

Exercise 6

1. In each of the following functions, express the function in terms of the Heaviside function and find the Laplace transform of the function.

$$(a) f(t) = \begin{cases} 1 & 0 \leq t < 7 \\ \cos t & t > 7 \end{cases}$$

$$(b) f(t) = \begin{cases} t & 0 \leq t < 3 \\ 1 - 3t & t > 3 \end{cases}$$

$$(c) f(t) = \begin{cases} -4 & 0 \leq t < 1 \\ 0 & 1 < t < 3 \\ e^{-t} & t > 3 \end{cases}$$

2. Find the inverse Laplace transform for each of the following functions.

$$(a) \frac{1}{s^2 + 4s + 12}$$

$$(b) \frac{1}{s^2 - 4s + 5}$$

$$(c) \frac{e^{-5s}}{s^3}$$

$$(d) \frac{e^{-2s}}{s^2 + 9}$$

$$(e) \frac{s - 4}{s^2 - 8s + 10}$$

$$(f) \frac{1}{s(s^2 + 16)} e^{-21s}$$

3. Solve the following IVPs.

$$(a) y'' + 4y = f(t) = \begin{cases} 0 & 0 \leq t < 4 \\ 3 & t > 4 \end{cases}, y(0) = 1, y'(0) = 0.$$

$$(b) y'' + 5y' + 6y = f(t) = \begin{cases} -2 & 0 \leq t < 3 \\ 0 & t > 3 \end{cases}, y(0) = 0, y'(0) = 0$$

$$(c) y'' - 4y' + 4y = f(t) = \begin{cases} t & 0 \leq t < 3 \\ t + 2 & t > 3 \end{cases}, y(0) = -2, y'(0) = 1$$

$$(d) y'' + 4y' + 5y = \deg(t - 1), y(0) = 0, y'(0) = 3$$

$$(e) y'' + 3y' + 2y = 10(\sin t + \delta(t - 1)), y(0) = 1, y'(0) = -1$$