C. Optical Properties of Materials

Contents

- light (9.1)
- refractive index, \( n \) (9.2)
- dispersion, \( n(\lambda) \) (9.3)
  - \( v_g, N_g \), zero dispersion (9.4)
- principles
  - Snell (9.6): direction
  - Fresnel (9.7): amplitudes, phases
- losses & origins
  - \( n-jK, \varepsilon_r'-j\varepsilon_r'', k'-jk'' \) (9.8)
  - lattice absorption (9.9)
  - VB → CB absorption (9.10)
  - scattering (9.11)
- case studies
  - optical fibre (9.12)
  - luminescence, phosphor (9.13)

Objectives

- What happens when light shines on a material?
- colors of materials
- Why are some materials transparent and others not?
- Optical applications:
  - optical fibers
  - optical amplification
  - luminescence, phosphors
  - nanostructures

9.1 Light

- optical properties of matter describe the interaction of light with matter
- types of interactions: scattering, reflection, absorption, transmission, diffraction...
- properties of light: wave-particle duality
  - for propagation & scattering, light = EM wave
  - for absorption & emission, photons = particle
- visible light has energy ~1.8-3.1 eV, wavelength ~400-700 nm (visible spectrum) vs.
- a much wider electromagnetic spectrum (Fig./Table 14.1) (L1)
- wavelength λ of interest: 10 nm (X-rays) – 100 µm. Selected applications:
  - germicide/cosmetic: 300-400 nm (UV)
  - display: 400-700 nm (VIS)
  - optical communication: 1.3 and 1.55 µm (IR)
- light is an oscillating electromagnetic (EM) field where electric component \( E \perp \) to magnetic component \( B \), both of which \( \perp \) to propagation direction \( k \) (Fig. 9.1) (L2-L3)
  - simplest description: monochromatic light travelling in space in 1D (Fig. 9.2)
  - more complete description must be in 3D (Fig. 9.3)
- light can be generated by electrons in atoms, molecules, solids (crystals) dropping from high to low energy levels/bands (L4-L5). Other courses explain the principle of light generation by semiconductor devices (2102385, SKJ), and lasers (2102589, SPY)
Figure 14.1 The electromagnetic spectrum; the visible region only occupies a small part of the whole, from approximately 400–700 nm.

Table 14.1 The visible spectrum

<table>
<thead>
<tr>
<th>Colour</th>
<th>$\lambda$/nm</th>
<th>$\nu$/Hz</th>
<th>Energy/J</th>
<th>Energy/eV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infrared</td>
<td>750</td>
<td>$4.00 \times 10^{14}$</td>
<td>$2.65 \times 10^{-19}$</td>
<td>1.65</td>
</tr>
<tr>
<td>Deep red</td>
<td>700</td>
<td>$4.28 \times 10^{14}$</td>
<td>$2.84 \times 10^{-19}$</td>
<td>1.77</td>
</tr>
<tr>
<td>Orange-red</td>
<td>650</td>
<td>$4.61 \times 10^{14}$</td>
<td>$3.06 \times 10^{-19}$</td>
<td>1.91</td>
</tr>
<tr>
<td>Orange</td>
<td>600</td>
<td>$5.00 \times 10^{14}$</td>
<td>$3.31 \times 10^{-19}$</td>
<td>2.07</td>
</tr>
<tr>
<td>Yellow</td>
<td>580</td>
<td>$5.17 \times 10^{14}$</td>
<td>$3.43 \times 10^{-19}$</td>
<td>2.14</td>
</tr>
<tr>
<td>Yellow-green</td>
<td>550</td>
<td>$5.45 \times 10^{14}$</td>
<td>$3.61 \times 10^{-19}$</td>
<td>2.25</td>
</tr>
<tr>
<td>Green</td>
<td>525</td>
<td>$5.71 \times 10^{14}$</td>
<td>$3.78 \times 10^{-19}$</td>
<td>2.36</td>
</tr>
<tr>
<td>Blue-green</td>
<td>500</td>
<td>$6.00 \times 10^{14}$</td>
<td>$3.98 \times 10^{-19}$</td>
<td>2.48</td>
</tr>
<tr>
<td>Blue</td>
<td>450</td>
<td>$6.66 \times 10^{14}$</td>
<td>$4.42 \times 10^{-19}$</td>
<td>2.75</td>
</tr>
<tr>
<td>Violet</td>
<td>400</td>
<td>$7.50 \times 10^{14}$</td>
<td>$4.97 \times 10^{-19}$</td>
<td>3.10</td>
</tr>
<tr>
<td>Ultraviolet</td>
<td>350</td>
<td>$8.57 \times 10^{14}$</td>
<td>$5.68 \times 10^{-19}$</td>
<td>3.54</td>
</tr>
</tbody>
</table>
“Light -”

an electromagnetic wave

\[ E_x \perp B_y \]

Fig. 9.1: An electromagnetic wave is a travelling wave which has time varying electric and magnetic fields which are perpendicular to each other and the direction of propagation, \( z \).

monochromatic plane wave:

in 1D:

\[ E_x = E_o \cos(\omega t - kz + \phi_o) \]

\( k \) is propagation constant or wavenumber

Which \((E, B)\) is more important?

- Electric and magnetic fields always co-exist by virtue of Faraday’s Law: time-varying \((B \text{ field } \Leftrightarrow E \text{ field})\)
- Most materials (atoms in the periodic table) only respond to \( E \), thus \( B \) is usually ignored
- Optical field refers to \( E \) field

light-related vocabularies:

- \textit{monochromatic}: single or very narrow range of \( \lambda \) (red laser yes, the sun no)
- \textit{coherent}: photons constituting light beam are in phase (laser yes, LED no)
- \textit{polarization}: direction of \( E \) field, light sources can be (*) unpolarized, (a) plane polarized, (b) elliptically polarized, (c) circularly polarized,
- propagation

monochromatic plane wave propagates in space at velocity $v$

1D (far from source)
plane wave
“planes”

for any given plane, the phase

\[ \phi = \omega t - kz + \phi_0 \]

is a constant which moves $dz$ at every $dt \to \text{phase velocity}$:

\[
\nu = \frac{dz}{dt} = \nu = \frac{\omega}{k} = \frac{\lambda}{f}
\]

\[
\begin{bmatrix}
  f &=& 1/T \\
  \omega &=& 2\pi / T \\
  k &=& 2\pi / \lambda
\end{bmatrix}
\]

3D (close to source)
spherical wave

\[
E(r,t) = E_o \cos(\omega t - k \cdot r + \phi_o)
\]

for any given plane, the phase

\[ \phi = \omega t - kz + \phi_0 \]

is a constant which moves $dz$ at every $dt \to \text{phase velocity}$:

\[
\nu = \frac{dz}{dt} = \nu = \frac{\omega}{k} = \frac{\lambda}{f}
\]

\[
\begin{bmatrix}
  f &=& 1/T \\
  \omega &=& 2\pi / T \\
  k &=& 2\pi / \lambda
\end{bmatrix}
\]
- generation

**GAS**

$H_2$

**SOLID**

semiconductors

direct

InP

indirect

Si
- generation by hot bodies (incandescence)

- heated elements/molecules: (●) electrons excited to E2 and relaxed to E1 (Fig. 14.4), releasing photons
- output spectrum depends solely on temperature $T$ described by Planck’s law of blackbody radiation

\[
S = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/kT} - 1}
\]

\[
S' = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}
\]

**Figure 14.4** Absorption and emission of radiation: (a) light absorption occurs when a photon excites an atom or molecule from a lower electronic energy $E_1$ to a higher energy $E_2$; (b) atoms can emit an identical photon via spontaneous emission when dropping from $E_2$ to $E_1$; (c) each electronic energy level in a molecule has additional associated energy levels due to molecular vibration and rotation.
Light-Matter Interaction

- incident light is mostly *Reflected, Absorbed, Transmitted* (refracted, non-normal incidence), and partly scattered (Fig. 14.13) (I1). Other types of interactions: *diffraction, fluorescent*...

- conservation of energy: (ignoring scattering and secondary effects)

\[ I_o = I_T + I_A + I_R \]

- optical materials/systems often designed to maximize or minimize \( R, A \) or \( T (\lambda) \) – see pix (I1)

- interaction of light with
  - *metals* (I2): reflection, mostly
  - *non-metals*: absorption, luminescence (I3)

- results of light-matter interactions:
  - *colours*: (I4) CdS, Hope, (I5) ruby
  - *opacity*: internal reflection by *microstructure* determines *opacity* (I5)
  - *refractive index*: light slowed down, bent (refracted) (I6)
Light-Matter Interaction

"interaction"

- Transmission
- Absorption
- Reflection

over view of applications

Reflection
Absorption
Cloaking
Transmission

- fluorescence/luminescence centre
- absorption centre
- scattering centre

**Figure 14.13** The interaction of light with a solid.
Light interaction with metals

**Absorption**

- Metals have a fine succession of energy states.
- Near-surface electrons absorb visible light.

**Reflection**

- Metals appear reflective (shiny) because
- Reflectivity $= \frac{I_R}{I_o}$ is $\approx 0.90-0.95$.
  (reflected light same frequency as incident)

**Colors**

Color of metals dictated by interfacial property (metal/insulator interface) called “surface plasmon resonance (SPR)”

since $I_A + I_R \approx 1 \rightarrow I_T \approx 0$; therefore, all metals are opaque (except thin foil $< 100$ nm, transmit visible light)
Light interaction with non-metals (insulators, semiconductors)

**Absorption** (conditional): if $h\nu > E_{\text{gap}}$

- If $E_G < 1.8$ eV, full absorption; color is *black* (Si@1.1, GaAs@1.4 eV)
- If $E_G > 3.1$ eV, no absorption; colorless (diamond@5.5, glass@9 eV), *transparent* (!), see (I5)
- If $E_G$ in between or has impurity level, partial absorption; material has a *color*.

**Reflection**: depends on $n$, ex.
- $R \approx 4\%$ for glass (insulator, $n \sim 1.4$)
- $R \approx 30\%$ for Si (semicond, $n \sim 3.5$)
- for details, see Fresnel (9.7)

Energy range of visible light:
- **blue light**: $h\nu = 3.1$ eV
- **red light**: $h\nu = 1.7$ eV

Energy of electron

incident photon energy $h\nu$

$E_{\text{gap}}$

Luminescence (re-emitted light)

unfilled states

filled states

- $R \approx 4\%$ for glass (insulator, $n \sim 1.4$)
- $R \approx 30\%$ for Si (semicond, $n \sim 3.5$)
- for details, see Fresnel (9.7)
Light-Matter Interaction: Results

- **Colour** determined by sum of frequencies of
  - transmitted light, scattered light
  - re-emitted light from electron transitions

- Ex. 1 (Semiconductor): **Cadmium Sulfide** (CdS)
  - $E_{\text{gap}} = 2.4$ eV,
  - absorbs higher energy visible light (blue, violet),
  - Red/yellow/orange is transmitted, gives color

- Ex. 2 (Insulator)
The Hope Diamond = C (+ % B)
  - $E_{\text{gap}} = 5.6$ eV
  - luminesce **after** UV exposure ($\lambda<220\text{nm}$)
  - phosphorescence
  (re-emission occurs with delay time > 1 s)

Origin of color:
**Boron** (0-8 ppm depends on position, av. 0.36 ppm)

[Links to source material]
http://nhminsci.blogspot.com/2012/05/hope-diamond-blue-by-day-red-by-night_22.html
Light-Matter Interaction: Results

- Ex. 3 (Insulator)
  Ruby = Sapphire (Al$_2$O$_3$) + Cr$_2$O$_3$
  - $E_{\text{gap}} > 3.1 \text{eV}$
  - pure sapphire is colorless
  - adding Cr$_2$O$_3$: (0.5-2) at. %
    - alters the band gap
    ① blue light is absorbed
    ② yellow is absorbed
    ③ red is transmitted
    - Result: Ruby is deep red.

Opacity/Transparency ($E_G>3.1 \text{eV}$)
Transparency of insulators dictated by microstructure:
① single-crystal
② poly-crystal (dense)
③ poly-crystal (porous)

Intrinsically transparent dielectrics can be made translucent/opaque by “interior” reflection/refraction/scattering.
Optical Properties

• Transmitted light distorts electron clouds.

- No transmitted light
- Transmitted light

• Result 1: Light is slower in a material vs vacuum.

\[
\text{Index of refraction (} n \text{) = } \frac{\text{speed of light in a vacuum}}{\text{speed of light in a material}}
\]

- Adding large, heavy ions (e.g. lead, Pb) can decrease the speed of light.

• Result 2: Intensity of transmitted light decreases with distance traveled (thick pieces less transparent) -- see later in Losses (9.8)

• Result 3: Light can be “bent” or “refracted”, see later Snell (9.6)

Refraction (\( \theta_i \neq 0 \)): a change in propagation direction of a wave when it changes a medium
## Table 14.3 Refractive indices

<table>
<thead>
<tr>
<th>Substance</th>
<th>Refractive index*</th>
<th>Substance</th>
<th>Refractive index*</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Vacuum)</td>
<td>1.0 (definition)</td>
<td>Dry air, 1 atm. 15°C</td>
<td>1.00027</td>
</tr>
<tr>
<td>Water</td>
<td>1.3324</td>
<td>Na$_3$AlF$_6$ (cryolite)</td>
<td>1.338**</td>
</tr>
<tr>
<td>MgF$_2$</td>
<td>1.382**</td>
<td>Fused silica (SiO$_2$)</td>
<td>1.4601</td>
</tr>
<tr>
<td>KCl (sylvite)</td>
<td>1.490</td>
<td>Crown glass</td>
<td>1.522</td>
</tr>
<tr>
<td>Extra light flint glass***</td>
<td>1.543</td>
<td>NaCl (halite)</td>
<td>1.544</td>
</tr>
<tr>
<td>Flint glass***</td>
<td>1.607</td>
<td>MgO (periclase)</td>
<td>1.735</td>
</tr>
<tr>
<td>Dense flint glass***</td>
<td>1.746</td>
<td>Al$_2$O$_3$ (corundum)</td>
<td>1.765**</td>
</tr>
<tr>
<td>ZrO$_2$ (zirconia)</td>
<td>2.160**</td>
<td>C (diamond)</td>
<td>2.418</td>
</tr>
<tr>
<td>CaTiO$_3$ (perovskite)</td>
<td>2.740</td>
<td>TiO$_2$ (rutile)</td>
<td>2.755**</td>
</tr>
</tbody>
</table>

*A value appropriate to the yellow light emitted by sodium atoms, the sodium D-lines, with an average wavelength 589.3 nm, is given.

**The refractive index varies with direction; the average value is given.

***The flint glasses contain significant amounts of lead oxide, PbO, as follows: extra light flint, 24 wt.% PbO; flint, 44 wt.% PbO; dense flint, 62 wt.% PbO.
9.2 Refractive Index \((n)\)

- *refractive index* \((n)\) measures how much light *slows down* in a material, with respect to vacuum. Speed of light in i) vacuum = \(c\), ii) material = phase velocity \(v\) (\(\uparrow\)), \(\therefore n \uparrow\)

- materials slow down light due to light-matter interaction: atoms/molecules/grains in optical/dielectric materials are *polarized* by the electric field component of light \(E\) (Reminder: *polarization mechanisms*), Fig. A (R1)

- propagation of light \((E)\) in a material is effectively *delayed* (phase lag) wrt. vacuum (from \(\uparrow\)) material permittivity \(\varepsilon_r \uparrow\)\(\rightarrow\) stronger dipoles (drag \(\uparrow\)) \(\rightarrow\) delays \(\uparrow\)

- atoms/molecules in medium are polarized at the *same frequency* as \(E\), hence the frequency of light does not change, but other wave propagation & parameters do change, Table A (R1). Typical wavefronts in films, Fig. B (R1)

- \(n\) is not a constant, but can change with incident *direction*, especially in some crystals (birefringent) which show *optical anisotropy* (9.14) (R2,R3)

- \(n\) is not a constant, but can change with incident *wavelength*, thus \(n(\lambda)\), see example (R4), for details next section 9.3 Dispersion

\[
\begin{align*}
\uparrow & \quad v = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_r \mu_o}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = \frac{c}{\sqrt{\varepsilon_r}} \\
\Downarrow & \quad n \equiv \frac{c}{v} = \sqrt{\varepsilon_r} \\
& \text{optical materials are non-magnetic} \quad (\mu_r \cong 1)
\end{align*}
\]
Dielectric properties of materials
- complex relative permittivity as a function of frequency
- contribution of various polarization mechanisms

\[ n = \sqrt{\varepsilon_r} \]

\[ n(\lambda) \leftrightarrow \varepsilon_r(f) \]

Table A: Wave propagation parameters

<table>
<thead>
<tr>
<th>vacuum</th>
<th>medium</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>( f )</td>
</tr>
<tr>
<td>( c = \lambda_o f )</td>
<td>( v = \lambda f )</td>
</tr>
<tr>
<td>( \lambda_o )</td>
<td>( \lambda = \lambda_o / n )</td>
</tr>
<tr>
<td>( k_o )</td>
<td>( k = nk_o )</td>
</tr>
</tbody>
</table>

\[ n \equiv \frac{c}{v} = \frac{\lambda_o f}{\lambda f} = \frac{\lambda_o}{\lambda} = \frac{2\pi / \lambda}{2\pi / \lambda_o} = \frac{k}{k_o} \]

Fig. A

Fig. B Wave travelling into optical films

- Estimate \( n \) in Film 1, Film 2
- Film 1 is lossy, 2 lossless
Optical anisotropy (9.14)

- most non-crystalline materials (glasses, liquids) and all cubic crystals are **optically isotropic** (they “look” the same in all directions)
- crystals are different: \( n \) of crystals depend on direction of electric field in the propagating light beam
- **Optically anisotropic** crystals are called **bi-refringent** because incident light beam may be doubly refracted (Figs. 9.25, 9.26)

Fig. 9.25: A line viewed through a cubic sodium chloride (halite) crystal (optically isotropic) and a calcite crystal (optically anisotropic).

Applications: polarising beam splitter, phase contrast microscopy...
$n$ is **frequency** dependent
$n$ maybe **direction** dependent

In **noncrystalline materials**, $n$ is **isotropic**. In **crystalline materials**, $n$ is **anisotropic**.

$$n = \sqrt{\varepsilon_r}$$

$$\varepsilon_r(\omega) = 1 + \frac{N\alpha(\omega)}{\varepsilon_0}$$

### Table 9.3
Principal refractive indices of some optically isotropic and anisotropic crystals (near 589 nm, yellow Na-D line).

<table>
<thead>
<tr>
<th>ISOTROPIC</th>
<th>$n = n_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass (crown)</td>
<td>1.510</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.417</td>
</tr>
<tr>
<td>Fluorite (CaF$_2$)</td>
<td>1.434</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UNIAXIAL - POSITIVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_e &gt; n_o$</td>
</tr>
<tr>
<td>Ice</td>
</tr>
<tr>
<td>Quartz</td>
</tr>
<tr>
<td>Rutile (TiO$_2$)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UNIAXIAL - NEGATIVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_o &gt; n_e$</td>
</tr>
<tr>
<td>Calcite (CaCO$_3$)</td>
</tr>
<tr>
<td>Tourmaline</td>
</tr>
<tr>
<td>Lithium niobate (LiNBO$_3$)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BIAXIAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_1$</td>
</tr>
<tr>
<td>Mica (muscovite)</td>
</tr>
</tbody>
</table>

---

wiki/Birefringence
Physical origin of **dispersion** relation:

\[ n = \sqrt{\varepsilon_r} \]

\[ n(\lambda) \leftarrow \varepsilon_r(f) \]

- **Examples**

<table>
<thead>
<tr>
<th>Material</th>
<th>(\varepsilon_r) (LF)</th>
<th>(\sqrt[\varepsilon_r\text{(LF)}])</th>
<th>(n) (optical)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamond</td>
<td>5.7</td>
<td>2.39</td>
<td>2.41 (at 590 nm)</td>
</tr>
<tr>
<td>Si</td>
<td>11.9</td>
<td>3.44</td>
<td>3.45 (at 2.15 (\mu)m)</td>
</tr>
<tr>
<td>SiO(_2)</td>
<td>3.84</td>
<td>2.00</td>
<td>1.46 (at 600 nm)</td>
</tr>
</tbody>
</table>

Dielectric resonance

- electronic
- dipolar

**How are these measured in practice?**

- **LF:** kHz; e.g. by C-V measurement
- **HF:** (Optical) \(10^{15}\) Hz; Snell’s

**case 1** (=): same polarization mechanism (▲)

**case 2** (≠): additional polarization mechanism (▲) at low frequency
9.3 Dispersion $n(\lambda)$

- *dispersion relations*: interrelations of wave properties ($\lambda, f, v, n$), most importantly $n(\lambda)$
- *dispersion* occurs when pure plane waves of different wavelengths have different propagation velocities, so that a wave packet of mixed wavelengths tends to spread out in space
- brief *physics* of dispersion (D1)
- the *cause*: imperfect light sources, they all have finite width $\Delta\lambda$, or FWHM (D2)
- the *effect*: group velocity and group index (9.4) (D3)
- the *results*: pulse spread, poor system performance (bit-error rate, BER $\uparrow$)
- the *remedy*: zero dispersion (D4)

---

*dispersion in optical fiber communication*

- *dispersion in prism* causes different colors to refract at different angles, splitting white light into a rainbow of colors
Brief Physics of dispersion relation

\[ n^2 = \varepsilon_r \]
\[ \varepsilon_r = 1 + \frac{N\alpha_e}{\varepsilon_o} \]
\[ \alpha_e = \frac{p_{\text{induced}}}{\xi} \]
\[ p_{\text{induced}} = Ze \cdot x = \frac{Ze^2 \xi}{m_e (\omega_0^2 - \omega^2)} \]

- in real crystals, atoms interact → very complicated. In general, a material can have polarization mechanisms each with a different resonance frequency. This leads to a semi-empirical dispersion relation:

\[ n^2(\lambda) = A + \frac{B_1 \lambda^2}{\lambda^2 - C_1} + \frac{B_2 \lambda^2}{\lambda^2 - C_2} + \frac{B_3 \lambda^2}{\lambda^2 - C_3} \]

\( B_{1,2,3}, C_{1,2,3} \) : Sellmeier coefficients

Sources: Batop.com, Wiki

- Example \( n(\lambda) \) for GaAs (0.89-4.1 \( \mu \text{m} \)) at 300K (obtained empirically):

\[ n^2 = 7.10 + \frac{3.78 \lambda^2}{\lambda^2 - 0.2767} \quad [\lambda \text{ in } \mu\text{m}] \]
**Dispersion** occurs when pure plane waves of different wavelengths have different propagation velocities, so that a wave packet of mixed wavelengths tends to spread out in space. (Wiki)

Light sources:
- broadband: Sun, incandescent – for general lighting
- narrowband: light-emitting diodes (LEDs), (LDs) – for communication

**Color** depends on material ($E_G$)

**Color purity** (full-width at half-maximum: FWHM) depends on device (structure / mechanism):
- LEDs (homojunction [p-n], spontaneous emission): FWHM $\sim$ 30 nm
- LDs (heterojunction [p+-n+], stimulated emission): FWHM $\sim$ few nm

FWHM = 0 (perfect monochrome) cannot be achieved.

no perfect monochromatic waves $\omega$ not single value

Heisenberg: $\Delta E \Delta t \geq \hbar$
The effect of dispersion: 9.4 Group velocity and group index

Dispersion occurs when pure plane waves of different wavelengths have different propagation velocities, so that a wave packet of mixed wavelengths tends to spread out in space. (Wiki)

Information:

- *phase* travels @ phase velocity \( (v) \)
- *\( E_{\text{max}} \)* travels @ group velocity \( (v_g) \)

Note that eyes/detectors measure intensity \( (E^2) \)

\[
\begin{align*}
\bar{v} &= c/n \text{ and } \omega = vk \\
\frac{v_g}{v} &= \frac{\delta\omega}{\delta k}
\end{align*}
\]

① in vacuum: \( n = 1 \) (\( \lambda \) independent)

\[ v = c \text{ and } \omega = ck \]

\[ \therefore v_g = \frac{\delta\omega}{\delta k} = c = \text{phase velocity for all } \lambda' \text{’s} \]

② in a medium: \( n > 1 \) (\( \lambda \) dependent)

\[
\begin{align*}
\bar{v} &= \frac{c}{n(\lambda)} \rightarrow v_g &= \frac{\delta\omega}{\delta k} \approx \frac{c}{n - \lambda \frac{dn}{d\lambda}}
\end{align*}
\]

\[ \therefore v_g = \frac{c}{N_g} \quad \text{\( N_g \) group index} \]
**Ex. 9.6:** find the phase velocity, group index, and group velocity of light (1300 nm) travelling in a pure silica glass.

\[ n = 1.447 \quad \rightarrow \quad v = 2.07 \times 10^8 \text{ m/s} \]
\[ N_g = 1.462 \quad \rightarrow \quad v_g = 2.052 \times 10^8 \text{ m/s} \]
9.6 Snell’s law and TIR

Requirement for wave propagation =

Constructive interference: wavefronts for reflected and refracted (transmitted) waves must be in phase with incidence wave

\[ \frac{\sin \theta_i}{\sin \theta_t} = \frac{n_2}{n_1} \]

Snell’s law (1621)

\[ AA' = AB' \sin \theta_i = v_2 t \quad \text{(1)} \]
\[ BB' = AB' \sin \theta_i = v_1 t \quad \text{(2)} \]

\[ \frac{\sin \theta_i}{\sin \theta_t} = \frac{n_2}{n_1} \]

\[ AA'' = AB' \sin \theta_r = v_1 t \quad \text{(3)} \]

\[ \theta_i = \theta_r \]

ALL other angles interfere destructively
from Snell’s law:
when \( \theta_i \rightarrow \theta_c \rightarrow (\theta_t \rightarrow 90^\circ) \)
\[
\theta_c = \sin^{-1}
\left(\frac{n_2}{n_1}\right)
\]

Fig. 9.9: Light wave travelling in a more dense medium strikes a less dense medium. Depending on the incidence angle with respect to \( \theta_c \), which is determined by the ratio of the refractive indices, the wave may be transmitted (refracted) or reflected. (a) \( \theta_i < \theta_c \)  (b) \( \theta_i = \theta_c \)  (c) \( \theta_i > \theta_c \) and total internal reflection (TIR).

\[
E_{r0,\perp}(y, z, t) = e^{-\alpha y} \exp j(\omega t - kz)
\]
\[
\alpha = \frac{2\pi n_2}{\lambda} \left[ \left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_i - 1 \right]^{1/2}
\]
penetration depth = \( 1/\alpha \)

An optical fiber link for transmitting digital information in communications. The fiber core has a higher refractive index so that the light travels along the fiber inside the fiber core by total internal reflection at the core-cladding interface.

TIR leads to wave propagation in a dielectric medium
9.7 Fresnel’s equations

- a set of four equations (Fresnel, 1818) used to determine the magnitudes and phases of reflected and transmitted waves at the interface of two transparent materials with different refractive indices, see (F1)

- the results (mainly reflection coefficients) are plotted in two example cases:
  - internal reflection, when \( n_1 > n_2 \), see (F2)
  - external reflection, when \( n_1 < n_2 \), see (F3)

- human eyes (and optical detectors) sense intensity \( (E^2) \), not the magnitude of the optical field \( (E) \), hence appropriate to plot reflectance and transmittance (F4)

- the equations provide mathematical guideline to the designs of some important normal-incidence \( (\theta_i = 0^\circ) \) applications:
  - low-reflection coating: aims to absorb all incident radiation, useful in solar cells and photodetectors, see anti-reflection coating (ARC) (F6)
  - high-reflection coating: aims to reflect all incident radiation, useful as mirrors, see quarter wave stack (F7)
  - various optical filters: low-pass, high-pass, bandpass (similar in principle to electronic filters)
Governing principles:

A. Snell’s law: \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \)

B. Electromagnetism: \( B_\perp = \left( \frac{n}{c} \right) E_\parallel \) and \( B_\parallel = \left( \frac{n}{c} \right) E_\perp \)

C. Boundary conditions:

\[ E_{\text{tangential}} (1) = E_{\text{tangential}} (2) \]
\[ B_{\text{tangential}} (1) = B_{\text{tangential}} (2) \]; for non magnetic media

\[
\begin{align*}
r_\perp &\equiv \frac{E_{r,\perp}}{E_{i,\perp}} = \frac{\cos \theta_i - \sqrt{n_1^2 - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{n_1^2 - \sin^2 \theta_i}} \\
t_\perp &\equiv \frac{E_{t,\perp}}{E_{i,\perp}} = \frac{2 \cos \theta_i}{\cos \theta_i + \sqrt{n_1^2 - \sin^2 \theta_i}}
\end{align*}
\]

\[
\begin{align*}
r_\parallel &\equiv \frac{E_{r,\parallel}}{E_{i,\parallel}} = \frac{\sqrt{n_1^2 - \sin^2 \theta_i} - n_1^2 \cos \theta_i}{\sqrt{n_1^2 - \sin^2 \theta_i} + n_1^2 \cos \theta_i} \\
t_\parallel &\equiv \frac{E_{t,\parallel}}{E_{i,\parallel}} = \frac{2n \cos \theta_i}{\sqrt{n_1^2 - \sin^2 \theta_i} + n_1^2 \cos \theta_i}
\end{align*}
\]

\[ r_\perp + 1 = t_\perp \quad ; \quad r_\parallel + nt_\parallel = 1 \]
Internal Reflection

\((n_1 > n_2)\)

Example:

\[ n_1 = 1.44 \text{ and } n_2 = 1 \rightarrow n = \frac{1}{1.44}, \quad \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) = 44^\circ \]

Magnitude of reflection coefficients

Phase changes in degrees

\[ \Delta \phi = 0 @ \theta_i = 0 \]

\[ \text{set } \theta_i = 0 \]

\[ \text{normal incidence: } r_{//} = r_\perp = \frac{n_1 - n_2}{n_1 + n_2} \]

\[ \theta_p = \tan^{-1} \left( \frac{n_2}{n_1} \right) = 34.8^\circ \quad \text{polarization angle} \]

\[ \text{Brewster's angle} \]
**External Reflection**  
\((n_1 < n_2)\)

Example:
\[ n_1 = 1 \quad \text{and} \quad n_2 = 1.44 \rightarrow n = \frac{1.44}{1}, \sqrt{n^2 - \sin^2 \theta_i} > 0 \rightarrow \text{no critical angle} \]

\[ \frac{n_1}{n_2} \sin \theta_1 = \sin \theta_2 \]
\[ (n_1 < n_2) \rightarrow (\sin \theta_2 < 1) \]
\[ \therefore \text{no } \theta_c \]

\[ \theta_p = \tan^{-1}(1.44) = 55.2^\circ \]

\[ \Delta \phi = \pi \at \theta_i = 0 \]

\[ \text{Fig. 9.12: The reflection coefficients } r_{//} \text{ and } r_{\perp} \text{ vs.} \]
\[ \text{angle of incidence } \theta_i \text{ for } n_1 = 1.00 \text{ and } n_2 = 1.44. \]

Transmitted light in both internal and external reflection does not experience phase shift (\(t\) is always +ve, \(\theta_i < \theta_c\))

(main concern is reflected light for propagation)
**Reflectance (R) and Transmittance (T)**

\[ R_\perp = \left| \frac{E_{r, \perp}}{E_{i, \perp}} \right|^2 = |r_\perp|^2 \quad \text{and} \quad R_{\parallel} = \left| \frac{E_{r, \parallel}}{E_{i, \parallel}} \right|^2 = |r_{\parallel}|^2 \]

\[ T_\perp = \frac{n_2 |E_{t, \perp}|^2}{n_1 |E_{i, \perp}|^2} = \frac{n_2 |t_\perp|^2}{n_1} \quad \text{and} \quad T_{\parallel} = \frac{n_2 |E_{t, \parallel}|^2}{n_1 |E_{i, \parallel}|^2} = \frac{n_2 |t_{\parallel}|^2}{n_1} \]

At normal incidence (\( \theta_i = 0 \)):

(practical angle in laser diodes, or cavity structures)

\[ R = R_{\parallel} = R_\perp = \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2 \]

\[ T = T_{\parallel} = T_\perp = \frac{4n_1n_2}{(n_1 + n_2)^2} \]

\[ R + T = 1 \]

Ex:

- \( n_i = 1.0, n_2 = 2.0 \)
- \( n_i = 2.0, n_2 = 1.0 \)

**Figure 14.18** The reflection of light: (a) the angle of incidence, \( \theta_i \) is equal to the angle of reflection, \( \theta_r \); (b) the coefficient of reflection at normal incidence, \( r \), relates the amplitude reflected to the incident amplitude, while the reflectivity, \( R \), relates the irradiance reflected to the incident irradiance.
**Internal Reflection**

**Ex. 9.8**

i) Find minimum angle for TIR

ii) Find penetration depth of evanescent wave in the upper medium when $\theta_i = 87°$ and $90°$

\[
\alpha = \frac{2m}{\lambda} \left[ \left( \frac{n_1}{n_2} \right)^2 \sin^2 \theta_i - 1 \right]^{1/2}
\]

<table>
<thead>
<tr>
<th>$\theta_i$ (°)</th>
<th>$1/\alpha$ (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>87</td>
<td>0.906</td>
</tr>
<tr>
<td>90</td>
<td>0.859</td>
</tr>
</tbody>
</table>

**Internal and External Reflection**

**Ex. 9.9**

Find reflection coefficients and reflectance in each case

<table>
<thead>
<tr>
<th>Air</th>
<th>Glass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$n$</td>
</tr>
<tr>
<td>1.5</td>
<td></td>
</tr>
</tbody>
</table>

Thicker than 5 * penetration depth
Ex. 9.10: \( n_1 = 1, \ n_3 = 3.5 \), calculate \( R \) without ARC layer.

Requirement: maximum light transmitted to 
PV for electricity generation
\( n_1 < n_2 < n_3 \)

Insert ARC layer to minimise reflection. A and B must be:
① of similar amplitude \( \rightarrow n_2 = (n_1n_3)^{1/2} \) *** Proof
② out of phase \( \rightarrow \ d = ? \)

Max Absorption (PV)
Ex. 9.11: Dielectric Mirrors
Q) Why do we need dielectric mirrors?
A) Bend light without absorption (cf metallic mirrors)

Design criterion: A,B,C interfere constructively.

Show that waves A, B and C interfere constructively.
note: \( \lambda_1 = \lambda_0 / n_1 \), \( \lambda_2 = \lambda_0 / n_2 \)
9.8 Loss and complex refractive index

- for transparent insulators ($E_G > 3.1$ eV), the light-matter interaction can be described simply by the \textit{(real) refractive index} $n$

- for metals (no $E_G$) and semiconductors ($E_G \sim$ few eV), large number of free electrons cause absorption (loss) of light and the light-matter interaction must be described by a \textit{complex refractive index} $N = n - jK$
  - loss quantified by \textit{absorption coefficient} $\alpha$, related to \textit{absorption index} $K$, the imaginary part of \textit{complex refractive index} $N$ (L1)
  - example ($n,K$) spectra of amorphous Si (a solar cell material) (L2), using a simple optical test setup (L3). Note relationship between \textit{optical} and \textit{dielectric} properties in (L2), with calculation example in (L4)

- loss of photons (between source and detector) is caused by \textit{absorption} and \textit{scattering}
  - two \textit{intrinsic absorption} mechanisms (anything “intrinsic” can’t be removed):
    - \textit{lattice absorption} (9.9): photons to phonons (heat), IR active
    - \textit{band-to-band absorption} (9.10): photons to carriers (EHPs), UV active
  - one \textit{intrinsic scattering} mechanism, Rayleigh (9.11), UV-VIS-IR active
Since $\varepsilon_r$ is complex $\rightarrow n$ is also complex $\rightarrow$ written as $N$

In fact, propagation constant $k$ is complex: $k \equiv k' - jk''$

\[ n \equiv \frac{c}{v} = \frac{\lambda_o f}{\lambda f} = \frac{\lambda_o}{\lambda} = \frac{2\pi / \lambda}{2\pi / \lambda_o} = \frac{k}{k_o} \]

(linear) Absorption Coefficient $\alpha$

Lossless: \[ E = E_o \exp j(\omega t - kz) \quad \text{Plane wave} \]
Lossy: \[ E = E_o \exp j(\omega t - (k' - jk'')z) \]
\[ = E_o \exp(-k''z) \exp j(\omega t - k'z) \]

Intensity: \[ I \propto |E|^2 \propto \exp(-2k''z) \]
\[ \text{or } I = I_o \exp(-\alpha z) \quad \text{Beer's law}^* \]
\[ \text{where } \alpha = 2k'' = 2k_o K \]

EXP. $\Phi$ determine $K(\lambda)$
from \textit{absorption} \textit{/transmission} \textit{measurements}
at normal incidence as a function of freq.

\[ N \equiv n - jK = \frac{k}{k_o} = \frac{k' - jk''}{k_o} \]

$n$ refractive index $= k'/k_o$
$K$ absorption index $= k''/k_o$
(extinction coefficient)

$k'$ describes propagation:
phase velocity $v = \omega/k'$

$k''$ describes absorption:
attenuation along $z$

$k_o$ is propagation constant in a vacuum (no loss, real #)

* Bouguer, Beer, Lambert
Hence, **complex refractive index**: 
\((n,K)\) collectively called **optical constants** but they vary with wavelength, hence not constants as name suggest.

\[ n^2 - K^2 = \varepsilon_r \quad \text{and} \quad 2nK = \varepsilon_r' \]

**a-Si**

![Graph showing real and imaginary parts of refractive index](image)

**EXP.** 
\(\odot\) determine \(R(\lambda)\) from **reflectance measurement** at normal incidence as a function of freq.

\[ R = \frac{1-N}{1+N} = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2} \]

from 9.7 (Fresnel)

Optical constants \((n\text{ and } K)\) can also be found from **ellipsometry** (a kind of reflection measurement, see JA Woollam.com), the principle is based on polarization and angle of incidence (Fresnel’s equations.)
Setup for \((n,K)\) measurements

- **Light Source:** Broadband light source
- **Monochromator:** Filters specific wavelengths
- **Beam Splitter:** 50:50 splitter
- **Material Under Test:** Sample under study
- **Photodetector:** Measures the output

Equations:

\[
\frac{I_r}{I_o}, R
\]

\[
\frac{I_t}{I_o}
\]

\[
\lambda, \text{photon energy}
\]

\[
R(n,K)
\]

\[
\alpha(k'', K)
\]

\[
\lambda, \text{photon energy}
\]
Ex. 9.12 Spectroscopic ellipsometry measurements on a Si crystal at 826.6 nm show that the real and imaginary parts of complex relative permittivity are 13.488 and 0.038. Find complex refractive index, absorption coefficient \( \alpha \) at this wavelength, and phase velocity.
9.9 Lattice absorption

- the electric component of the lightwave \((E)\) interacts with ions in the lattice

- the interaction is strongest when the frequency of the lightwave (color, energy) match the natural lattice vibration frequency \((\sim 10^{12} \text{ Hz} , \text{in the IR region})\)

Lattice absorption through a crystal. The field in the EM wave oscillates the ions, which consequently generate "mechanical" waves in the crystal; energy is thereby transferred from the wave to lattice vibrations.
9.10 Band-to-band absorption

- at sufficiently high energy, light interacts with *valence electrons* in the bonds, freeing them ($\rightarrow$ EHPs)
- electrons are transferred from (valence) band-to-(conduction) band, requiring minimum photon energy:

$$E_G = \frac{hc}{\lambda} \rightarrow \lambda_{\text{cut-off}} (\mu m) = \frac{1.24}{E_G (eV)}$$

- for *direct materials*, Fig. (a), the interaction only involve photons & electrons, but for (b) *indirect materials*, Fig. (b), phonons are involved too (conservations of energy & momentum)
- (band-to-band) absorption spectra, $\alpha(\lambda)$, show sudden change around $\lambda$ ($\blacktriangle$) of materials (B1)
- optical detection systems performance limited mainly by choice of material and device (B2)

Direct materials:

$$hv = E_G$$

(a) GaAs (Direct bandgap)

(b) Si (Indirect bandgap)

Indirect materials:

$$hv = E_G + h \nu$$

phonon energy
\[ \alpha (= 2k'' = 2k_0K) \]

\[ I(x) = I_0 \exp(-\alpha x) \]

Most photon (63\% or 1-1/e) absorbed over penetration depth \( \delta = 1/\alpha \)

---

**Table 9.2** Band gap energy \( E_g \) at 300 K, cut-off wavelength \( \lambda_g \) and type of bandgap (D = Direct and I = Indirect) for some photodetector materials.

<table>
<thead>
<tr>
<th>Semiconductor</th>
<th>( E_g ) (eV)</th>
<th>( \lambda_g ) (µm)</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>InP</td>
<td>1.35</td>
<td>0.91</td>
<td>D</td>
</tr>
<tr>
<td>GaAs(<em>{0.88})Sb(</em>{0.12})</td>
<td>1.15</td>
<td>1.08</td>
<td>D</td>
</tr>
<tr>
<td>Si</td>
<td>1.12</td>
<td>1.11</td>
<td>I</td>
</tr>
<tr>
<td>In(<em>{0.7})Ga(</em>{0.3})As(<em>{0.64})P(</em>{0.36})</td>
<td>0.89</td>
<td>1.4</td>
<td>D</td>
</tr>
<tr>
<td>In(<em>{0.53})Ga(</em>{0.47})As</td>
<td>0.75</td>
<td>1.65</td>
<td>D</td>
</tr>
<tr>
<td>Ge</td>
<td>0.66</td>
<td>1.87</td>
<td>I</td>
</tr>
<tr>
<td>InAs</td>
<td>0.35</td>
<td>3.5</td>
<td>D</td>
</tr>
<tr>
<td>InSb</td>
<td>0.18</td>
<td>7</td>
<td>D</td>
</tr>
</tbody>
</table>

Si photodiodes cannot be used for optical communication @ 1.3, 1.55 µm. Ge OK.

**Ex 9.17**: A GaAs LED emits at 860nm. Si photodetector is to be used. What should be the thickness of Si that absorbs most of the radiation? Given \( \alpha(Si) \) @ 860nm is \( 6 \times 10^4 \) m\(^{-1}\).
Photodetectors
- materials: energy gap ($E_G$) dictates detection range
- devices: structure (p-n, p-i-n, APD, ...) dictates speed ($RC$), sensitivity
- systems: available as 0D (point), 1D (line), 2D (area) detection
- commercial products: embedded in tablet, smartphone, robots, cars...

Silicon (UV-VIS-NIR)

III-V, II-VI (NIR, MIR)
9.11 Light scattering in materials

- scattering is due to small particles embedded (solid) or suspended (liquid, gas) in transparent media
- scattering reduces light intensity in the main direction by radiating part of the incident energy in all directions; the radiation pattern (angle dependent intensity: \( I(\theta) \)) depends on the relative size between the incident wavelength \( \lambda \) and the particle size:
  - particle size < \( \lambda/10 \) → Rayleigh scattering (see ▶)
  - particle size ~ \( \lambda \) → Mie scattering
- colour of sky: blue (daytime, △), pink/red (sunrise/sunset, ◇) due to
  i) eye sensitivity (peaks around green, 555 nm)
  ii) the visible spectrum (bell shape, range 400-700 nm), and
  iii) \( 1/\lambda^4 \) — shorter \( \lambda \) (blue) scattered more, longer \( \lambda \) (red) forwarded more, by atmospheric dusts
- attenuation in optical fiber (9.12) is fundamentally limited by iii)

\[ I = I_0 \frac{8\pi^4 N \alpha^2}{\lambda^4 R^2} \left( 1 + \cos^2 \theta \right) \]

Scattering at right angles is half the forward intensity for Rayleigh scattering

\[ \text{The strong wavelength dependence of Rayleigh scattering enhances the short wavelengths, giving us the blue sky.} \]

https://bilimfili.com/hiperfizik/hbase/atmos/blusky.html#c1
9.12 Attenuation in optical fibers

Glass optical fiber (GOF)

- Optical fiber materials are mainly glass (A0) and polymer (A1), optical transmission through fibers always suffer from loss, quantified by attenuation (dB/km), see ▼

- *Intrinsic* loss mechanisms in GOF: lattice absorption (9.9) and Rayleigh scattering (9.11), the latter due to non-crystalline state of glass (small density fluctuation)

- *Extrinsic* loss mechanisms: hydroxyl (OH−), metallic (Fe2+) impurity, negligible by 1979

- Comparison between electrical (Cu) vs optical (GOF, POF) signal transmission, Table 1 (A2)

- When nature gives you a *loss*, it also gives you a *gain* (EDFA) (A3)

- Not shown, below 500 nm, $\alpha \uparrow$ sharply since photons excite electrons from VB $\rightarrow$ CB (9.10)

- Loss @ 1.5 µm (dB/km): 0.2 in 1979, now = 0.16, intrinsic limit. (Source: Tilley 2011)
Plastic optical fiber (POF)

Material
PMMA: Poly (methyl methacrylate)
($C_5O_2H_8)_n$

Other names:
Acrylic glass, Plexiglas

Figure 1. Loss spectrum for a poly(methyl methacrylate) fiber (a) shows minimums at 530, 570, and 650 nm, all in the visible range. Loss spectrum for a perfluorinated fiber (b) is broader (650 to 1,300 nm) and the loss is less than 50 dB/km.

Figure 3. Connector, light source, and plastic optical fiber are easily interfaced.
Benefits of optical fibers in communications systems:
1. transmission distance
2. transmission speed
3. information density
4. no electromagnetic interference (EMI)

Speed:
- light in space: \( c = 3 \times 10^8 \) m/s
- light in glass: \( \sim 0.67c \)
- electrons in Cu: \( \sim 0.01c \)
- electrons in Si: \( \sim 0.001c \)  
  \( (v_{sat} = 10^7 \text{ cm/s at 300 K}) \)
Erbium doped fiber amplifier (EDFA)

- active region of EDFA system consists of ~30-m section of fiber doped with Er$^{3+}$ (Fig. 14.35a)
- how it works: weak optical signal enters (1), and after being pumped by laser diodes (LD) (2), leaves as output (3) with increased intensity: input (1) + pump (2) $\rightarrow$ output (3)
- material: Erbium (III) ion (Er$^{3+}$), not atom. The electronic structure of Er$^{3+}$ is [Xe] 4f$^{11}$, the 4f-shell is well shielded (by the outer 5s, 5p shells) from the environment (SiO$_2$), thus energy levels (not bands): ground state $^4$I$_{15/2}$ (GS), and next 2 excited states $^4$I$_{13/2}$, $^4$I$_{11/2}$ (Fig. 14.35b)
- physics: LD 980nm shone $\rightarrow$ (4) Er$^{3+}$ excited from $^4$I$_{15/2}$ to $^4$I$_{13/2}$, but (5) quickly relaxes by non-radiative radiation to $^4$I$_{13/2}$ and stays there for a long time, when an input photon (1) $\lambda$~1480 nm arrives, it entices the excited Er$^{3+}$ (at $^4$I$_{13/2}$) to return to GS by *stimulated emission* (6), think laser. Thus, 1 photon in, 2 photons out. Gain!
- effectively, power transfers from LD (2) to output (3), which is an amplified version of input (1)

![Diagram of Erbium Doped Fiber Amplifier](image)

*Figure 14.35* Signal amplification: (a) an incoming weak signal is amplified on passage through a length of fibre in which the core has been doped with Er$^{3+}$ ions; (b) pump wavelengths of 980nm and 1480nm (upward-pointing arrows) excite the Er$^{3+}$ ions into the $^4$I$_{13/2}$ state which transfers energy to the signal (downward-pointing arrow).
9.13. Luminescence

- **Incandescence**: emission of radiation from thermal source (L5)
- **Luminescence**: emission of radiation by a material due to *absorption and conversion of energy* (non-thermal source) (Fig. 15.9). Note the emission is always lower in energy than excitation.

- Luminescence classified by excitation source (M1)
  
  (a) **Photoluminescence** (PL): luminescence from materials excited by photons. Ex. PL fibers in banknotes
  
  (b) **Cathodoluminescence** (CL): luminescence from materials excited by energetic electrons. Ex. CRT

- Luminescence classified by time (emission after excitation)
  
  - < 10 ns: *fluorescence*
  
  - > 10 ns: *phosphorescence*

- Luminescent materials (*phosphors*):
  
  - fluorescent lamps, TV screens (M1,2)
  
  - white LEDs (M2)
  
  - quantum dots (QDs) (M3-M5)

---

**Figure 15.9**

Energy changes during luminescence. (1) Electron-hole pairs are created by exciting electrons to the conduction band or to traps. (2) Electrons can be thermally excited from one trap to another or into the conduction band. (3) Electrons can drop to upper activator (donor) levels and then subsequently to lower acceptor levels, emitting visible light.
Optical Properties

Table 9.4  Selected phosphor examples

<table>
<thead>
<tr>
<th>Phosphor</th>
<th>Activator</th>
<th>Useful Emission</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y₂O₃:Eu³⁺</td>
<td>Eu³⁺</td>
<td>Red</td>
</tr>
<tr>
<td>BaMgAl₁₀O₁₇:Eu²⁺</td>
<td>Eu²⁺</td>
<td>Blue</td>
</tr>
<tr>
<td>CeMgAl₁₁O₁₉:Tb³⁺</td>
<td>Tb³⁺</td>
<td>Green</td>
</tr>
<tr>
<td>Y₃Al₅O₁₂:Ce³⁺</td>
<td>Ce³⁺</td>
<td>Yellow</td>
</tr>
<tr>
<td>Sr₂SiO₄:Eu³⁺</td>
<td>Eu³⁺</td>
<td>Yellow</td>
</tr>
<tr>
<td>ZnS:Ag⁺</td>
<td>Ag⁺</td>
<td>Blue</td>
</tr>
<tr>
<td>Zn₀.₆₈Cd₀.₃₂S:Ag⁺</td>
<td>Ag⁺</td>
<td>Green</td>
</tr>
<tr>
<td>ZnS:Cu⁺</td>
<td>Cu⁺</td>
<td>Green</td>
</tr>
</tbody>
</table>

halophosphate Ca₁₀F₂P₆O₂₄: (Cl⁻,Sb³⁺,Mn²⁺)

(a) Photoluminescence
(b) Cathodoluminescence
(c) A typical phosphor = host + activators
Mercury (Hg) emission spectrum (for fluorescent lamp)
- white light from fluorescent lamp comes from the phosphor (RGB mixture) excited by sharp lines of Hg ions (Hg atoms previously excited by energetic electrons)
- the spectrum of Hg (left) and energy level diagram (right)

Application: **White LEDs**

(a) A typical “white” LED structure. (b) The spectral distribution of light emitted by a white LED. Blue luminescence is emitted by the GaInN chip and “yellow” phosphorescence or luminescence is produced by a phosphor. The combined spectrum looks “white”.

**Optical Properties**

InGaN chip: blue emission
Phosphor (YAG): yellow emission

**Mercury (Hg) emission spectrum (for fluorescent lamp)**
- white light from fluorescent lamp comes from the phosphor (RGB mixture) excited by sharp lines of Hg ions (Hg atoms previously excited by energetic electrons)
- the spectrum of Hg (left) and energy level diagram (right)
Quantum Dots (QDs)

- **bulk** materials, carriers free to move in three dimensions (3D)
- **nanomaterials**, carriers motion limited in 1 or more dimensions. The three types of quantum nanostructures: quantum wells (2D), quantum wires (1D), quantum dots (0D) (Fig. 3.4) (M4)
- electron energy levels: see equations [1D], [3D] (M4). Optical properties (absorption, emission) of nanostructures depend on material (atoms), bonding (lattice constant), and size:
  - for quantum wells, size means thickness, emission at energy $E > E_G$ (Fig. 14.38) (M5)
  - for QDs, size depends on shape, but generally $a \times b \times c$ (length x width x height). See energy diagram (Fig. 14.41) and size-dependent emission spectra (Fig. 14.40) (M5)
- quantum dots maybe synthesized by various techniques: wet chemistry, chemical vapor deposition (CVD), molecular beam epitaxy (MBE)
(Quantum) Nanostructures

- structures with one or more dimensions in the same scale as de Broglie wavelength of electrons (nm in metals, 10s nm in semiconductors)
- electrons confined in such structures can have only certain energies with certain wavelength dictated by size. For confinement in:
  - 1 dimension (Fig. 2.17), i.e. a well of width $a$, the discrete energy levels is $[1D]$ 
  - 3 dimensions, in a $a \times b \times c$ dot, the discrete energy levels is $[3D]$ 
- Size matters: electron energy is inversely proportional to size ($E \propto 1/a^2$)
Figure 14.38 Interband transitions (schematic) between electron (upper) and hole (lower) sub-bands.

Figure 14.40 Photoluminescent colours emitted by CdS quantum dots.

Figure 14.41 Quantum dot colours: (a) the change in band structure of a quantum dot as the diameter falls; (b) fluorescence colours of different diameter dots.