

Exercise 2

1. Solve the following equations.

(a) $(1 - x^2)y'' - 2xy' + 2y = 0.$

(b) $(1 - r^2)\phi'' - 2r\phi' + 12\phi = 0.$

2. Solve the equation

$$(4 - x^2)y'' - 2xy' + 6y = 0$$

by transforming the equation into the variable $z = x/2$. *Hint.* You may have to apply the chain rule and calculate y', y'' in terms of $dy/dz, d^2y/dz^2$.

3. Verify that for each of the following ODE, $a = 0$ is a regular singular point. Find the roots of the indicial equation. Then use the Frobenius method to solve the equation about $a = 0$. Use the larger root of the indicial equation.

(a) $xy'' + 2y' + xy = 0.$

(b) $xy'' + y = 0.$

(c) $xy'' + (2x + 1)y' + (x + 1)y = 0.$

(d) $4x^2y'' + 4xy' - y = 0.$