Exercise 9

1. For each of the following functions find its Fourier integral representation.

   (a) $f(x) = \begin{cases} x & -\pi < x < \pi \\ 0 & |x| > \pi \end{cases}$

   (b) $f(x) = \begin{cases} k & -10 < x < 10 \\ 0 & |x| > 10 \end{cases}$

   (c) $f(x) = \begin{cases} -1 & -\pi < x < 0 \\ 1 & 0 < x < \pi \\ 0 & |x| > \pi \end{cases}, p = 2\pi$

   (d) $f(x) = e^{-|x|}$.

2. For each of the following functions, find the Fourier transform. The following formula can be used:

$$\mathcal{F}[e^{-kx^2}](\omega) = \frac{1}{\sqrt{2k}} e^{-\omega^2/4k}, \quad \mathcal{F}\left[\frac{1}{k^2 + x^2}\right](\omega) = \frac{1}{k}\sqrt{\frac{\pi}{2}} e^{-k|\omega|}$$

   (a) $f(x) = \begin{cases} 1 & 0 < x < 1 \\ -1 & -1 < x < 0 \\ 0 & |x| > 1 \end{cases}$.

   (b) $f(x) = \dfrac{1}{1 + x^2}$

   (c) $f(x) = 5[H(t-3) - H(t-11)]$

   (d) $f(x) = 5e^{-3(x-5)^2}$

3. In each of the following functions, find the inverse Fourier transform of the function.

   (a) $9e^{-(\omega+4)^2/32}$

   (b) $e^{(20-4\omega)i}/(3 - (5-\omega)i)$

   (c) $10\sin(3\omega)/(\omega + \pi)$