Exercise 1

1. For each of the following equations, show that the given number a is an ordinary point.

   (a) $y'' + xy' + y = 0$, $a = 0$.

   (b) $y'' + x^2y' + xy = 0$, $a = 0$.

   (c) $y'' + x^2y = 0$, $a = 0$.

   (d) $(x^2 - 1)y'' - 6xy' + 12y = 0$, $a = 0$.

   (e) $y'' - 2(x + 3)y' - 3y = 0$, $a = -3$.

   (f) $(x^2 - 2x)y'' + 5(x - 1)y' + 3y = 0$, $a = 1$.

2. Find the power series solution about $a = 0$ for the following ODEs.

   (a) $(1 + x)y' = y$

   (b) $y'' - y' + xy = 0$

   (c) $y'' + (1 + x^2)y = 0$

   (d) $y' + 2y' + xy = 0$

3. Solve the following initial value problems.

   (a) $y'' + xy' + y = 0, \quad y(0) = 1, y'(0) = 0.$

   (b) $y'' + xy' - 2y = 0, \quad y(0) = 1, y'(0) = 0$