

Exercise 4

1. Find the following Laplace transforms.

(a) $\mathcal{L}[3 \sin(5t) - \cos(2t)]$

(b) $\mathcal{L}\left[\cos\left(t + \frac{\pi}{3}\right)\right]$

(c) $\mathcal{L}[e^{2t-5}]$

(d) $\mathcal{L}[2 \cos(3)]$

(e) $\mathcal{L}\left[3 + \frac{1}{e^{2t}}\right]$

(f) $\mathcal{L}[t^2(1 - 3t)]$

2. Find the following inverse Laplace transforms.

(a) $\mathcal{L}^{-1}\left[\frac{1}{3s} + \frac{2}{s^2}\right]$.

(b) $\mathcal{L}^{-1}\left[\frac{1}{4s + 5}\right]$

(c) $\mathcal{L}^{-1}\left[\frac{2}{s^2 + 3}\right]$

(d) $\mathcal{L}^{-1}\left[\frac{3s}{(s - 4)(s^2 + 1)}\right]$

3. Let $y(t)$ be the solution of the IVP

$$2y'' - 3y' + 4y = f(t), \quad y(0) = 1, y'(0) = 0.$$

For each of the following $f(t)$, find the Laplace transform $Y(s) = \mathcal{L}[y(t)]$.

(a) $f(t) = 5e^{-2t}$

(b) $f(t) = t^2$

(c) $f(t) = \cos(3t)$

4. Solve the following IVP:

$$y'' + 3y' + 2y = e^{-t} + 2t, \quad y(0) = y'(0) = 1.$$