-TCO for electron side, p-TCO for hole side – think light-emitting glass (new industry?)

Table 2.1  Typical optical and electrical properties found for thin films of ZnO, In$_2$O$_3$ and SnO$_2$, measured at 300 K. μ values in parentheses correspond to the typical μ obtained in single crystals.

<table>
<thead>
<tr>
<th></th>
<th>$E_G$ (eV)</th>
<th>$\sigma$ (Ω cm)$^{-1}$</th>
<th>$N$ (cm$^{-3}$)</th>
<th>$\mu$ (cm$^2$ V$^{-1}$ s$^{-1}$)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZnO</td>
<td>3.2 – 3.4 (dir)</td>
<td>$&lt; 10^4$</td>
<td>$&lt; 10^{21}$</td>
<td>5 – 50 (200 – 400)</td>
<td>[7-9, 39–41]</td>
</tr>
<tr>
<td>In$_2$O$_3$</td>
<td>2.7 (ind), 3.5 – 3.7 (ind)</td>
<td>$&lt; 10^4$</td>
<td>$&lt; 10^{21}$</td>
<td>10 – 50 (160)</td>
<td>[7-9, 39, 42]</td>
</tr>
<tr>
<td>SnO$_2$</td>
<td>3.6 – 4.3 (dir)</td>
<td>$&lt; 10^3$</td>
<td>$&lt; 10^{20}$</td>
<td>5 – 30 (240)</td>
<td>[8, 9, 39, 43]</td>
</tr>
</tbody>
</table>

**Figure 2.2** Schematics proposed by Nomura et al. of the orbitals composing the CBM on covalent semiconductors with $sp^3$ orbitals and ionic semiconductors with ns orbitals (nHF): a) covalent crystalline; b) covalent amorphous; c) ionic crystalline; d) ionic amorphous. Reproduced with permission from [52] Copyright (2004) Macmillan Publishing Ltd.

**Figure 3.4** a) Chemical bond between an oxide ion and a cation that has a closed-shell electronic configuration; b) representation of the more important defects in Cu$_2$O; c) a simple electronic model proposed by Brattain, with a compensated semiconductor with one acceptor level at 0.3 eV and a deep donor level at 0.9 eV from $V_B$. Adapted with permission from ref [87] Copyright (2010) American Institute of Physics.

... polymers / organic / plastic electronics

- “polymers” means repeating unit of monomers or \([\text{monomer}]_n\); “organic” means carbon-based
- material advantages: low-cost processings (plastics), and light weight \((^{1}\text{H}, ^{6}\text{C} \text{ vs } ^{13}\text{Al}, ^{14}\text{Si}, ^{29}\text{Cu})\)
- first conducting polymer: polyacetylene \((\text{C}_2\text{H}_2)_n = n \text{ units of acetylene (C}_2\text{H}_2)\) (Fig. 13.2)
- electronic conduction possible due to \textit{conjugated} double and single bond (Figs. 13.2b-c, 13.1)
- \textit{conjugated} system: a system of atoms covalently bonded (connected \textbf{p} orbitals) with alternating single and multiple bonds, with \textit{delocalized electrons} in a molecule, and in general lowers the overall energy of the molecule and increases stability. Conjugated systems may be cyclic, acyclic, linear or mixed.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{polymer_structures.png}
\caption{Scheme 13.1 Part of a conjugated hydrocarbon molecule (schematic), in which carbon atoms are linked alternately by single and double bonds: C, carbon; H, hydrogen.}
\end{figure}
plastics/polymers

insulating

non-conjugated

polyethylene

polypropylene

polyvinyl chloride

semiconducting

conjugated: cyclic, acyclic, linear, mixed

pentacene

anthracene

polypyrrole

rubrene-OFET with the highest charge mobility

polyacetylene

Figure 1. Main polyaniline structures n+m = 1, x = half degree of polymerization
- when two atoms are brought together, \textit{bonding} will occur if electron in each has opposite spin (Fig. 2.3a,b), otherwise \textit{anti-bonding} will occur (Fig. 2.3c,d)

- two main bonding types (name $\sigma$, $\pi$ represent shape of resulting bond $s$-like or $p$-like)
  - $\sigma$-bond: $s-p$ (Fig. 2.4a), end-on $p-p$ (Fig. 2.4b), electrons fixed (shared between two connecting nuclei)
  - $\pi$-bond: sideways-on $p-p$ (Fig. 2.5), electrons delocalized (not belong to particular atom)

- for large molecules (polymers) need to consider hybridized bonds ($sp$, $sp^2$, $sp^3$—see next slide)

- polymers that are potentially conducting must have monomers with delocalized $\pi$-bondings (Figs. 2.9-2.11)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figs.png}
\caption{Isolated hydrogen atoms can have electrons in an (a) antiparallel, or (c) parallel, spin arrangement. (b) When orbitals (a) overlap, the electron density accumulates between the nuclei to form a covalent bond. (d) When orbitals (c) overlap, the electron density is low between the nuclei and no bond forms.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figs.png}
\caption{A covalent $\sigma$ bond formed by the overlap of an $s$ orbital and an end-on $p$ orbital when the two electrons have antiparallel spins. (b) A covalent $\sigma$ bond formed by the overlap of two end-on $p$ orbitals when the electrons have antiparallel spins.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figs.png}
\caption{Two sideways-on $p$ orbitals containing electrons with antiparallel spins. (b) A $\pi$ bond formed by the sideways-on overlap of $p$ orbitals.}
\end{figure}
orbital hybridization*

atoms bond (approximately) in such as away that the orbital overlaps are maximum

bonding between

$$s^1 + p^1$$

$$s^1 + p^2$$

$$s^1 + p^3 \text{ (or } s^2 + p^2\text{)}$$

*concerned mostly with

$^6\text{C}: 1s^2\ 2s^2\ 2p^2$ and $^1\text{H}: 1s^1$
Figure 2.9  Bonding in ethyne (acetylene), C$_2$H$_2$. (a, b) Overlap of the 1s orbitals of H with the sp-hybrid orbitals on C results in a σ-bonded linear molecule. (c, d) Overlap of the 2p$_z$ and 2p$_x$ orbitals on C results in the formation of two π bonds with lobes surrounding the C—C σ bond.

Figure 2.10  Bonding in ethene (ethylene), C$_2$H$_4$. (a, b) Overlap of the 1s orbitals of H with the sp$^2$-hybrid orbitals on C results in a σ-bonded molecule. (c, d) Overlap of the 2p$_y$ orbitals on C results in the formation of a π bond.

Figure 2.11  Bonding in benzene, C$_6$H$_6$. (a) Overlap of the 1s orbitals of H with the sp$^3$-hybrid orbitals on C results in a σ-bonded hexagonal molecule. (b) Overlap of the 2p$_y$ orbitals on C results in the formation of π bonds with lobes above and below the plane of the C—H hexagon.

C$_6$H$_6$

Benzene
Molecular formula

Kekulé Structures
(Resonance Forms)

Planar Hexagon
Bond Length 140 pm

Sigma Bonds
sp$^2$-Hybridized orbitals

6 p$_y$ orbitals
delocalized π system

Benzenoid ring
Simplified depiction
... polyacetylene \((\text{C}_2\text{H}_2)_n\) revisited

- In polymer forms, they take either *trans-* or *cis-* configuration (Fig. 13.20)
- The two configurations are similar (alternating double [short] and single [long] bonds), but different (periodicity) \(\rightarrow\) different energy gaps (Fig. 13.20)
- Doping is possible, with orders of magnitude increase in conductivity (Fig. 13.21) by
  - n-type: dope with alkali (Group I)
  - p-type: dope with halogens (Group VII)

*Figure 13.20* Distortion of a polymer chain: (a) delocalised orbitals along a polymer chain lead to equally spaced atoms and a half-filled energy band; (b) Peierls distortion in *trans*-polyacetylene; (c) Peierls distortion in *cis*-polyacetylene; both leading to alternating short and long bonds and a band structure similar to that of an intrinsic semiconductor.

*Figure 13.21* Variation of the conductivity of polyacetylene with iodine dopant concentration. The conductivity changes from a value typical of an insulator to that associated with a metal.
... glasses & polymers

\[ \sigma = \sigma_0 \exp\left( -\frac{E_\sigma}{kT} \right) \]

- Activation energy  
- Conductivity is thermally activated 
- Soda silica glass: 24%Na2O-76%SiO2
- Borosilicate, As3.0Te3.0Si1.2Ge1.0 glass
- 24%Na2O-88%SiO2  
  - Tg < 600 °C
- PVC
- PVAc
- Tg: glass transition temperature (softening point)

\[ E_\sigma \sim 0.7 \text{ eV} \]

Corresponds to \( \mu \sim 10^{-10} \text{ cm}^2/\text{V.s} \) for [c.f. Ag: 40 and Si: 1500 cm^2/V.s]
2.9 Thin Metal Films in IC Interconnects

- $\rho$ (material property) generally does not depend on dimensions, unless $\text{dim} \sim \text{mfp}$
- shrinking electronics results in nm-scale interconnection
- conduction electrons are scattered by the usual mechanisms (lattice scattering, impurity scattering), plus additional ones (at grain boundaries and surfaces)

<table>
<thead>
<tr>
<th>Metal Level</th>
<th>Stack A (SoC)</th>
<th>Stack B (HPCs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1x</td>
<td>M0 - M3</td>
<td>M0 - M3</td>
</tr>
<tr>
<td>2x</td>
<td>M4 - M9</td>
<td>M4 - M7</td>
</tr>
<tr>
<td>3.2x</td>
<td>M10 - M11</td>
<td>M8 - M11</td>
</tr>
<tr>
<td>6.4x</td>
<td>-</td>
<td>M12 - M13</td>
</tr>
<tr>
<td>9x</td>
<td>-</td>
<td>M14 - M15</td>
</tr>
<tr>
<td>18x</td>
<td>M12 - M13</td>
<td>-</td>
</tr>
<tr>
<td>60x</td>
<td>-</td>
<td>M16 - M17</td>
</tr>
</tbody>
</table>

source: Intel, IEDM Dec 2017

reminder: mfp (Cu) 40 nm, slide #6

source: Intel, IEDM Dec 2017
$\rho$ (film) > $\rho$ (bulk) due to:

1. **grain boundary scattering**

   $\rho$ is a function of structure/morphology

Manufacturing method:
- thin films: deposited by evaporation $\rightarrow$ grainy
- bulk: thermal processes, annealing

Matthiessen’s:

$$\rho = \rho_T + \rho_R$$

$$\rho_R \propto \frac{1}{\tau} + \frac{1}{\tau_\perp}$$

mfp in single crystal

$$\frac{\rho}{\rho_0} \approx 1 + \frac{l}{d}$$

mean grain size
## 2. Surface Scattering

Significant in very thin films only, when thickness is comparable to the mean free path (usually 10 nm or less—note that mfp of electrons in Cu @ room temperature is ~ 37nm)

Thin film (< 10 nm) formed by:
- deposition → discontinuous → lose conduction ($\rho \to \infty$)
- epitaxy → $\tau_\perp \downarrow$ → very resistive ($\rho \uparrow \uparrow$)

Table 2.6 Typical resistivity values for thin metal films (polycrystalline) in microelectronics.

<table>
<thead>
<tr>
<th>Metal</th>
<th>Bulk $\rho$ (Ω m) ($\times 10^8$)</th>
<th>Film $\rho$ (Ω m) ($\times 10^8$)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>2.67</td>
<td>2.8 - 3.3</td>
<td>Vacuum evaporated</td>
</tr>
<tr>
<td>Gold</td>
<td>2.2</td>
<td>2.4</td>
<td>Vacuum evaporated or sputtered</td>
</tr>
<tr>
<td>Nickel</td>
<td>6.9</td>
<td>12</td>
<td>Vacuum evaporated or sputtered</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>5.7</td>
<td>10</td>
<td>Sputtered and annealed</td>
</tr>
</tbody>
</table>

$$ \frac{D}{l_B} < 0.1 $$

$l_B$ = mean free path in bulk material

\[ \rho = \rho_T + \rho_R \]
\[ \rho_R \propto \frac{1}{\tau_{//}} + \frac{1}{\tau_{\perp}} \]

\[ \frac{\rho}{\rho_0} \approx 1 + \frac{l}{\pi D} \]

Film thickness ($D$) vs mean free path ($l$)
- on board (PCB), $D >> l$, use bulk $\rho$
- in chip (IC), $D << l$, be careful with $\rho$
(a) \( \rho_{\text{film}} \) of the Cu polycrystalline films vs. reciprocal mean grain size (diameter), 1/d. Film thickness \( D = 250 \text{ nm} - 900 \text{ nm} \) does not affect the resistivity. The straight line is \( \rho_{\text{film}} = 17.8 \text{ n}\Omega \text{ m} + (595 \text{ n}\Omega \text{ m nm})(1/d) \),

(b) \( \rho_{\text{film}} \) of the Cu thin polycrystalline films vs. film thickness \( D \). In this case, annealing (heat treating) the films to reduce the polycrystallinity does not significantly affect the resistivity because \( \rho_{\text{film}} \) is controlled mainly by surface scattering.


\[
\rho \approx \rho_T + \rho_{\text{grain}} + \rho_{\text{surface}}
\]

varies slightly with deposition technique, substrate, etc...

\[ \rho_{\text{surface}} \text{ improve by annealing} \]
\[ \rho_{\text{grain}} \text{ improve by increasing thickness} \]
2.8 Skin Effect

\( \rho \) is a **material** property, normally should not depend on excitation (voltage, electric field), except when excitation is ac with very high frequency.

Fig. 2.29: Illustration of the skin effect. A hypothetical cut produces a hollow outer cylinder and a solid inner cylinder. Cut is placed where it would give equal current in each section. The two sections are in parallel so that the currents in (b) and (c) sum to that in (a).

\[
J = J_S e^{-d/\delta}
\]

Skin depth is due to the circulating eddy currents cancelling the current flow in the center of a conductor and reinforcing it in the skin.

**Lenz's law:** the current induced in a circuit due to a change in the magnetic field is so directed as to oppose the change in flux or to exert a mechanical force opposing the motion.

\[
L = \frac{\Phi}{i}.
\]
HF signals prefer low impedance path → current confined to skin

From solving Maxwell’s eq

\[ \frac{1}{\sqrt{\omega \sigma \mu / 2}} = \delta = \text{Skin depth} \]

- \( \mu = \mu_r \mu_o \)
  - magnetic permeability
- \( \sigma = \text{frequency independent} \) (dc up to 1/\( \tau \), mean free time, see Ex. 2.2)

\[ A \approx \pi a^2 - \pi (a - \delta)^2 \]
\[ \approx 2\pi a \delta \]
\[ \therefore r_{ac} = \frac{\rho}{A} \approx \frac{\rho}{2\pi a \delta} \]

(per unit length)

Fig. 2.30: At high frequencies, the core region exhibits more inductive impedance than the surface region, and the current flows in the surface region of a conductor defined approximately by the skin depth, \( \delta \).

Implications:
1. HF signal Tx should not use solid conductor → use pipes (waveguide) to reduce weight & cost.
2. Ag-coated copper wire for audio applications.
Q1) what is the skin depth of Cu wire at 50 Hz?
A1) 9.3 mm

Q2) what is the largest diameter of Cu wire available commercially?
A2) Wiki (AWG: American wire gauge)

Q3) materials suitable as magnetic core for LF vs HF applications?
A3) magnets (Fe,...), conductivity
Ex 2.26: What is the change in the dc resistance of a Cu wire of radius 1 mm for an ac signal at 10 MHz? And at 1 GHz?
[Cu has $\rho_{dc} = 1.7 \times 10^{-8} \, \Omega \cdot m$ or $\sigma_{dc} = 5.9 \times 10^{7} \, \Omega^{-1} \cdot m^{-1}$ and a relative permeability $\mu_r \sim 1$. Given that $\mu_o = 4\pi \times 10^{-7} \, H/m$.]

Soln:

$r_{dc} = \frac{1}{\rho_{dc} \cdot \frac{A}{\pi r^2}}$

$r_{ac} = \frac{1}{2 \pi \mu_r \mu_o \omega \cdot \frac{A}{\pi r^2}}$

$r_{ac} / r_{dc} = \frac{1}{2 \pi \mu_r \mu_o \omega}$

<table>
<thead>
<tr>
<th>@</th>
<th>$\delta(\mu m)$</th>
<th>$r_{ac}/r_{dc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10MHz</td>
<td>20.7</td>
<td>24.2</td>
</tr>
<tr>
<td>1GHz</td>
<td>2.07</td>
<td>242</td>
</tr>
</tbody>
</table>

Note: $H = \frac{Wb}{A} = \frac{V_s}{\Lambda} = \Omega \cdot s$
SUMMARY

**Electrical Properties $\sigma$**

- $\rho = \rho_I(T) + \rho_1$
- $\rho = \rho_0 \left( \frac{T}{T_c} \right)^n$
- $\rho_I = CX(1 - X)$
- $\sigma = ne\mu_e + pe\mu_h$
- $\sigma = \sum n_q \mu_q$
- $\sigma = \sigma_0 \exp \left( -\frac{E_0}{kT} \right)$

**Thermal Properties $\kappa$**

- $\frac{\kappa}{\sigma} = TC_{WPL}$

**Skin effect**

- $E$: very high frequency

**Hall effect**

- + magnetic field B

**Input:** E

**Output:** J

**Materials**

- Metals: electrons
- Non-metals: phonons
- Semiconductors
- Insulators

**Non-metals**

- $\rho_{film} > \rho_{bulk}$

**Equations**

- $J = \sigma E$
- $\sigma = ne\mu$
- $\mu = e\tau / m$

**Drude Model**

**Matthiessen's Rule**

**Nordheim's Rule**

**Mixture Rules**

**Phase Diagram**
2.6 Thermal Conduction

Metals
thermal carriers: electrons

• metals conduct electricity and heat well
• In metals, free electrons play role in heat conduction
• In nonmetals, heat conducted by lattice vibrations (phonons).

Heat flow \( \propto A \)
\( \propto \frac{\Delta T}{x} \)
\[
\therefore Q' = \kappa A \frac{\Delta T}{x}
\]
\[
K = K_e + K_l
\]

Fig. 2.18: Thermal conduction in a metal involves transferring energy from the hot region to the cold region by conduction electrons. More energetic electrons (shown with longer velocity vectors) from the hotter regions arrive at cooler regions and collide there with lattice vibrations and transfer their energy. Lengths of arrowed lines on atoms represent the magnitudes of atomic vibrations.
Fig. 2.19: Heat flow in a metal rod heated at one end. Consider the rate of heat flow, $dQ/dt$, across a thin section $\delta x$ of the rod. The rate of heat flow is proportional to the temperature gradient $\delta T/\delta x$ and the cross sectional area $A$.

<table>
<thead>
<tr>
<th>Rate of heat flow</th>
<th>Rate of charge flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q' = \frac{dQ}{dt} = -A \kappa \frac{\delta T}{\delta x}$</td>
<td>$I = \frac{dq}{dt} = -A \sigma \frac{\delta V}{\delta x}$</td>
</tr>
</tbody>
</table>

Fourier’s law of thermal conduction

Ohm’s law, Drude ($J = \sigma E$)

*note: $I=JA$
$J=\sigma E$*
In metals, electrons participate in charge ($\sigma$) and heat ($\kappa$) transport processes $\rightarrow \sigma$ and $\kappa$ must be related!

**WFL law**

\[ \frac{\kappa}{\sigma} = T C_{WFL} \]

Lorenz number or Wiedemann-Franz-Lorenz coefficient

\[ C_{WFL} = 2.44 \times 10^{-8} \text{ W} \Omega \text{K}^{-2} \]

(= $\pi^2 k^2 / 3 e^2$)

In pure metal:

\[ \rho \propto T \iff \frac{1}{\sigma} \propto T \]

\[ \therefore \kappa \propto T \]
Fig. 2.21: Thermal conductivity vs. temperature for two pure metals (Cu and Al) and two alloys (brass and Al-14%Mg). Data extracted from *Thermophysical Properties of Matter*, Vol. 1: Thermal Conductivity, Metallic Elements and Alloys, Y.S. Touloukian et. al (Plenum, New York, 1970).

Note: $\kappa_{\text{alloy}} < \kappa_{\text{element}}$ (remember Nordheim?)
Non-metals: thermal carriers: phonons

Fig. 2.22: Conduction of heat in insulators involves the generation and propagation of atomic vibrations through the bonds that couple the atoms. (An intuitive figure.)

<table>
<thead>
<tr>
<th>Coupling</th>
<th>in</th>
<th>( \kappa ) (W / m-K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOOD</td>
<td>Diamonds</td>
<td>1000</td>
</tr>
<tr>
<td>POOR</td>
<td>Polymers</td>
<td>&lt; 1</td>
</tr>
</tbody>
</table>
Table 2.5 Typical thermal conductivities of various classes of materials at 25 °C.  
\( \kappa (W \text{ m}^{-1} \text{ K}^{-1}) \)

<table>
<thead>
<tr>
<th>Pure metal</th>
<th>Nb</th>
<th>Fe</th>
<th>Zn</th>
<th>W</th>
<th>Al</th>
<th>Cu</th>
<th>Ag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>52</td>
<td>80</td>
<td>113</td>
<td>178</td>
<td>250</td>
<td>390</td>
<td>420</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Metal alloys</th>
<th>Stainless Steel</th>
<th>55Cu-45Ni</th>
<th>70Ni-30Cu</th>
<th>Steel</th>
<th>Bronze</th>
<th>Brass</th>
<th>Dural</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12 - 16</td>
<td>19.5</td>
<td>25</td>
<td>50</td>
<td>80</td>
<td>125</td>
<td>147</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ceramics and glasses</th>
<th>Glass-borosilicate</th>
<th>Silica-fused S(_3)N(_4)</th>
<th>Alumina</th>
<th>Saphire</th>
<th>Beryllia</th>
<th>Diamond</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>1.5</td>
<td>20</td>
<td>30</td>
<td>37</td>
<td>260</td>
<td>(\sim)1000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Polymers</th>
<th>Polypropylene</th>
<th>PVC</th>
<th>Polycarbonate</th>
<th>Nylon</th>
<th>Teflon</th>
<th>Polyethylene</th>
<th>Polyethylene</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.12</td>
<td>0.17</td>
<td>0.22</td>
<td>0.24</td>
<td>0.25</td>
<td>0.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>
phonons not coupled

phonons bounced back

Unharmonic interaction

Direction of heat flow

$\kappa(T)$ of non-metals

# Thermal resistance ($\theta$)

## Ohm’s Law

\[ I = \frac{\Delta V}{R} = \frac{\Delta V}{L/A\sigma} \]

## Fourier’s Law

\[ Q' = \frac{\Delta T}{\theta} = \frac{\Delta T}{L/Ak} \]

---

**Fig. 2.23:** Conduction of heat through a component in (a) can be modeled as a thermal resistance $\theta$ shown in (b) where $Q' = \Delta T/\theta$.  

Electrical & Thermal Properties
Ex. 2.19: A brass disk of electrical resistivity 50 nΩ·m conducts heat from a heat source to a heat sink at a rate of 10 W. If its diameter is 20mm and thickness is 30mm, what is the temperature drop across the disk?

\[ C_{WFL} = 2.44 \times 10^{-8} \text{ WΩK}^{-2} \]

Ans: 6.52 K