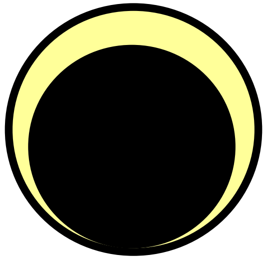
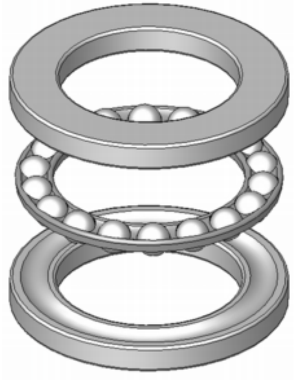
Bearing: A device that supports, guides, and reduces the friction of motion between fixed and moving machine parts.



Journal bearing

Ball bearing

Roller bearing

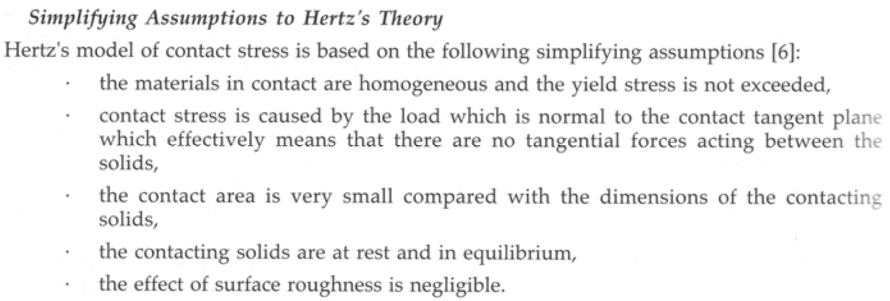


Thrust bearing

7.2-7.3 Contact stresses

Contact stress causes deformation, plastic or elastic. The contact area will change depending on the magnitude of the contact stress. Therefore, it is very imporant to calculate the actual stress at the point of contact, the so-called contact stress

By theory of elasticity, developed by Hertz, in 1881, when he was 23 years old!



Simplifying Assumptions to Hertz's Theory

Hertz's model of contact stress is based on the following simplifying assumptions [6]:

- the materials in contact are homogeneous and the yield stress is not exceeded,
- contact stress is caused by the load which is normal to the contact tangent plane which effectively means that there are no tangential forces acting between the solids,
- the contact area is very small compared with the dimensions of the contacting solids,
- the contacting solids are at rest and in equilibrium,
- the effect of surface roughness is negligible.

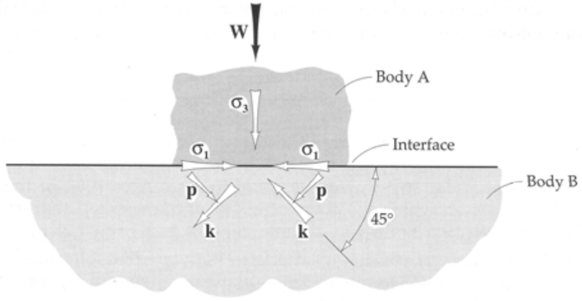


FIGURE 7.1 Stress status in a static contact; σ_1, σ_3 are the principal stresses, **p** is the hydrostatic pressure and **k** is the shear yield stress of the material.

In a static contact, the interface is the principal plane (no shear stress). Maximum stress stress occurs at 45 degree to the normal stress, and it can cause deformation by movement of dislocation.

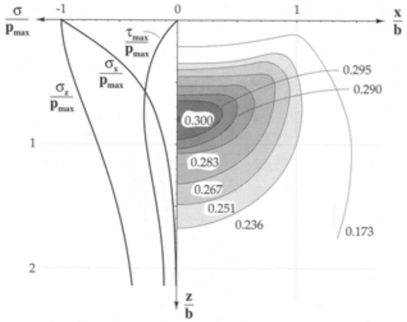
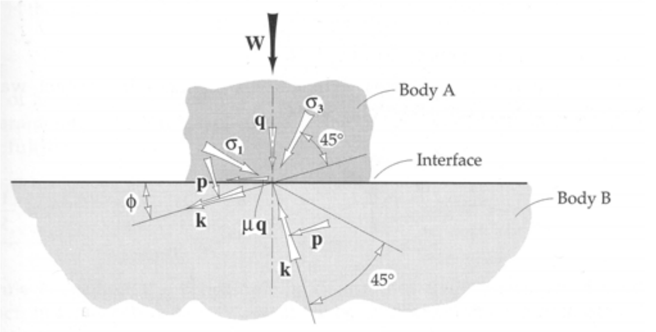


FIGURE 7.2 Subsurface stress field for two cylinders in static contact; $\mathbf{p_{max}}$ is the maximum contact pressure and **b** is the half width of the contact rectangle [9].

Below contacting surface, sigma x, sigma z, and shear stress varies with depth. Variation depends on the magnitude of the load. However, generally max shear stress occurs at about 0.6a, where a is radius of contact area.

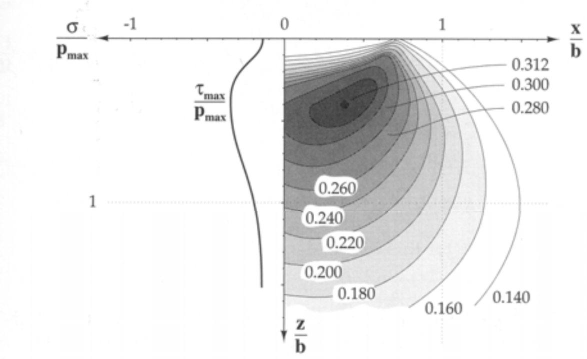
Contact stress under sliding. Sliding generates friction force which is tangential force at interface. Principal plan rotates by angle psi.

$$\phi = 1/2\cos^{-1}(\mu q/k)$$

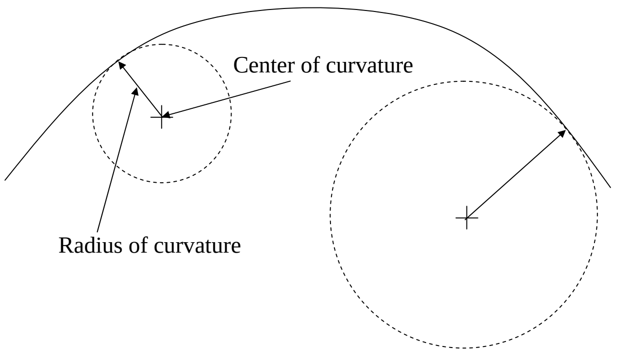


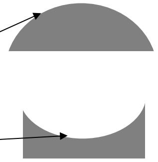
Stresses in a contact with sliding; σ_1, σ_3 are the principal stresses, **p** is the hydrostatic pressure, **k** is the shear yield stress of the material, μ is the coefficient of friction, **q** is the stress normal to the interface or compressive stress due to load, ϕ is the angle by which the planes of principal stress are rotated from the corresponding zero friction positions to balance the frictional stress.

Distribution of shear stress, in sliding contact, under contacting surface. The calculation is for friction coeff. of 0.2. To be noted is the location of maximum shear stress moves closer to the surface, comparing to the static contact.



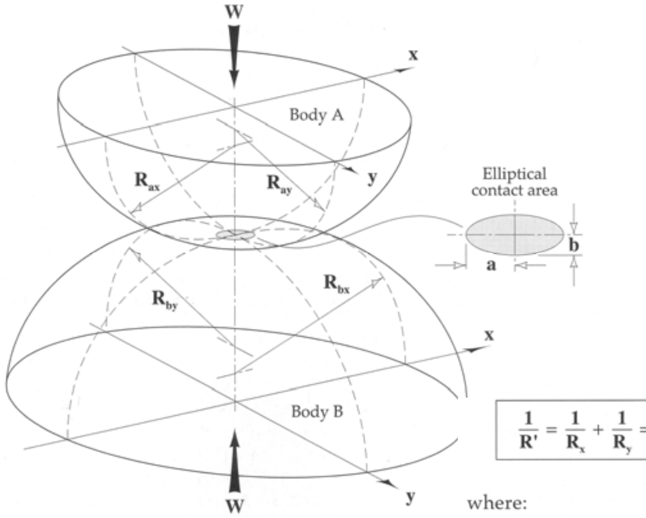
Subsurface stress field for a cylinder sliding on a plane; $\mathbf{p_{max}}$ is the maximum contact pressure and **b** is the half width of the contact rectangle [9].





Convex: Center of curvature is inside the material (positive curvature)
Concave: Center of curvature is outside the material (negative curvature)

Two elastic bodies with convex surface in contact.



$$\boxed{\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y} = \frac{1}{R_{ax}} + \frac{1}{R_{bx}} + \frac{1}{R_{ay}} + \frac{1}{R_{by}}}$$

where:

$$\frac{1}{R_x} = \frac{1}{R_{ax}} + \frac{1}{R_{bx}}$$

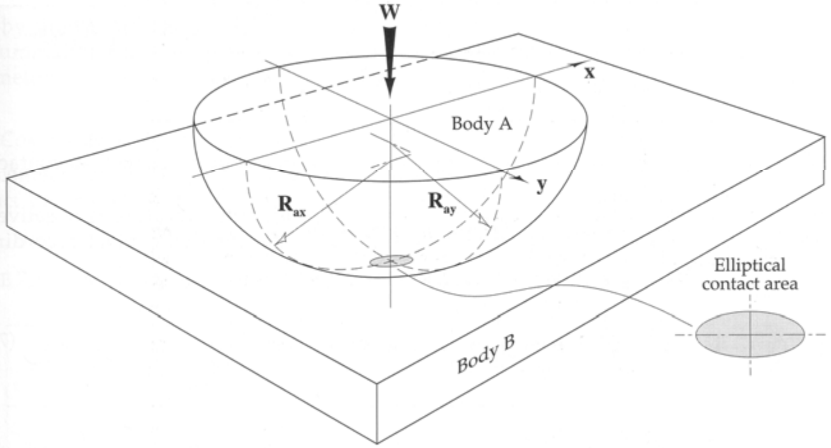
$$\frac{1}{R_y} = \frac{1}{R_{ay}} + \frac{1}{R_{by}}$$

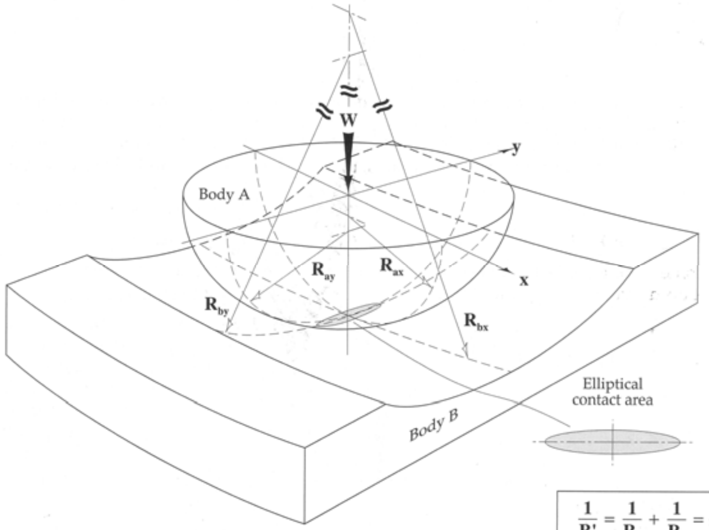
R = Radius of curvature
R’= Reduced radius of curvature

Reduced radius of curvature:

$$\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y} = \frac{1}{R_{ax}} + \frac{1}{R_{bx}} + \frac{1}{R_{ay}} + \frac{1}{R_{by}}$$

$$R_{bx} = R_{by} = \infty$$

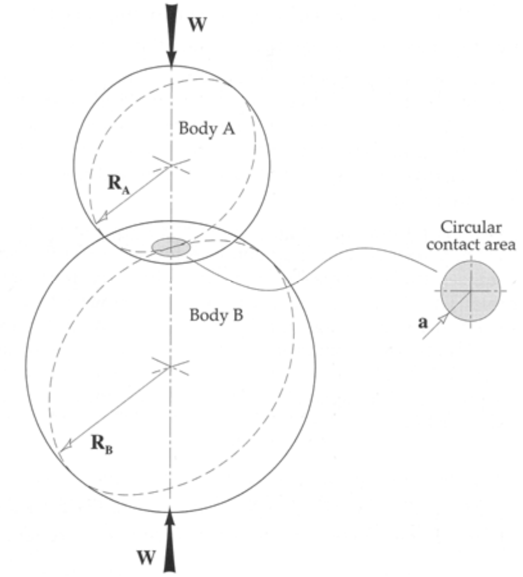




$$\boxed{\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y} = \frac{1}{R_{ax}} - \frac{1}{R_{bx}} + \frac{1}{R_{ay}} - \frac{1}{R_{by}}}$$

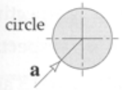
or

$$\boxed{\frac{1}{R'} = \left(\frac{1}{R_{ax}} + \frac{1}{R_{ay}}\right) - \left(\frac{1}{R_{bx}} + \frac{1}{R_{by}}\right)}$$



$$\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y} = \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_A} + \frac{1}{R_B} = 2\left(\frac{1}{R_A} + \frac{1}{R_B}\right)$$

TABLE 7.1 Formulae for contact parameters between two spheres.

Contact area dimensions	Maximum contact pressure	Average contact pressure	Maximum deflection	Maximum shear stress
$a = \left(\frac{3WR'}{E'}\right)^{1/3}$ circle 	$p_{max} = \frac{3W}{2\pi a^2}$ Hemispherical pressure distribution	$p_{average} = \frac{W}{\pi a^2}$	$\delta = 1.0397\left(\frac{W^2}{E'^2R'}\right)^{1/3}$	$\tau_{max} = \frac{1}{3}p_{max}$ at a depth of $z = 0.638a$

where:

a	is the radius of the contact area [m];
W	is the normal load [N];
p	is the contact pressure (Hertzian stress) [Pa];
δ	is the total deflection at the centre of the contact (i.e., $\delta = \delta_A + \delta_B$; where '$\delta_A$' and '$\delta_B$' are the maximum deflections of body '**A**' and '**B**', respectively) [m];
τ	is the shear stress [Pa];
z	is the depth under the surface where the maximum shear stress acts [m];
E'	is the reduced Young's modulus [Pa];
R'	is the reduced radius of curvature [m].

$$\frac{1}{E'} = \frac{1}{2}\left[\frac{1-\upsilon_A^2}{E_A} + \frac{1-\upsilon_B^2}{E_B}\right]$$

EXAMPLE

Find the contact parameters for two steel balls. The normal force is W = 5 [N], the radii of the balls are $R_A = 10 \times 10^{-3}$ [m] and $R_B = 15 \times 10^{-3}$ [m]. The Young's modulus for both balls is $E = 2.1 \times 10^{11}$ [Pa] and the Poisson's ratio of steel is $\upsilon = 0.3$.

- *Reduced Radius of Curvature*

Since $R_{ax} = R_{ay} = R_A = 10 \times 10^{-3}$ [m] and $R_{bx} = R_{by} = R_B = 15 \times 10^{-3}$ [m] the reduced radii of curvature in the '**x**' and '**y**' directions are:

$$\frac{1}{R_x} = \frac{1}{R_{ax}} + \frac{1}{R_{bx}} = \frac{1}{10 \times 10^{-3}} + \frac{1}{15 \times 10^{-3}} = 166.67 \qquad \boxed{\Rightarrow R_x = 6 \times 10^{-3}\,[\mathrm{m}]}$$

$$\frac{1}{R_y} = \frac{1}{R_{ay}} + \frac{1}{R_{by}} = \frac{1}{10 \times 10^{-3}} + \frac{1}{15 \times 10^{-3}} = 166.67 \qquad \boxed{\Rightarrow R_y = 6 \times 10^{-3}\,[\mathrm{m}]}$$

Note that $\mathbf{1/R_x = 1/R_y}$, i.e., condition (7.3) is satisfied (circular contact), and the reduced radius of curvature is:

$$\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y} = 166.67 + 166.67 = 333.34 \qquad \boxed{\Rightarrow R' = 3 \times 10^{-3}\,[\mathrm{m}]}$$

- *Reduced Young's Modulus*

$$\frac{1}{E'} = \frac{1}{2}\left[\frac{1-\upsilon_A^2}{E_A} + \frac{1-\upsilon_B^2}{E_B}\right] = \frac{1}{2}\left[\frac{1-0.3^2}{2.1 \times 10^{11}} + \frac{1-0.3^2}{2.1 \times 10^{11}}\right] \qquad \boxed{\Rightarrow E' = 2.308 \times 10^{11}\,[\mathrm{Pa}]}$$

- *Contact Area Dimensions*

$$a = \left(\frac{3WR'}{E'}\right)^{1/3} = \left(\frac{3 \times 5 \times (3 \times 10^{-3})}{2.308 \times 10^{11}}\right)^{1/3} \qquad \boxed{= 5.799 \times 10^{-5}\,[\mathrm{m}]}$$

- *Maximum and Average Contact Pressures*

$$p_{max} = \frac{3W}{2\pi a^2} = \frac{3 \times 5}{2\pi(5.799 \times 10^{-5})^2} \qquad \boxed{= 709.9\,[\mathrm{MPa}]}$$

$$p_{average} = \frac{W}{\pi a^2} = \frac{5}{\pi(5.799 \times 10^{-5})^2} \qquad \boxed{= 473.3\,[\mathrm{MPa}]}$$

- *Maximum Deflection*

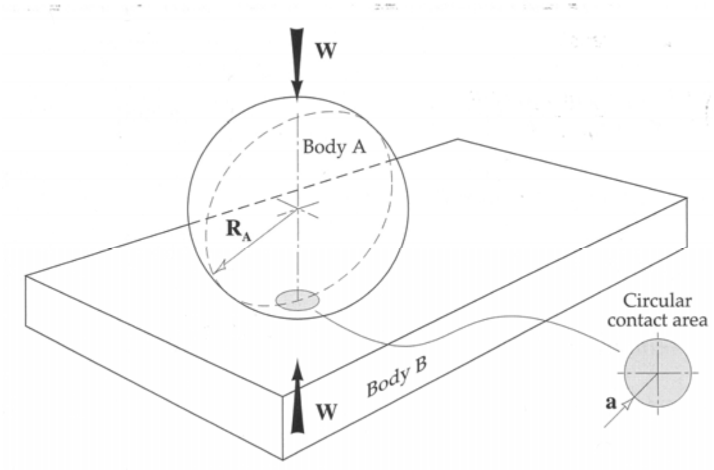
$$\delta = 1.0397\left(\frac{W^2}{E'^2R'}\right)^{1/3} = 1.0397\left(\frac{5^2}{(2.308 \times 10^{11})^2 3 \times 10^{-3}}\right)^{1/3} \qquad \boxed{= 5.6 \times 10^{-7}\,[\mathrm{m}]}$$

- *Maximum Shear Stress*

$$\tau_{max} = \frac{1}{3}p_{max} = \frac{1}{3}709.9 \qquad \boxed{= 236.6\,[\mathrm{MPa}]}$$

- *Depth at which Maximum Shear Stress Occurs*

$$z = 0.638a = 0.638 \times (5.799 \times 10^{-5}) \qquad \boxed{= 3.7 \times 10^{-5}\,[\mathrm{m}]}$$



Contact between a sphere and a flat surface.

$$\boxed{\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y} = \frac{1}{R_A} + \frac{1}{\infty} + \frac{1}{R_A} + \frac{1}{\infty} = \frac{2}{R_A}}$$

EXAMPLE

Find the contact parameters for a steel ball on a flat steel plate. The normal force is W = 5 [N], the radius of the ball is $R_A = 10 \times 10^{-3}$ [m], the Young's modulus for ball and plate is $E = 2.1 \times 10^{11}$ [Pa] and the Poisson's ratio is $\upsilon = 0.3$.

- *Reduced Radius of Curvature*

Since the radii of the ball and the plate are $R_{ax} = R_{ay} = 10 \times 10^{-3}$ [m] and $R_{bx} = R_{by} = \infty$ [m], respectively, the reduced radii of curvature in '**x**' and '**y**' directions are:

$$\frac{1}{R_x} = \frac{1}{R_{ax}} + \frac{1}{R_{bx}} = \frac{1}{10 \times 10^{-3}} + \frac{1}{\infty} = 100 \qquad \boxed{\Rightarrow R_x = 0.01\,[\mathrm{m}]}$$

$$\frac{1}{R_y} = \frac{1}{R_{ay}} + \frac{1}{R_{by}} = \frac{1}{10 \times 10^{-3}} + \frac{1}{\infty} = 100 \qquad \boxed{\Rightarrow R_y = 0.01\,[\mathrm{m}]}$$

Condition (7.3), i.e., $\mathbf{1/R_x = 1/R_y}$ is satisfied (circular contact), and the reduced radius of curvature is:

$$\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y} = 100 + 100 = 200 \qquad \boxed{\Rightarrow R' = 5 \times 10^{-3}\,[\mathrm{m}]}$$

- *Reduced Young's Modulus*

$$\boxed{E' = 2.308 \times 10^{11}\,[\mathrm{Pa}]}$$

- *Contact Area Dimensions*

$$a = \left(\frac{3WR'}{E'}\right)^{1/3} = \left(\frac{3 \times 5 \times (5 \times 10^{-3})}{2.308 \times 10^{11}}\right)^{1/3} \qquad \boxed{= 6.88 \times 10^{-5}\,[\mathrm{m}]}$$

- *Maximum and Average Contact Pressures*

$$p_{max} = \frac{3W}{2\pi a^2} = \frac{3 \times 5}{2\pi(6.88 \times 10^{-5})^2} \qquad \boxed{= 504.4\,[\mathrm{MPa}]}$$

$$p_{average} = \frac{W}{\pi a^2} = \frac{5}{\pi(6.88 \times 10^{-5})^2} \qquad \boxed{= 336.2\,[\mathrm{MPa}]}$$

- *Maximum Deflection*

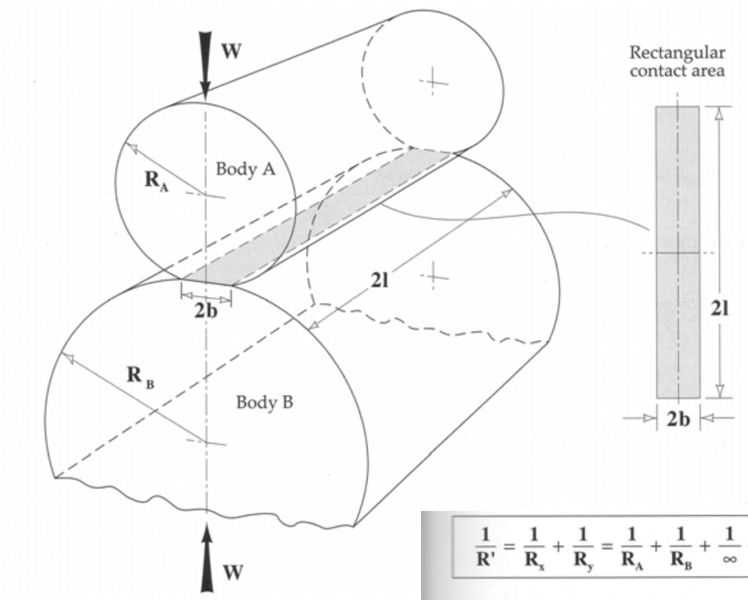
$$\delta = 1.0397\left(\frac{W^2}{E'^2 R'}\right)^{1/3} = 1.0397\left(\frac{5^2}{(2.308 \times 10^{11})^2 5 \times 10^{-3}}\right)^{1/3} \qquad \boxed{= 4.7 \times 10^{-7}\,[\mathrm{m}]}$$

- *Maximum Shear Stress*

$$\tau_{max} = \frac{1}{3} p_{max} = \frac{1}{3} 504.4 \qquad \boxed{= 168.1\,[\mathrm{MPa}]}$$

- *Depth at which Maximum Shear Stress Occurs*

$$z = 0.638a = 0.638 \times (6.88 \times 10^{-5}) \qquad \boxed{= 4.4 \times 10^{-5}\,[\mathrm{m}]}$$

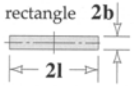


$$\boxed{\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y} = \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{\infty} + \frac{1}{\infty} = \frac{1}{R_A} + \frac{1}{R_B}}$$

where:

$$\frac{1}{R_x} = \frac{1}{R_A} + \frac{1}{R_B} \quad \text{and} \quad \frac{1}{R_y} = 0$$

TABLE 7.2 Formulae for contact parameters between two parallel cylinders.

Contact area dimensions	Maximum contact pressure	Average contact pressure	Maximum deflection	Maximum shear stress
$\mathbf{b} = \left(\frac{4WR'}{\pi l E'}\right)^{1/2}$ rectangle 	$p_{max} = \frac{W}{\pi b l}$ Elliptical pressure distribution	$p_{average} = \frac{W}{4bl}$	$\delta = 0.319\left(\frac{W}{E'l}\right) \times \left[\frac{2}{3} + \ln\left(\frac{4R_AR_B}{b^2}\right)\right]$	$\tau_{max} = 0.304\,p_{max}$ at a depth of $z = 0.786\,b$

EXAMPLE

Find the contact parameters for two parallel steel rollers. The normal force is $W = 5$ [N], radii of the rollers are $R_A = 10 \times 10^{-3}$ [m] and $R_B = 15 \times 10^{-3}$ [m], Young's modulus for both rollers is $E = 2.1 \times 10^{11}$ [Pa] and the Poisson's ratio is $\upsilon = 0.3$. The length of both rollers is $2l = 10 \times 10^{-3}$ [m].

- *Reduced Radius of Curvature*

Since the radii of the cylinders are $R_{ax} = R_A = 10 \times 10^{-3}$ [m], $R_{ay} = \infty$ and $R_{bx} = R_B = 15 \times 10^{-3}$ [m], $R_{by} = \infty$, respectively, the reduced radii of curvature in the 'x' and 'y' directions are:

$$\frac{1}{R_x} = \frac{1}{R_{ax}} + \frac{1}{R_{bx}} = \frac{1}{10 \times 10^{-3}} + \frac{1}{15 \times 10^{-3}} = 166.67 \qquad \boxed{\Rightarrow R_x = 6 \times 10^{-3}\,[\mathrm{m}]}$$

$$\frac{1}{R_y} = \frac{1}{R_{ay}} + \frac{1}{R_{by}} = \frac{1}{\infty} + \frac{1}{\infty} = 0 \qquad \boxed{\Rightarrow R_y = \infty\,[\mathrm{m}]}$$

Since $1/R_x > 1/R_y$ condition (7.3) is satisfied and the reduced radius of curvature is:

$$\frac{1}{R'} = \frac{1}{R_x} = 166.67 \qquad \boxed{\Rightarrow R' = 6 \times 10^{-3}\,[\mathrm{m}]}$$

- *Reduced Young's Modulus*

$$\boxed{E' = 2.308 \times 10^{11}\,[\mathrm{Pa}]}$$

- *Contact Area Dimensions*

$$b = \left(\frac{4WR'}{\pi l E'}\right)^{1/2} = \left(\frac{4 \times 5 \times (6 \times 10^{-3})}{\pi \times (5 \times 10^{-3}) \times (2.308 \times 10^{11})}\right)^{1/2} \qquad \boxed{= 5.75 \times 10^{-6}\,[\mathrm{m}]}$$

- *Maximum and Average Contact Pressures*

$$p_{max} = \frac{W}{\pi b l} = \frac{5}{\pi \times (5.75 \times 10^{-6}) \times (5 \times 10^{-3})} \qquad \boxed{= 55.4\,[\mathrm{MPa}]}$$

$$p_{average} = \frac{W}{4bl} = \frac{5}{4 \times (5.75 \times 10^{-6}) \times (5 \times 10^{-3})} \qquad \boxed{= 43.5\,[\mathrm{MPa}]}$$

- *Maximum Deflection*

$$\delta = 0.319\left[\frac{W}{E'l}\right]\left[\frac{2}{3} + \ln\left(\frac{4R_AR_B}{b^2}\right)\right]$$

$$= 0.319\left[\frac{5}{(2.308 \times 10^{11}) \times (5 \times 10^{-3})}\right]\left[\frac{2}{3} + \ln\left(\frac{4 \times (10 \times 10^{-3}) \times (15 \times 10^{-3})}{(5.75 \times 10^{-6})^2}\right)\right]$$

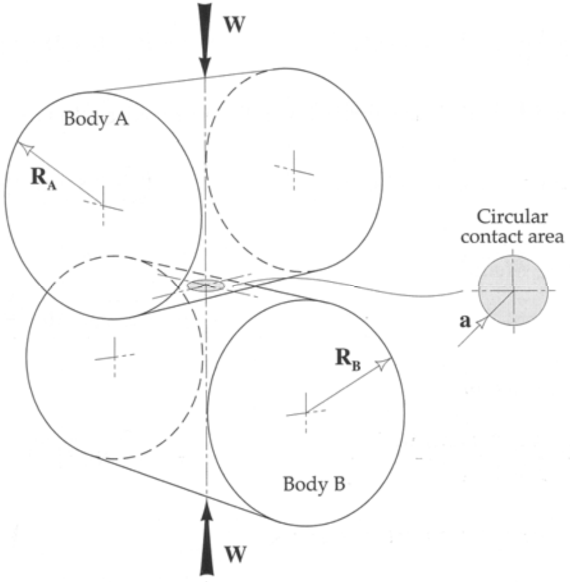
$$\boxed{= 2.40 \times 10^{-8}\,[\mathrm{m}]}$$

- *Maximum Shear Stress*

$$\tau_{max} = 0.304\,p_{max} = 0.304 \times 55.4 \qquad \boxed{= 16.8\,[\mathrm{MPa}]}$$

- *Depth at which Maximum Shear Stress Occurs*

$$z = 0.786\,b = 0.786 \times (5.75 \times 10^{-6}) \qquad \boxed{= 4.5 \times 10^{-6}\,[\mathrm{m}]}$$



$$\boxed{\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y} = \frac{1}{\infty} + \frac{1}{R_B} + \frac{1}{R_A} + \frac{1}{\infty} = \frac{2}{R_A}}$$

EXAMPLE

Find the contact parameters for two steel wires of the same diameter crossed at 90°. This configuration is often used in fretting wear studies. The normal force is $W = 5$ [N], radii of the wires are $R_A = R_B = 1.5 \times 10^{-3}$ [m], the Young's modulus for both wires is $E = 2.1 \times 10^{11}$ [Pa] and the Poisson's ratio is $\upsilon = 0.3$.

- *Reduced Radius of Curvature*

Since the radii of the wires are $R_{ax} = \infty$, $R_{ay} = R_A = 1.5 \times 10^{-3}$ [m], and $R_{bx} = R_B = 1.5 \times 10^{-3}$ [m], $R_{by} = \infty$, respectively, the reduced radii of curvature in the '**x**' and '**y**' directions are:

$$\frac{1}{R_x} = \frac{1}{R_{ax}} + \frac{1}{R_{bx}} = \frac{1}{\infty} + \frac{1}{1.5 \times 10^{-3}} = 666.67 \qquad \boxed{\Rightarrow R_x = 0.0015\,[\mathrm{m}]}$$

$$\frac{1}{R_y} = \frac{1}{R_{ay}} + \frac{1}{R_{by}} = \frac{1}{1.5 \times 10^{-3}} + \frac{1}{\infty} = 666.67 \qquad \boxed{\Rightarrow R_y = 0.0015\,[\mathrm{m}]}$$

Since $\mathbf{1/R_x = 1/R_y}$ condition (7.3) is satisfied and the reduced radius of curvature is:

$$\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y} = 666.67 + 666.67 = 1333.34 \qquad \boxed{\Rightarrow R' = 7.5 \times 10^{-4}\,[\mathrm{m}]}$$

- *Reduced Young's Modulus*

$$\boxed{E' = 2.308 \times 10^{11}\,[\mathrm{Pa}]}$$

- *Contact Area Dimensions*

$$a = \left(\frac{3WR'}{E'}\right)^{1/3} = \left(\frac{3 \times 5 \times (7.5 \times 10^{-4})}{2.308 \times 10^{11}}\right)^{1/3} \qquad \boxed{= 3.65 \times 10^{-5}\,[\mathrm{m}]}$$

- *Maximum and Average Contact Pressures*

$$p_{max} = \frac{3W}{2\pi a^2} = \frac{3 \times 5}{2\pi(3.65 \times 10^{-5})^2} \qquad \boxed{= 1791.9\,[\mathrm{MPa}]}$$

$$p_{average} = \frac{W}{\pi a^2} = \frac{5}{\pi(3.65 \times 10^{-5})^2} \qquad \boxed{= 1194.6\,[\mathrm{MPa}]}$$

- *Maximum Deflection*

$$\delta = 1.0397\left(\frac{W^2}{E'^2 R'}\right)^{1/3}$$

$$= 1.0397\left(\frac{5^2}{(2.308 \times 10^{11})^2 \times (7.5 \times 10^{-4})}\right)^{1/3} \qquad \boxed{= 8.9 \times 10^{-7}\,[\mathrm{m}]}$$

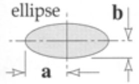
- *Maximum Shear Stress*

$$\tau_{max} = \frac{1}{3} p_{max} = \frac{1}{3} 1791.9 \qquad \boxed{= 597.3\,[\mathrm{MPa}]}$$

- *Depth at which Maximum Shear Stress Occurs*

$$z = 0.638a = 0.638 \times (3.65 \times 10^{-5}) \qquad \boxed{= 2.3 \times 10^{-5}\,[\mathrm{m}]}$$

TABLE 7.4 Approximate formulae for contact parameters between two elastic bodies [7].

Contact area dimensions	Maximum contact pressure	Maximum deflection	Simplified elliptical integrals
$a = \left(\frac{6\bar{k}^2\bar{\varepsilon}WR'}{\pi E'}\right)^{1/3}$ $b = \left(\frac{6\bar{\varepsilon}WR'}{\pi\bar{k}E'}\right)^{1/3}$	$p_{max} = \frac{3W}{2\pi ab}$	$\delta = \bar{\xi}\left[\left(\frac{4.5}{\bar{\varepsilon}R'}\right)\left(\frac{W}{\pi\bar{k}E'}\right)^2\right]^{1/3}$	$\bar{\varepsilon} = 1.0003 + \frac{0.5968R_x}{R_y}$ $\bar{\xi} = 1.5277 + 0.6023\ln\left(\frac{R_y}{R_x}\right)$
ellipse 	Average contact pressure		Ellipticity parameter
	$p_{average} = \frac{W}{\pi ab}$		$\bar{k} = 1.0339\left(\frac{R_y}{R_x}\right)^{0.636}$

EXAMPLE

Find the contact parameters for a steel ball in contact with a groove on the inside of a steel ring (as shown in Figure 7.7). The normal force is W = 50 [N], radius of the ball is $R_{ax} = R_{ay} = R_A = 15 \times 10^{-3}$ [m], the radius of the groove is $R_{bx} = 30 \times 10^{-3}$ [m] and the radius of the ring is $R_{by} = 60 \times 10^{-3}$ [m]. The Young's modulus for both ball and ring is $E = 2.1 \times 10^{11}$ [Pa] and the Poisson's ratio is $\upsilon = 0.3$.

- *Reduced Radius of Curvature*

Since the radii of the ball and the grooved ring are $R_{ax} = 15 \times 10^{-3}$ [m], $R_{ay} = 15 \times 10^{-3}$ [m] and $R_{bx} = -30 \times 10^{-3}$ [m] (concave surface), $R_{by} = -60 \times 10^{-3}$ [m] (concave surface), respectively, the reduced radii of curvature in the '**x**' and '**y**' directions are:

$$\frac{1}{R_x} = \frac{1}{R_{ax}} + \frac{1}{R_{bx}} = \frac{1}{15 \times 10^{-3}} + \frac{1}{-30 \times 10^{-3}} = 33.33 \qquad \boxed{\Rightarrow R_x = 0.03\,[\mathrm{m}]}$$

$$\frac{1}{R_y} = \frac{1}{R_{ay}} + \frac{1}{R_{by}} = \frac{1}{15 \times 10^{-3}} + \frac{1}{-60 \times 10^{-3}} = 50.0 \qquad \boxed{\Rightarrow R_y = 0.02\,[\mathrm{m}]}$$

Since $\mathbf{1/R_x < 1/R_y}$ condition (7.3) is not satisfied. According to the convention it is necessary to transpose the directions of the coordinates, so '$\mathbf{R_x}$' and '$\mathbf{R_y}$' become:

$$R_x = 0.02\,[\mathrm{m}] \quad \text{and} \quad R_y = 0.03\,[\mathrm{m}]$$

and the reduced radius of curvature is:

$$\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y} = 50.0 + 33.33 = 83.33 \qquad \boxed{\Rightarrow R' = 0.012\,[\mathrm{m}]}$$

- *Reduced Young's Modulus*

$$\boxed{E' = 2.308 \times 10^{11}\,[\mathrm{Pa}]}$$

- *Simplified Elliptical Integrals*

$$\bar{\varepsilon} = 1.0003 + \frac{0.5968\,R_x}{R_y} = 1.0003 + \frac{0.5968 \times 0.02}{0.03} \qquad \boxed{= 1.3982}$$

$$\bar{\xi} = 1.5277 + 0.6023 \ln\left(\frac{R_y}{R_x}\right) = 1.5277 + 0.6023 \ln\left(\frac{0.03}{0.02}\right) \qquad \boxed{= 1.7719}$$

- *Contact Area Dimensions*

$$a = \left(\frac{6\bar{k}^2\bar{\varepsilon}WR'}{\pi E'}\right)^{1/3} = \left(\frac{6 \times 1.3380^2 \times 1.3982 \times 50 \times 0.012}{\pi(2.308 \times 10^{11})}\right)^{1/3} \qquad \boxed{= 2.32 \times 10^{-4}\,[\mathrm{m}]}$$

$$b = \left(\frac{6\bar{\varepsilon}WR'}{\pi\bar{k}E'}\right)^{1/3} = \left(\frac{6 \times 1.3982 \times 50 \times 0.012}{\pi \times 1.3380 \times (2.308 \times 10^{11})}\right)^{1/3} \qquad \boxed{= 1.73 \times 10^{-4}\,[\mathrm{m}]}$$

- *Maximum and Average Contact Pressures*

$$p_{max} = \frac{3W}{2\pi ab} = \frac{3 \times 50}{2\pi(2.32 \times 10^{-4}) \times (1.73 \times 10^{-4})} \qquad \boxed{= 594.8\,[\mathrm{MPa}]}$$

$$p_{average} = \frac{W}{\pi ab} = \frac{50}{\pi(2.32 \times 10^{-4}) \times (1.73 \times 10^{-4})} \qquad \boxed{= 396.5\,[\mathrm{MPa}]}$$

- *Maximum Deflection*

$$\delta = \bar{\xi}\left[\left(\frac{4.5}{\bar{\varepsilon}R'}\right)\left(\frac{W}{\pi\bar{k}E'}\right)^2\right]^{1/3}$$

$$= 1.7719\left[\left(\frac{4.5}{1.3982 \times 0.012}\right)\left(\frac{50}{\pi 1.3380 \times (2.308 \times 10^{11})}\right)^2\right]^{1/3} \qquad \boxed{= 1.6 \times 10^{-6}\,[\mathrm{m}]}$$