

## 4.4. Integration of Transcendental Functions

### Some Basic Formulas

$$1. \int e^x dx = e^x + C$$

$$2. \int a^x dx = \frac{a^x}{\ln a} + C$$

$$3. \int \frac{1}{x} dx = \ln|x| + C$$

*Proof.* By differentiation formulas:

$$\frac{d}{dx} [e^x] = e^x \Rightarrow \int e^x dx = e^x + C$$

$$\frac{d}{dx} [a^x] = a^x \ln a \Rightarrow \frac{d}{dx} \left[ \frac{a^x}{\ln a} \right] = a^x$$

$$\Rightarrow \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\frac{d}{dx} [\ln|x|] = \frac{1}{x} \Rightarrow \int \frac{1}{x} dx = \ln|x| + C$$

**EXAMPLE.** Find

$$\int (3e^x - 2x) dx$$

**EXAMPLE.** Find

$$\int \left( x - \frac{5}{x} \right) dx$$

**EXAMPLE (Simplify).** Find

$$\int \frac{3^{2x}}{2^x} dx$$

**Formulas with  $u$ -substitution**

$$1. \int e^u u' dx = e^u + C$$

$$2. \int \frac{u'}{u} dx = \ln|u| + C$$

$$3. \int a^u u' dx = \frac{a^u}{\ln a} + C$$

**EXAMPLE.** Find the integral

$$\int x e^{-x^2} dx$$

**EXAMPLE.** Find the integral

$$\int (e^{x+1} - 2e^{3x}) dx$$

**EXAMPLE** ( $\frac{1}{a^x} = \left(\frac{1}{a}\right)^x$ ). Find the integral

$$\int \frac{1}{2^{4x-1}} dx$$



**EXAMPLE.** Find the integral

$$\int \frac{1}{1 + 3x} dx$$

**EXAMPLE** ( $u = \sqrt{x}$ ). By expressing

$$\tan x = \frac{\sec x \tan x}{\sec x}$$

show the formula

$$\int \tan x \, dx = \ln|\sec x| + C.$$

**EXAMPLE.** Use the formula  $\int \tan x \, dx = \ln|\sec x| + C$  to find

$$\int \frac{\tan(1/\sqrt{x})}{x\sqrt{x}} \, dx$$

**EXAMPLE.** By expressing

$$\frac{1}{x^2 + a^2} = \frac{1}{a} \cdot \frac{1/a}{1 + \left(\frac{x}{a}\right)^2}$$

show the formula

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$

**EXAMPLE.** Use the formula  $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$  to find the integral

$$\int \frac{x+9}{x^2+9} dx$$