There is another test for a *local extrema* using the second derivative.

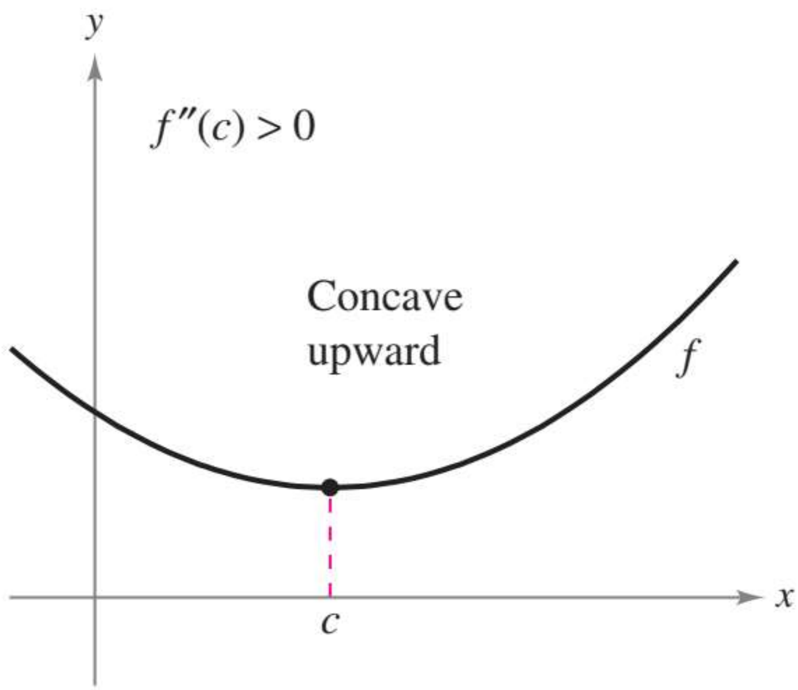
Test for local extrema (by f'').

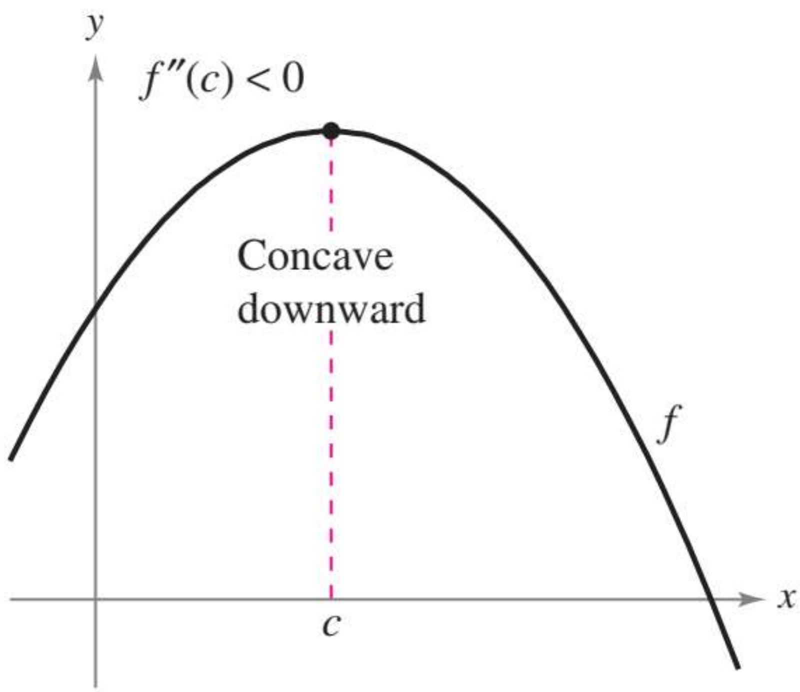
Suppose $f'(c) = 0$.

1. If $f''(c) > 0$ then $f(c)$ is a ***local minimum*.**
2. If $f''(c) < 0$ then $f(c)$ is a ***local maximum*.**
3. If $f''(c) = 0$, ***no conclusion*** can be made.

   Have to use the former test.

Of course, this second derivative can be applied if $f''(c)$ exists at the critical number c.


y
f″(c) > 0
Concave
upward
f
x
c


y
f″(c) < 0
Concave
downward
f
x
c

EXAMPLE. Find the local extrema of

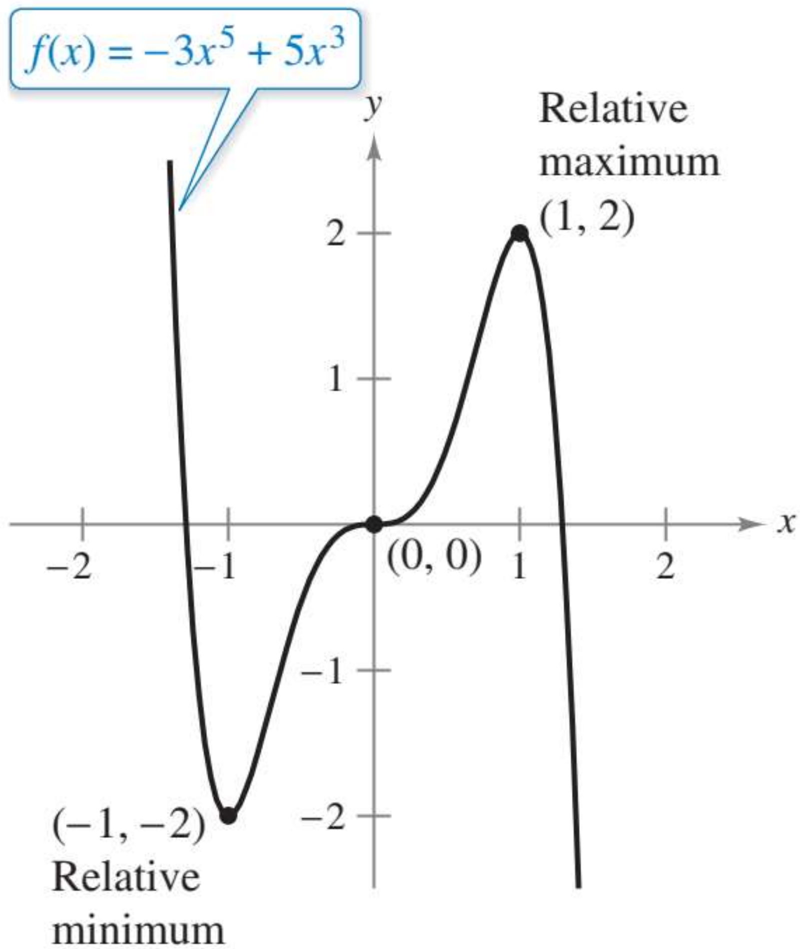
$$f(x) = x^4 - 4x^3 + 2$$

EXAMPLE. Find the local extrema of

$$f(x) = x \ln x$$

EXAMPLE. Find the local extrema of

$$f(x) = -3x^5 + 5x^3$$



Optimization Problems.

We study optimization problems. They involve finding certain variable numbers that give the max or min of certain quantity.

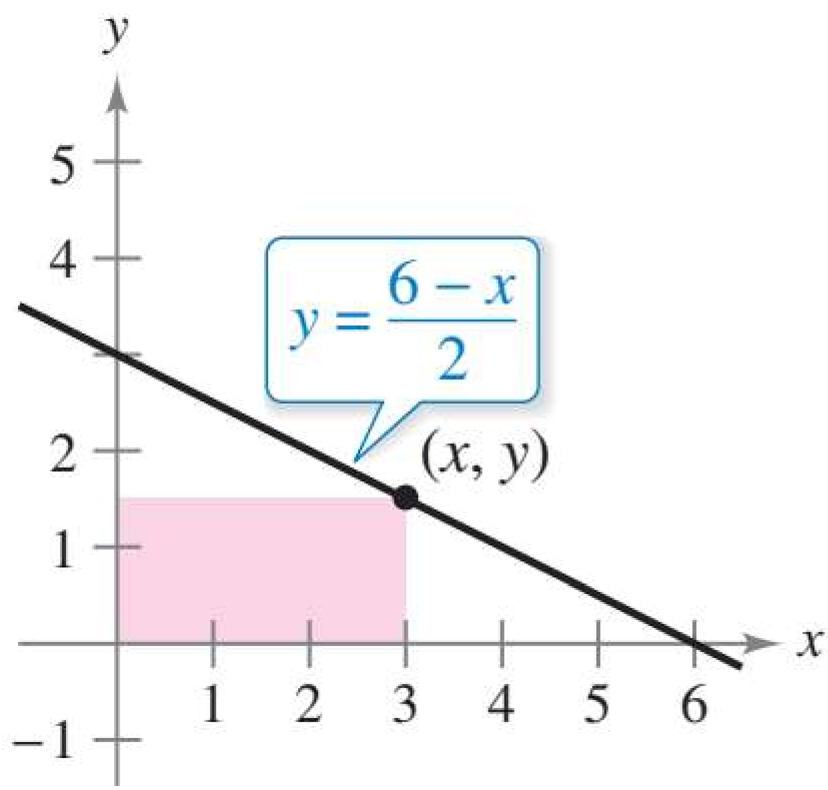
Find x such that $f(x) \Rightarrow$ max or min.

Some guidelines

1. identify variables and quantities
2. set up ***primary equation***, the equation for the quantity to optimize.
3. set up ***secondary equations***, equations that involve independent variables.

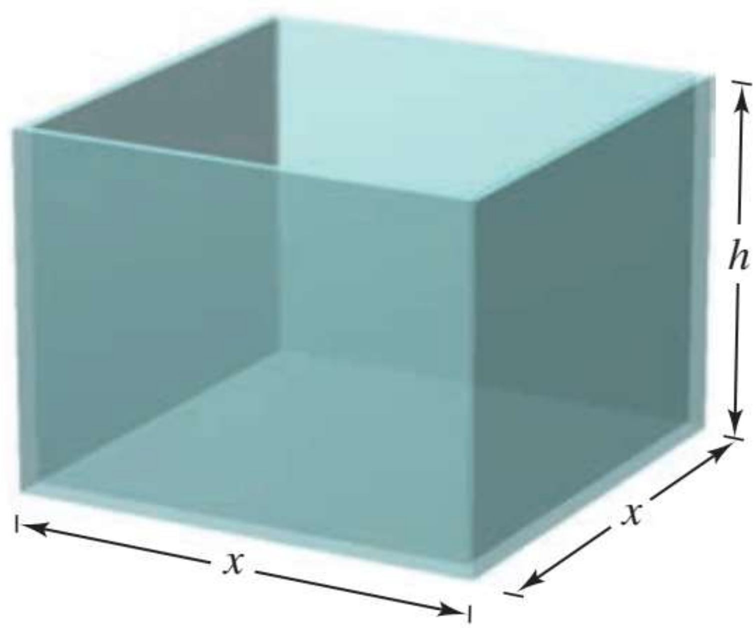
4. reduce independent variable into ***one*** from the secondary equations.
5. determine ***feasible domain***, domain for the variable in 4 so all quantities make sense.
6. use calculus technique to find the max/min.

EXAMPLE. A rectangle is bounded by the x- and y-axes and the graph of $y = (6 - x)/2$. What length and width should the rectangle have so that its area is a maximum?



EXAMPLE. Which points on the graph of $y = 4 - x^2$ are closest to the point $(0,2)$?

EXAMPLE. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches, as shown. What dimensions will produce a box with maximum volume?



EXAMPLE. Find the dimensions of the rectangle of maximum area, with sides parallel to the coordinate axes, that can be inscribed in the ellipse given by

$$\frac{x^2}{144} + \frac{y^2}{16} = 1.$$