SEMICONDUCTOR PHYSICS

Electron momentum: $\mathsf{p} = m^*\mathsf{v} = \hbar\mathbf{k} = \dfrac{h}{\lambda}$ Planck: $E = h\nu = \hbar\omega$

Kinetic: $E = \dfrac{1}{2}m^*\mathsf{v}^2 = \dfrac{1}{2}\dfrac{\mathsf{p}^2}{m^*} = \dfrac{\hbar^2}{2m^*}\mathbf{k}^2$ (3–4) Effective mass: $m^* = \dfrac{\hbar^2}{d^2E/d\mathbf{k}^2}$ (3–3)

Total electron energy $= P.E. + K.E. = E_c + E(\mathbf{k})$

Fermi-Dirac e^- distribution: $f(E) = \dfrac{1}{e^{(E-E_F)/kT}+1} \cong e^{(E_F-E)/kT}$ for $E \gg E_F$ (3–10)

Equilibrium: $n_0 = \displaystyle\int_{E_c}^{\infty} f(E)N(E)dE = N_c f(E_c) = N_c e^{-(E_c-E_F)/kT}$ (3–15)

$N_c = 2\left(\dfrac{2\pi m_n^* kT}{h^2}\right)^{3/2}$ (3–16a) $N_v = 2\left(\dfrac{2\pi m_p^* kT}{h^2}\right)^{3/2}$ (3–20)

$p_0 = N_v[1 - f(E_v)] = N_v e^{-(E_F-E_v)/kT}$ (3–19)

$n_i = N_c e^{-(E_c-E_i)/kT}, \quad p_i = N_v e^{-(E_i-E_v)/kT}$ (3–21)

$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT} = 2\left(\dfrac{2\pi kT}{h^2}\right)^{3/2}(m_n^* m_p^*)^{3/4} e^{-E_g/2kT}$ (3–23), (3–26)

Equilibrium: $n_0 = n_i e^{(E_F-E_i)/kT}$, $p_0 = n_i e^{(E_i-E_F)/kT}$ (3–25) $n_0 p_0 = n_i^2$ (3–24)

Steady state: $n = N_c e^{-(E_c-F_n)/kT} = n_i e^{(F_n-E_i)/kT}$, $p = N_v e^{-(F_p-E_v)/kT} = n_i e^{(E_i-F_p)/kT}$ (4–15) $np = n_i^2 e^{(F_n-F_p)/kT}$ (5–38)

$\mathscr{E}(x) = -\dfrac{d\mathscr{V}(x)}{dx} = \dfrac{1}{q}\dfrac{dE_i}{dx}$ (4–26)

Poisson: $\dfrac{d\mathscr{E}(x)}{dx} = -\dfrac{d^2\mathscr{V}(x)}{dx^2} = \dfrac{\rho(x)}{\epsilon} = \dfrac{q}{\epsilon}(p - n + N_d^+ - N_a^-)$ (5–14)

$\mu \equiv \dfrac{q\bar{t}}{m^*}$ (3–40a) Drift: $\mathsf{v}_d \cong \dfrac{\mu\mathscr{E}}{1+\mu\mathscr{E}/\mathsf{v}_s}\begin{cases} = \mu\mathscr{E} \text{ (low fields, ohmic)} \\ = \mathsf{v}_s \text{ (high fields, saturated vel.)}\end{cases}$ (Fig. 6–9)

Drift current density: $\dfrac{I_x}{A} = J_x = q(n\mu_n + p\mu_p)\mathscr{E}_x = \sigma\mathscr{E}_x$ (3–43)

Conduction current:

$$J_n(x) = q\mu_n n(x)\mathscr{E}(x) + qD_n\frac{dn(x)}{dx}$$

drift diffusion (4–23)

$$J_p(x) = q\mu_p p(x)\mathscr{E}(x) - qD_p\frac{dp(x)}{dx}$$

$J_{\text{total}} = J_{\text{conduction}} + J_{\text{displacement}} = J_n + J_p + C\dfrac{dV}{dt}$

Continuity: $\dfrac{\partial p(x,t)}{\partial t} = \dfrac{\partial\delta p}{\partial t} = -\dfrac{1}{q}\dfrac{\partial J_p}{\partial x} - \dfrac{\delta p}{\tau_p}$ $\quad \dfrac{\partial\delta n}{\partial t} = \dfrac{1}{q}\dfrac{\partial J_n}{\partial x} - \dfrac{\delta n}{\tau_n}$ (4–31)

For steady state diffusion: $\dfrac{d^2\delta n}{dx^2} = \dfrac{\delta n}{D_n\tau_n} \equiv \dfrac{\delta n}{L_n^2}$ $\quad \dfrac{d^2\delta p}{dx^2} = \dfrac{\delta p}{L_p^2}$ (4–34)

Diffusion length: $L \equiv \sqrt{D\tau}$ Einstein relation: $\dfrac{D}{\mu} = \dfrac{kT}{q}$ (4–29)

p-n JUNCTIONS

Equilibrium: $V_0 = \dfrac{kT}{q}\ln\dfrac{p_p}{p_n} = \dfrac{kT}{q}\ln\dfrac{N_a}{n_i^2/N_d} = \dfrac{kT}{q}\ln\dfrac{N_aN_d}{n_i^2}$ (5–8)

$\dfrac{p_p}{p_n} = \dfrac{n_n}{n_p} = e^{qV_0/kT}$ (5–10) $W = \left[\dfrac{2\epsilon(V_0 - V)}{q}\left(\dfrac{N_a + N_d}{N_aN_d}\right)\right]^{1/2}$ (5–57)

One-sided abrupt p^+-n: $x_{n0} = \dfrac{WN_a}{N_a + N_d} \simeq W$ (5–23b) $V_0 = \dfrac{qN_dW^2}{2\epsilon}$

$\Delta p_n = p(x_{n0}) - p_n = p_n(e^{qV/kT} - 1)$ (5–29)

$\delta p(x_n) = \Delta p_n e^{-x_n/L_p} = p_n(e^{qV/kT} - 1)e^{-x_n/L_p}$ (5–31b)

Ideal diode: $I = qA\left(\dfrac{D_p}{L_p}p_n + \dfrac{D_n}{L_n}n_p\right)(e^{qV/kT} - 1) = I_0(e^{qV/kT} - 1)$ (5–36)

Non-ideal: $I = I_0'(e^{qV/\mathbf{n}kT} - 1)$ $(\mathbf{n} = 1 \text{ to } 2)$ (5–74)

With light: $I_{\text{op}} = qAg_{\text{op}}(L_p + L_n + W)$ (8–1)

Capacitance: $C = \left|\frac{dQ}{dV}\right|$ (5–55)

Junction depletion: $C_j = \epsilon A\left[\frac{q}{2\epsilon(V_0 - V)}\frac{N_d N_a}{N_d + N_a}\right]^{1/2} = \frac{\epsilon A}{W}$ (5–62)

Stored charge exp. hole dist.: $Q_p = qA\int_0^\infty \delta p(x_n)dx_n = qA\Delta p_n\int_0^\infty e^{-x_n/L_p}dx_n = qAL_p\Delta p_n$ (5–39)

$I_p(x_n = 0) = \frac{Q_p}{\tau_p} = qA\frac{L_p}{\tau_p}\Delta p_n = qA\frac{D_p}{L_p}p_n(e^{qV/kT} - 1)$ (5–40), (5–29)

$G_s = \frac{dI}{dV} = \frac{qAL_p p_n}{\tau_p}\frac{d}{dV}(e^{qV/kT}) = \frac{q}{kT}I$ (5–65)

Long p$^+$-n: $i(t) = \frac{Q_p(t)}{\tau_p} + \frac{dQ_p(t)}{dt}$ (5–47)

MOS-n CHANNEL

Oxide: $C_i = \frac{\epsilon_i}{d}$ Depletion: $C_d = \frac{\epsilon_s}{W}$ MOS: $C = \frac{C_i C_d}{C_i + C_d}$ (6–36)

Threshold: $V_T = \underbrace{\Phi_{ms} - \frac{Q_i}{C_i}}_{\text{Flat band}} - \frac{Q_d}{C_i} + 2\phi_F$ (6–38)

Inversion: $\phi_s(\text{inv.}) = 2\phi_F = 2\frac{kT}{q}\ln\frac{N_a}{n_i}$ (6–15) $W = \left[\frac{2\epsilon_s\phi_s}{qN_a}\right]^{1/2}$ (6–30)

$Q_d = -qN_a W_m = -2(\epsilon_s qN_a\phi_F)^{1/2}$ (6–32) At V_{FB}: $C_{FB} = \frac{C_i\, C_{\text{debye}}}{C_i + C_{\text{debye}}}$

Debye screening length: $L_D = \sqrt{\frac{\epsilon_s kT}{q^2 p_0}}$ (6–25) $C_{\text{debye}} = \frac{\epsilon_s}{L_D}$ (6–40)

Substrate bias: $\Delta V_T \simeq \frac{\sqrt{2\epsilon_s qN_a}}{C_i}(-V_B)^{1/2}$ (n channel) (6–63b)

$I_D = \frac{\overline{\mu}_n ZC_i}{L}\left\{(V_G - V_{FB} - 2\phi_F - \tfrac{1}{2}V_D)V_D - \frac{2}{3}\frac{\sqrt{2\epsilon_s qN_a}}{C_i}\left[(V_D + 2\phi_F)^{3/2} - (2\phi_F)^{3/2}\right]\right\}$ (6–50)

$I_D \simeq \frac{\overline{\mu}_n ZC_i}{L}\left[(V_G - V_T)V_D - \tfrac{1}{2}V_D^2\right]$ (6–49)

Saturation: $I_D(\text{sat.}) \simeq \tfrac{1}{2}\overline{\mu}_n C_i\frac{Z}{L}(V_G - V_T)^2 = \frac{Z}{2L}\overline{\mu}_n C_i V_D^2(\text{sat.})$ (6–53)

$g_m = \frac{\partial I_D}{\partial V_G}$; $g_m(\text{sat.}) = \frac{\partial I_D(\text{sat.})}{\partial V_G} \simeq \frac{Z}{L}\overline{\mu}_n C_i(V_G - V_T)$ (6–54)

For short L: $I_D \simeq ZC_i(V_G - V_T)v_s$ (6–60)

Subthreshold slope: $S = \frac{dV_G}{d(\log I_D)} = \frac{kT}{q}\ln 10\left[1 + \frac{C_d + C_{it}}{C_i}\right]$ (6–66)

BJT-p-n-p

$I_{Ep} = qA\frac{D_p}{L_p}\left(\Delta p_E \,\text{ctnh}\,\frac{W_b}{L_p} - \Delta p_C \,\text{csch}\,\frac{W_b}{L_p}\right)$ (7–18)

$\Delta p_E = p_n(e^{qV_{EB}/kT} - 1)$
$\Delta p_C = p_n(e^{qV_{CB}/kT} - 1)$ (7–8)

$I_C = qA\frac{D_p}{L_p}\left(\Delta p_E \,\text{csch}\,\frac{W_b}{L_p} - \Delta p_C \,\text{ctnh}\,\frac{W_b}{L_p}\right)$

$I_B = qA\frac{D_p}{L_p}\left[(\Delta p_E + \Delta p_C)\tanh\frac{W_b}{2L_p}\right]$ (7–19)

$B = \frac{I_C}{I_{Ep}} = \frac{\text{csch}\, W_b/L_p}{\text{ctnh}\, W_b/L_p} = \text{sech}\,\frac{W_b}{L_p} \simeq 1 - \left(\frac{W_b^2}{2L_p^2}\right)$ (7–26)

(Base transport factor)

$\gamma = \frac{I_{Ep}}{I_{En} + I_{Ep}} = \left[1 + \frac{L_p^n n_n \mu_n^p}{L_n^p p_p \mu_p^n}\tanh\frac{W_b}{L_p^n}\right]^{-1} \simeq \left[1 + \frac{W_b n_n \mu_n^p}{L_n^p p_p \mu_p^n}\right]^{-1}$ (7–25)

(Emitter injection efficiency)

$\frac{i_C}{i_E} = B\gamma \equiv \alpha$ (7–3)

(Common base gain)

$\frac{i_C}{i_B} = \frac{B\gamma}{1 - B\gamma} = \frac{\alpha}{1 - \alpha} \equiv \beta$ (7–6)

(Common emitter gain)

$\frac{i_C}{i_B} = \beta = \frac{\tau_p}{\tau_t}$ (7–7)

(For $\gamma = 1$)