Exercise 7

1. Find the Fourier series for each of the following 2π-periodic functions.

   (a) $f(x) = \begin{cases} x & |x| < \pi/2 \\ 0 & |x| > \pi/2 \end{cases}, p = 2\pi$

   (b) $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 \leqslant t < \pi \end{cases}, p = 2\pi$

   (c) $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}, p = 2\pi$

   (d) $f(x) = x^2$ for $|x| < \pi$, $p = 2\pi$

2. Find the Fourier series for each of the following $2L$-periodic functions.

   (a) $f(x) = \begin{cases} -x & - < x < 0 \\ x & 0 < x < 1 \end{cases}.$

   (b) $f(x) = \begin{cases} -4 - x & -4 < x < 0 \\ 4 - x & 0 < x < 4 \end{cases}$

   (c) $f(x) = \cos(\pi x)$, $(-\frac{1}{2} < x < \frac{1}{2})$, $p = 1$

3. In each of the following periodic functions, use the convergence theorem to determine the sum of the Fourier series of the function.

   (a) $f(x) = \begin{cases} 2x & -3 < x < -2 \\ 0 & -2 < x < 1 \\ x^2 & 1 < x < 3 \end{cases}, p = 6$

   (b) $f(x) = \begin{cases} 2x - 2 & -\pi < x < 1 \\ 3 & 1 < x < \pi \end{cases}, p = 2\pi$

   (c) $f(x) = \begin{cases} x^2 & -\pi < x < 0 \\ 2 & 0 < x < \pi \end{cases}, p = 2\pi$

4. Use the Fourier series expansion of $f(x) = x^2$, $(-1 < x < 1)$, $p = 2$, to show that

$$1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{25} + \cdots = \frac{\pi^2}{6}$$