4. Distributed Loads

2142111 Statics, 2011/2

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Objectives

Students must be able to

Course Objective
- Include distributed loads into equilibrium analyses

Chapter Objectives
- Describe the characteristics and determine the centroids, centers of mass and centers of gravity by integration and composite body methods
- Apply the Pappus Theorems for surface and volume of revolution
- Describe the characteristics and determine the first moment of area, second moment of area and polar moment of inertia by integration, parallel-axis theorem and perpendicular-axis theorem
- Determine the resultant of loads (force/couple) with line, area and volume distribution by integration and area/volume analogy
Objectives Students must be able to #2

- Analyze bodies/structures with distributed loads for unknown loads/reactions by appropriate FBDs

For fluid statics
- Describe the characteristics and determine hydrostatic and aerostatic pressures as distributed loads
- Determine the resultant of fluid statics by integration, volume analogy and block-of-fluid methods
- Describe and determine the buoyancy and stability of floating bodies
- Analyze bodies/structures with fluid statics for unknown loads/reactions by appropriate FBDs
Objectives Students must be able to #3

For flexible cables

- State the assumptions and geometrical definitions of flexible cables
- Appropriately approximate real-life cables into parabolic or catenary cables by load distribution
- Prove and apply profile, length and tension formula for parabolic & catenary cables
- Identify and utilize techniques for obtaining numerical solutions of parabolic & catenary cables
Contents

- Centroid, Center of Mass and Center of Gravity
- Pappus Theorems
- First Moment of Area, Moment of Inertia, Polar Moment of Inertia
- Distributed Loads
- Fluid Statics
- Flexible Cables
Software Helps

- 3M Software
  - Maple – Computer Center
  - MatLab – Computer Center
  - Mathematica

- Maple
  - Select Maple in the ‘Start Menu’
  - Type in commands, then ‘Enter’
  - Use ‘Help Menu’ for command templates
Centers of Gravity

\[ F = \bar{x} \left( \frac{x_1 + x_2 + x_3 + x_4}{4} \right) \]
Centers of Gravity, CG

- Weight of a body can be represented by an equivalent force acting at its center of gravity $G$.
- Assume a uniform and parallel force field due to gravitational attraction for most problems.
- Weight $W = mg$ where $m$ is the mass of the body and $g$ is the magnitude of gravitational acceleration.
- The center of gravity is a unique point which is a function of weight distribution only.
Center

**CG Principle of Moments**

\[ M_y = \sum_{i=1}^{N} \tilde{x}_i dW_i = \bar{x}W \]

\[ \bar{x} = \frac{\sum_{i=1}^{N} \tilde{x}_i dW_i}{W} \]

If \( dW_i \to 0 \):

\[ \bar{x} = \frac{\int \tilde{x} \, dW}{W}, \quad \bar{y} = \frac{\int \tilde{y} \, dW}{W} \]
\[ W = W_1 + W_2 \]
\[ W\bar{x} = W_1\tilde{x}_1 + W_2\tilde{x}_2 \]
\[ W\bar{y} = W_1\tilde{y}_1 + W_2\tilde{y}_2 \]
Center CG Composite Bodies #2

\[ W = W_1 - W_2 \]
\[ W\bar{x} = W_1\bar{x}_1 - W_2\bar{x}_2 \]
\[ W\bar{y} = W_1\bar{y}_1 - W_2\bar{y}_2 \]
Centers of Mass, CM

- An object’s distribution of mass can be represented by an equivalent mass acting at its center of mass.
- The center of mass of is a unique point which is a function solely of mass distribution.
- Centers of mass coincides with $G$ as long as the gravity field is treated as uniform and parallel.

\[
\bar{x} = \frac{\int x\,dm}{m} \\
\bar{y} = \frac{\int y\,dm}{m}
\]
Centroids

- If the density $\rho$ is constant and and gravity field is uniform and parallel, $G$ and center of mass coincide with the centroid of the body.
- The centroid $C$ is the geometrical center or the weighted average position of an object.
- Locating the centroid by averaging the ‘moments’ of elements of objects about axes.
- The centroid lies on the axis of symmetry.
- Geometry of the body is the only factor that influence the position of the centroid.
Centroids Formula

For a line of length $L$

$$\bar{x} = \frac{\int \tilde{x} \, dL}{L}, \quad \bar{y} = \frac{\int \tilde{y} \, dL}{L}, \quad \ldots$$

For a surface with area $A$

$$\bar{x} = \frac{\int \tilde{x} \, dA}{A}, \quad \bar{y} = \frac{\int \tilde{y} \, dA}{A}, \quad \ldots$$

For a body of volume $V$

$$\bar{x} = \frac{\int \tilde{x} \, dV}{V}, \quad \bar{y} = \frac{\int \tilde{y} \, dV}{V}, \quad \ldots$$
If a body has an axis of symmetry, its centroid lies on this axis.
Find centroid C of area A

\[ x = cy^2 \quad \rightarrow \quad y = \left( \frac{x}{c} \right)^{1/2} \]

at \( x = a, \ y = b \) \quad \rightarrow \quad a = cb^2, \ c = \frac{a}{b^2} \]

\[ dA = y \ dx = \left( \frac{x}{c} \right)^{1/2} \ dx \]

\[ A = \int_{A} dA = \int_{0}^{a} \left( \frac{x}{c} \right)^{1/2} \ dx = \frac{2}{3} \frac{a^{3/2}}{c^{1/2}} = \frac{2}{3} \ ab \]
Find $\bar{y}$ by moment of area about $x$ axis

$$A\bar{y} = \int_A \bar{y} \, dA = \int_0^a \frac{y}{2} (y \, dx)$$

$$= \frac{1}{2} \int_0^a y^2 \, dx = \frac{1}{2c} \int_0^a x \, dx$$

$$= \frac{1}{2c} \left[ \frac{x^2}{2} \right]^a_0 = \frac{b^2}{2a} \left[ \frac{1}{2} x^2 \right]^a_0 = \frac{ab^2}{4}$$

$$\bar{y} = \frac{ab^2}{4A} = \frac{ab^2}{4} \cdot \frac{3}{2ab} = \frac{3b}{8}$$

Ans
Example  Centroids 1 #3

Find $\bar{x}$ by moment of area about $y$ axis

$$A\bar{x} = \int_A x \, dA = \int_0^a x \left( \frac{x}{c} \right)^{1/2} \, dx$$

$$= \frac{1}{c^{1/2}} \int_0^a x^{3/2} \, dx = \frac{2}{5c^{1/2}} \left[ x^{5/2} \right]_0^a$$

$$= \frac{2a^{5/2}}{5c^{1/2}} = \frac{2a^2 b}{5}$$

$$\bar{x} = \frac{2a^2 b}{5A} = \frac{2a^2 b}{5} \frac{3}{2ab} = \frac{3a}{5} \quad \text{Ans}$$
Example Centroids 2 #1

Find centroid $C$ of line $L$, given $a = b = 100$ mm

$x = cy^2 \quad \rightarrow \quad \frac{dx}{dy} = 2cy$

d$L$ = $\sqrt{dx^2 + dy^2}$

$= \sqrt{(dx/dy)^2 + 1} \ dy$

$L = \int_L dL = \int_0^b \sqrt{1 + (dx/dy)^2} \ dy$

$= \int_0^b \sqrt{1 + (2cy)^2} \ dy$

$= 147.9$ mm
Example Centroids 2 #2

Find $\bar{y}$ by taking moment of line about $x$ axis

\[
L\bar{y} = \int_{L} \bar{y} \, dL = \int_{0}^{b} y \sqrt{1 + (2cy)^2} \, dy
\]

$L\bar{y} = 8483.6 \, \text{mm}^2 \rightarrow \bar{y} = 57.4 \, \text{mm} \quad \text{Ans}

Find $\bar{x}$ by taking moment of line about $y$ axis

\[
L\bar{x} = \int_{L} \bar{x} \, dL = \int_{0}^{b} cy^2 \sqrt{1 + (2cy)^2} \, dy
\]

$L\bar{x} = 6063.4 \, \text{mm}^2 \rightarrow \bar{x} = 41.0 \, \text{mm} \quad \text{Ans}
Example  Centroids 2 #3

\[ \int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[ x \sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2}) \right] \]

\[ \int x \sqrt{x^2 \pm a^2} \, dx = \frac{1}{3} \left[ (x^2 \pm a^2)^{3/2} \right] \]

\[ \int x^2 \sqrt{x^2 \pm a^2} \, dx = \frac{1}{4} x (x^2 \pm a^2)^{3/2} \pm \frac{1}{8} a^2 x \sqrt{x^2 \pm a^2} - \frac{1}{8} a^4 \ln(x + \sqrt{x^2 \pm a^2}) \]
Example Centroids 3 #1

Find centroid $C$ of arc $L$

By symmetry, $C$ lies on $x$ axis.

Length of arc $L = 2r\alpha$

$$L\bar{x} = \int_L \bar{x}(dL) = \int_{-\alpha}^{\alpha} \bar{x} r \cos \theta(r \, d\theta)$$

$$2r\alpha\bar{x} = 2r^2 \int_{-\alpha}^{\alpha} \cos \alpha \, d\theta = 2r^2 \sin \alpha$$

$$\bar{x} = \frac{r \sin \alpha}{\alpha} \quad \text{Ans}$$
Example Centroids 4 #1

Built around 2560 BC, the Great Pyramid of Khufu (Cheops) is one of the Seven Wonders of the Ancient World. It was 481 ft high; the horizontal cross section of the pyramid is square at any level, with each side measuring 751 ft at the base. By discounting any irregularities, find the position of Pharaoh’s burial chamber, which is located at the heart [centroid] of the pyramid.

(http://ce.eng.usf.edu/pharos/wonders/pyramid.html)
Center

Example Centroids 4 #2

By symmetry about x and y axes, C lies on z axis.
At $x = 0, z = h$ and $x = a, z = 0$

$$z = mx + C = \frac{h - 0}{0 - a} x + h = -\frac{h}{a} x + h$$

$$x = -\frac{a}{h} (z - h) = \frac{a}{h} (h - z)$$

$$dV = dz = \frac{4a^2}{h^2} (h - z)^2 dz$$

$$V = \int_V dV = \int_0^h (2x)^2 \ dz = \frac{4a^2h}{3}$$

$$\bar{z} = \frac{h}{4} = 120.25 \text{ ft} \quad \text{Ans}$$
Example  Centroids 5 #1

Find centroid C of the body.
**Example Centroids 5 #2**

<table>
<thead>
<tr>
<th>Comp</th>
<th>$V$ (mm$^3$)</th>
<th>$\bar{x}$ (mm)</th>
<th>$\bar{y}$ (mm)</th>
<th>$\bar{z}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box</td>
<td>$8.08 \times 10^6$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cylinder</td>
<td>$2.26 \times 10^6$</td>
<td>185</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rod</td>
<td>$17.67 \times 10^3$</td>
<td>0</td>
<td>175</td>
<td>0</td>
</tr>
<tr>
<td>Sphere</td>
<td>$0.524 \times 10^6$</td>
<td>0</td>
<td>275</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>$10.88 \times 10^6$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\bar{x} = \frac{\sum V_i \bar{x}_i}{\sum V_i} \quad \bar{x} = 38.5 \text{ mm}
\]
\[
\bar{y} = \frac{\sum V_i \bar{y}_i}{\sum V_i} \quad \bar{y} = 13.52 \text{ mm}
\]
\[
\bar{z} = \frac{\sum V_i \bar{z}_i}{\sum V_i} \quad \bar{z} = 0 \text{ mm} \quad \text{Ans}
\]
Pappus Theorems Formula #1

- The area of a surface of revolution equals the product of the length of the generating curve and the distance traveled by the centroid of the curve in generating the surface area.

For surface area generated by complete revolution:
\[ S = (2\pi)(\bar{x})L \]

For incomplete revolution:
\[ S = \theta \bar{x}L \]
Pappus Theorems  

The volume of a body of revolution equals the product of the generating area and the distance traveled by the centroid of the area in generating the volume.

For volume generated by complete revolution

\[ V = (2\pi)(\bar{x})A \]

\[ V = 2\pi \bar{x}A \]

For incomplete revolution

\[ V = \theta \bar{x}A \]
Pappus Theorems

- Also called the Pappus - Guldinus Theorem
- The theorem require that the generating curves and areas do not cross the axis about which they rotates.
Example Pappus Theorems 1 #1

Find centroid $C$ of a quarter circular area with radius $r$.

By symmetry, $\bar{x} = \bar{y}$

Quarter-circular area $A = \pi r^2 / 4$

Volume of hemisphere $V = 2\pi r^3 / 3$

From Pappus 2nd theorem

$[V = 2\pi \bar{x}A] \quad 2\pi r^3 / 3 = 2\pi \bar{x}(\pi r^2 / 4)$

$\bar{x} = \bar{y} = \frac{4r}{3\pi} \quad \text{Ans}$
Summary

- Centroid, CM and CG are centers of geometry, mass and gravity.
  - Centroid and CM coincide if the density $\rho$ is constant.
  - CM and CG coincide if $g$ is constant.

- Calculation by moment
  - Integration
  - Composite body

- Pappus theorems for bodies generated by revolutions
Moments

Moment of ...

- moment of force $F$ about $z$ axis = $rF$
- moment of line $L$ about $z$ axis = $rL$
- moment of area $A$ about $z$ axis = $rA$
- moment of volume $V$ about $z$ axis = $rV$
First Moment of Area $Q_x$ & $Q_y$

- The first moment of area $Q$
  - The first moment of area with respect to $x$ axes
    

$$Q_x = \int y \, dA = \bar{y}A$$

- The first moment of area with respect to $y$ axes

$$Q_y = \int x \, dA = \bar{x}A$$
The second moment of area or the area moment of inertia $I$

- The second moment of area with respect to $x$ axes
  \[ I_{xx} = \int y^2 \, dA \]

- The second moment of area with respect to $y$ axes
  \[ I_{yy} = \int x^2 \, dA \]
Polar Moment of Inertia $J$

\[ J = \int_A r^2 \, dA = \int_A (x^2 + y^2) \, dA \]

\[ = I_{xx} + I_{yy} \]
\[ I_{xx} = \int_A (y' + d_y)^2 \, dA \]
\[ = \int_A (y')^2 \, dA + 2d_y \int_A y' \, dA + d_y^2 \int_A dA \]
\[ = I_{x'x'} + 0 + A d_y^2 \]

\[ I_{yy} = I_{y'y'} + A d_x^2 \]

\[ J = I_{xx} + I_{yy} \]
Example Second Moment of Area 1 #1

Find the second moments of area and polar moment of inertia about axes passing through the center

\[ I_{xx} = \int_A y^2 \, dA = \int_A y^2 2x \, dy \]
\[ = \int_A y^2 (2\sqrt{a^2 - y^2}) \, dy = \frac{\pi a^4}{4} \]

By symmetry

\[ I_{yy} = I_{xx} = \frac{\pi a^4}{4} \]

\[ J = I_{xx} + I_{yy} = \frac{\pi a^4}{2} \]
Example Second Moment of Area 1 #2

Find the second moments of area and polar moment of inertia about axes $x_c - y_c$

\[
I_{x_c x_c} = I_{xx} + Ad_y = \frac{\pi a^4}{4} + \pi a^2 a^2 = \frac{5\pi a^4}{4}
\]

\[
I_{y_c y_c} = I_{yy} + Ad_x = \frac{\pi a^4}{4} + \pi a^2 a^2 = \frac{5\pi a^4}{4}
\]

\[
J = I_{x_c x_c} + I_{y_c y_c} = \frac{5\pi a^4}{2}
\]
## Moments

### Table Centroid and Moment of Inertia #1

<table>
<thead>
<tr>
<th>Area</th>
<th>$(\bar{x}, \bar{y})$</th>
<th>$I_{xx}$</th>
<th>$I_{yy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$bh$</td>
<td>$(0,0)$</td>
<td>$\frac{bh^3}{12}$</td>
<td>$\frac{hb^3}{12}$</td>
</tr>
<tr>
<td>$\frac{\pi D^2}{4}$</td>
<td>$(0,0)$</td>
<td>$\frac{\pi D^4}{64}$</td>
<td>$\frac{\pi D^4}{64}$</td>
</tr>
</tbody>
</table>

![Diagram of a rectangle with centroid C](image1.png)

![Diagram of a circle with centroid C](image2.png)
# Table: Centroid and Moment of Inertia #2

<table>
<thead>
<tr>
<th>Area</th>
<th>$(\bar{x}, \bar{y})$</th>
<th>$I_{xx}$</th>
<th>$I_{yy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$bh/2$</td>
<td>$(0, h/3)$</td>
<td>$bh^3/36$</td>
<td>$hb^3/48$</td>
</tr>
<tr>
<td>$\pi ab$</td>
<td>$(0, 0)$</td>
<td>$\pi ab^3/4$</td>
<td>$\pi ba^3/4$</td>
</tr>
</tbody>
</table>
Example Hibbeler 6-68 Mech of Mat #1

Find the second moment of area with respect to the neutral axis (the horizontal axis which passes through the centroid of the area)
Moments

Example Hibbeler 6-68 Mech of Mat #2

Find the centroid location

\[ C = C(\bar{y}, \bar{z}) \]

By symmetry, \( \bar{z} = 0 \)

\[ \bar{y}_1 = (10 + 0.5) \text{ in.} \]

\[ A_1 = (6 \text{ in.})(1 \text{ in.}) \]

\[ \bar{y}_2 = 5 \text{ in.} \]

\[ A_2 = (1 \text{ in.})(10 \text{ in.}) \]

\[ \bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A_1 + A_2} \]

\[ = 7.0625 \text{ in.} \]
Example Hibbeler 6-68 Mech of Mat #3

Find \( I \) with respect to (wrt) horizontal axis passing through \( C \)

\[
I_{zzc} = I_{zz1} + (Ad_y)_1 + I_{zz2} + (Ad_y)_2
\]

\[
= \frac{1}{12} (6 \text{ in.})(1 \text{ in.})^3 + (6 \text{ in.}^2)(10.5 \text{ in.} - 7.0625 \text{ in.})^2 + \frac{1}{12} (1 \text{ in.})(10 \text{ in.})^3 + (10 \text{ in.}^2)(7.0625 \text{ in.} - 5 \text{ in.})^2
\]

\[
I_{zzc} = 197.2708 \text{ in.}^4 = 197 \text{ in.}^4
\]
Distributed Loads

**Loads** Concentration vs. Distribution

**Concentrated Load**
- acts at a point,
- does not exist in the exact sense,
- acceptable approximation when contact area is small.

**Distributed Loads**
- distributed over line, area, volume.
Distributed Loads

**Loads Hand Tools**

- Blacksmith Hammer
- Wood Chisel
Distributed Loads

**Loads Distribution Types**

- **Line Distribution**
  - Intensity \( w \) (N/m) = force per unit length

- **Area Distribution**
  - Intensity (N/m\(^2\) or Pa) = force per unit area
  - Action of fluid force ➔ pressure
  - Internal intensity of force in solid ➔ stress

- **Volume Distribution**
  - Intensity (N/m\(^3\)) = body force per unit volume
  - Specific weight \( \rho g \) is the intensity of gravitational attraction.
Distributed loads can be represented by an equivalent force-couple system, consisting of:

- Resultant force $F_R$
  - Magnitude
  - Direction
  - Line of action
- Resultant couple $M_R$
  - Magnitude
  - Direction
Distributed Loads

**Line Distribution Descriptions**

- The distributed loads may be described by functions of positions.

Let \( w = (0.01^2 - x^2) \) MN/m (\( x \) in m)
Distributed Loads

**Line Distribution Resultants**

\[ F_R = \int w \, dx \]
\[ M = \int wx \, dx = F_R \bar{x} \]

---

magnitude of \( F \)

location of \( F \)
Consider right half the load

$$w = (0.01^2 - x^2) \times 10^6 \text{ N/m} \ (x \text{ in m})$$

$$F_R = \int w \, dx = 10^6 \int_0^{0.01} (0.01^2 - x^2) \, dx$$

$$= 10^6 \left[ 0.01^2 x - \frac{x^3}{3} \right]_0^{0.01} = 2/3 \text{ N}$$

$$M = \int wx \, dx = 10^6 \int_0^{0.01} (0.01^2 - x^2)x \, dx$$

$$= 10^6 \left[ (0.01^2 x^2/2) - \frac{x^4}{4} \right]_0^{0.01}$$

$$= 2.5 \times 10^{-3} \text{ N} \cdot \text{m}$$

$$M = \bar{x}F_R \quad \rightarrow \quad \bar{x} = 3.75 \text{ mm}$$
Distributed Loads

**Line Distribution Area Analogy**

If \( y = w \)

\[
F_R = \int w \, dx
\]

\[
\bar{x} = \int xw \, dx / F_R
\]

The single equivalent force \( F_R \) exerted by the line distributed load is equal to the “area” \( A \) and acts through the centroid of \( A \) between the loading curve and the x axis.
Example: Area Analogy 1 #1

Determine the reactions at A and C.
Example Area Analogy 1 #2

\[ F_1 = (200 \text{ N/m})(5 \text{ m}) = 1000 \text{ N} \]
\[ F_2 = (100 \text{ N/m})(6 \text{ m}) = 600 \text{ N} \]
\[ F_3 = 0.5(100 \text{ N/m})(6 \text{ m}) = 300 \text{ N} \]
Example Area Analogy 1 #3

\[ \sum F_x = 0 \]
\[-F_1 + A_x = 0 \]
\[ A_x = 1000 \text{ N} \quad \text{Ans} \]

\[ \sum M_A = 0 \quad \text{cW}^+ \]
\[ F_1(2.5 \text{ m}) - F_3(2 \text{ m}) - F_2(3 \text{ m}) + C_y(6 \text{ m}) = 0 \]
\[ C_y = -16.667 \text{ N} \quad \text{Ans} \]

\[ \sum F_y = 0 \]
\[ A_y + C_y - F_2 - F_3 = 0 \]
\[ A_y = 916.67 \text{ N} \quad \text{Ans} \]
Example Area Analogy 2 #1

If the cable can sustain tension of up to 600 N, determine the maximum $w$. 

![Diagram of a cable system with distributed loads and a 30° angle at point A.]
Example Area Analogy 2 #2

\[
\sum M_c = 0 \quad \text{(\#)}
\]

\[-(15w \text{ N})(7.5 \text{ m}) - (7.5w \text{ N})(20 \text{ m}) + T(15 \text{ m}) + T \sin 30^\circ(30 \text{ m}) = 0\]

\[w = \frac{30T}{262.5} \text{ N/m}\]

Given \(T_{\text{max}} = 600 \text{ N}\), thus \(w_{\text{max}} = \frac{30T_{\text{max}}}{262.5} \text{ N/m}\)

\[w_{\text{max}} = 68.57 \text{ N/m} \quad \text{Ans}\]
Distributed Loads

Example Area Analogy 3 #1

Find support reactions

\[ F = \int_A dA = \int_0^{1m} (x)^{1/2} dx \]
\[ = \left[ \frac{2(x)^{3/2}}{3} \right]_0^{1m} = \frac{2}{3} \text{ kN} \]

\[ \bar{x}F = \int_0^{1m} x(x)^{1/2} dx \]
\[ = \left[ \frac{2(x)^{5/2}}{5} \right]_0^{1m} = 0.4 \text{ kN} \cdot \text{m} \]

\[ \bar{x} = 0.6 \text{ m} \]
Distributed Loads

Example Area Analogy 3 #2

Consider FBD of beam

\[
\begin{align*}
\sum F_x &= 0 \\
C_x &= 0 \quad \text{Ans} \\
\sum F_y &= 0 \\
C_y + \frac{2}{3} \text{kN} &= 0 \\
C_y &= -0.667 \text{kN} \quad \text{Ans} \\
\sum M_C &= 0 \quad \text{Ans} \\
M + (0.6 \text{ m})(\frac{2}{3} \text{kN}) &= 0 \\
M &= -0.4 \text{kN} \cdot \text{m} \quad \text{Ans}
\end{align*}
\]
Distributed Loads

Area Distribution Resultants

\[ F_R = \int p \, dA \]
\[ M = \int x \, p \, dA = F_R \bar{x} \]
Find the equivalent concentrated load.
**Example Hibbeler 9-121 #2**

General equation of a plane

\[ ax + by + cp + d = 0 \quad (1) \]

- \[ P_1(0 \text{ ft}, 0 \text{ ft}, 40 \text{ lb/ft}^2) \]
  \[ 40c + d = 0 \]
- \[ P_2(5 \text{ ft}, 0 \text{ ft}, 30 \text{ lb/ft}^2) \]
  \[ 5a + 30c + d = 0 \]
- \[ P_3(5 \text{ ft}, 10 \text{ ft}, 10 \text{ lb/ft}^2) \]
  \[ 5a + 10b + 10c + d = 0 \]
- \[ P_4(0 \text{ ft}, 10 \text{ ft}, 20 \text{ lb/ft}^2) \]
  \[ 10b + 20c + d = 0 \]

\[ a = 2c, \quad b = 2c, \quad d = -40c \]

Subst into (1)

\[ 2cx + 2cy + cp - 40c = 0 \]
\[ 2x + 2y + p - 40 = 0 \]
\[ p = -2x - 2y + 40 \text{ lb/ft}^2 \]
Distributed Loads

**Example Hibbeler 9-121 #3**

\[ dF_R = p \, dA = (-2x - 2y + 40) \, dx \, dy \]

\[ F_R = \int_0^5 \int_0^{10} (-2x - 2y + 40) \, dy \, dx = 1250 \text{ lb} \quad \text{Ans} \]
Distributed Loads

Example Hibbeler 9-121 #4

\[ \bar{x} = \frac{\int_{F_R} x \, dF_R}{F_R} = 2.33 \text{ ft} \]

\[ \bar{y} = \frac{\int_{F_R} y \, dF_R}{F_R} = 4.33 \text{ ft} \quad \text{Ans} \]

\[ \int_{F_R} x \, dF_R = \int_{0}^{5} \int_{0}^{10} (-2x - 2y + 40)x \, dy \, dx = 2916.67 \text{ lb} \cdot \text{ft} \]

\[ \int_{F_R} y \, dF_R = \int_{0}^{5} \int_{0}^{10} (-2x - 2y + 40)y \, dy \, dx = 5416.67 \text{ lb} \cdot \text{ft} \]
If \( z = p \)

\[
F = \int p \, dA
\]

\[
\bar{x} = \frac{\int xp \, dA}{F}
\]

\[
\bar{y} = \frac{\int yp \, dA}{F}
\]

The single equivalent force \( F \) exerted by the area distributed load is equal to the “volume” \( V \) and acts through the centroid of \( V \).
Fluid Statics

- Required in studies and designs of pressure vessels, piping, ships, dams and off-shore structures, etc.

- Topics
  - Definitions
  - Fluid pressure
  - Hydrostatic pressure on submerged surfaces
  - Buoyancy
  - Air pressure
Fluid Statics Definitions

- Fluid is any continuous substance which, when at rest, is unable to support shear force.
- Fluid Statics studies pressure of fluid at rest.
  - Hydrostatics – stationary liquid
  - Aerostatics – stationary gas
- Pascal’s Law: the pressure at any given point in a fluid is the same in all directions.
- Pressure $p$ in fluid at rest is a function of vertical dimension and its density $\rho$.
- Resultant force on a body from pressure acts at the center of pressure $P$. 
Fluid Statics Fluid Pressure #1

Consider forces on control volume in x direction

\[ \sum F_x = 0 \]

\[ (p)dA + \rho_w g (dAdx) - (p + dp)dA = 0 \]

\[ dp = \rho_w g (dx) \]

\[ W = \rho_w g V = \rho_w g (dAdx) \]
Fluid Statics Fluid Pressure #2

In \( x \) direction only

\[ dp = \rho_w g \, dx \]

\[
\int_{p_0}^{p} dp = \int_{0}^{x} \rho_w g \, dx
\]

\[
p = p_0 + \rho_w g x
\]

\( p_0 \) = pressure at \( x = 0 \)

\[ F = \int_A p \, dA \]
Find the resultant force and its position on the submerged face AB. It is given that the width of the plate is $b$ and the fluid has a constant density $\rho$.

**Methods**

- Integration
- Volume analogy
- Equilibrium of block of fluid *new*
Consider equilibrium of the block of fluid above the plate:

\[ \sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_O = 0 \]
Example Fluid Statics 1 #1

Find the resultant force and its position on the dam with 1 m width. The density of water is 1000 kg/m$^3$. 

\[ p_1 \]
\[ p_2 \]
\[ F \]
\[ L \]
\[ P \]
\[ x \]
\[ y \]
Example Fluid Statics 1 #2

Fluid Statics

Integration

\[ y = \frac{-x}{2.5} + 1 \]

\[ F_x = \int_A p \, dA_y = \int_0^1 \rho gx \, (1 \, dy) \]
\[ = \int_{2.5}^0 \frac{\rho gx}{-2.5} \, (dx) \]
\[ = \frac{\rho g}{2.5} \left[ \frac{x^2}{2} \right]_0^{2.5} = 12250 \text{ N} \]

\[ F_y = \int_A p \, dA_x = \int_0^{2.5} \rho gx \, (1 \, dx) \]
\[ = \rho g \left[ \frac{x^2}{2} \right]_0^{2.5} = 30625 \text{ N} \]

\[ F^2 = F_x^2 + F_y^2 \]
\[ F = 32984 \text{ N} = 33.0 \text{ kN} \quad \text{Ans} \]
Fluid Statics

Example Fluid Statics 1 #3

\[ y = \frac{-x}{2.5} + 1 \]

\[ F_x \overline{y} = \int_A p y \, dA_y = \int_0^1 \rho g x y (1 \, dy) \]

\[ = \int_{-2.5}^{2.5} \rho g x \left( -\frac{x}{2.5} + 1 \right) \, dx \]

\[ = \rho g \left[ \frac{x^3}{3} \right]_{-2.5}^{2.5} \]

\[ = 4083.3 \text{ N} \cdot \text{m} \]

\[ \overline{y} = 0.33333 \text{ m} \]

\[ F_y \overline{x} = \int_A p x \, dA_x = \int_0^{2.5} \rho g x^2 (1 \, dx) \]

\[ = \rho g \left[ \frac{x^3}{3} \right]_0^{2.5} \]

\[ = 51042 \text{ N} \cdot \text{m} \]

\[ \overline{x} = 1.6667 \text{ m} \]

\[ P(\overline{x}, \overline{y}) = (1.67, 0.333) \text{ m} \quad \text{Ans} \]
Fluid Statics

Example Fluid Statics 1 #4

At $x = 0$, $p_1 = 0$

$x = 2.5$ m, $p_2 = 2.5 \rho g$ N

$$F = \frac{1}{2} (p_1 + p_2) LW$$

$$= \frac{1}{2} \times 2.5 \times 1000 \times 9.8 \times \sqrt{2.5^2 + 1^2} \times 1$$

$$F = 32.98 \text{ kN} \quad \text{Ans}$$

$$P(x, y) = \left(\frac{2}{3} 2.5, \frac{1}{3} 1\right)$$

$$P(x, y) = (1.67, 0.333) \text{ m} \quad \text{Ans}$$
Example Fluid Statics 1 #5

Fluid Statics

\[ W = \rho g V = 1000 \times 9.8 \times \frac{1}{2} \times 2.5 \times 1 = 12250 \text{ N} \]

\[ F_1 = \frac{1}{2} \rho g h H = \frac{1}{2} \times 1000 \times 9.8 \times 2.5 \times 2.5 = 30625 \text{ N} \]
Fluid Statics

Example Fluid Statics 1 #6

\[
\begin{align*}
\sum F_x &= 0 \\
W - F_x &= 0 \\
F_x &= 12250 \text{ N} \\
\sum F_y &= 0 \\
F_1 - F_y &= 0 \\
F_y &= 30625 \text{ N} \\
F^2 &= F_x^2 + F_y^2 \\
F &= 32984 \text{ N} = 33.0 \text{ kN} \quad \text{Ans}
\end{align*}
\]
Example Fluid Statics 1 #7

\[
\begin{align*}
\sum M_O &= 0 \ U+ \\
-F_y \bar{x} + F_x \bar{y} - W &\frac{1}{3} + F_1 \frac{2.5 \times 2}{3} = 0 \\
2 \times 2.5^2 &\frac{3}{3} - \frac{1}{3} + \bar{y} - 2.5 \bar{x} \\
\text{From } y &= \frac{-x}{2.5} + 1, \quad \bar{y} = \frac{-\bar{x}}{2.5} + 1 \\
\bar{x} &= \frac{5}{3}, \quad \bar{y} = \frac{1}{3} \\
P(\bar{x}, \bar{y}) &= (5/3, 1/3) \text{ m Ans}
\end{align*}
\]
Example Fluid Statics 2 #1

Cross section of a long channel is shown. Each of the bottom plates, hinged at B, has a mass of 250 kg per meter of channel length. Find force per meter of channel length acting on each plate at B. The density of water is 1000 kg/m³.
Example Fluid Statics 2 #2

At $B$, $p_1 = 0.9 \rho g$ N
At $A$, $p_2 = 1.8 \rho g$ N
Width of channel = $w$
Symmetry of forces at $B$: $B_y = 0$

\[
\begin{align*}
A_x & \quad A_y \\
p_1 & \quad B_y \\
p_2 & \quad B_x \\
B & \quad 0.9 \text{ m} \\
0.6 \text{ m} & \quad 1.2 \text{ m} \\
mg = 250gw & \\
C_x & \quad C_y
\end{align*}
\]
Example Fluid Statics 2 #3

Volume Analogy

\[ F_1 = p_1 1.5w = 1.35 \rho gw \text{ N} \]
\[ F_2 = 0.5(p_2 - p_1)1.5w = 0.675 \rho gw \text{ N} \]

\[ \sum M_A = 0 \Rightarrow \]
\[ 0.9B_x - 250gw0.6 - 1.35 \rho gw0.75 - 0.675 \rho gw0.5 = 0 \]
\[ 0.9B_x = 1500gw \Rightarrow B_x = 1666.7 \rho gw \text{ kN} \]

Force per meter acting at \( B \) = 1.67\( g \) = 16.3 kN/m \( \text{Ans} \)
Buoyancy

Definition

The resultant force exerted on the surface of an object immersed in a fluid:

- is equal and opposite to the weight of displaced fluid,
- pass through the center of buoyancy $B$ (center of mass of the displaced fluid).

$$F_B = pA = \left(\rho_w gh\right)A = \left(\rho_w hA\right)g = \rho_w V_f g$$
Fluid Statics

Buoyancy Stability

stable

unstable
Flexible Cables

- Found in suspension bridges, transmission lines, etc.
- Topics
  - Assumptions of flexible cables
  - Types of cable loadings
  - Geometrical definition
  - Parabolic cables
  - Catenary cables
Flexible Cables Assumptions

- Bear load only in tension
- Negligible displacement due to stretching
  - Inextensible
- Perfectly flexible
  - Negligible bending resistance
  - Tangential tension along the cable
Flexible Cables Concentrated Loads

- Concentrated / discrete load
- Weight of the cable is negligible.
Cables Geometrical Definitions

\[ S = \text{length (m)} \]
\[ L = \text{span (m)} \]
\[ h = \text{sag, dip (m)} \]

\[ w = \text{load intensity (N/m)} \]
\[ \theta = \text{tangential angle at } x \text{ (rad)} \]
\[ T = \text{tension at } x \text{ (N)} \]
\[ T_0 = \text{tension at lowest point (N)} \]
Parabolic Cables

Longest Suspension Bridge
Akashi Kaikyo Bridge, Japan
3910 m total span
(http://www.hsba.go.jp/bridge/)
Parabolic Cables Analysis

- $w = \text{uniform vertical load per unit of horizontal length}$

---

\[ \sum F_x = 0 \quad T \cos \theta - T_0 = 0 \quad \rightarrow \quad T \cos \theta = T_0 \quad (1) \]

\[ \sum F_y = 0 \quad T \sin \theta - wx = 0 \quad \rightarrow \quad T \sin \theta = wx \quad (2) \]

\[ \tan \theta = \frac{w}{T_0} \quad x = \frac{dy}{dx} \quad \rightarrow \quad \int_0^y dy = \int_0^x \frac{w}{T_0} x \, dx \]

\[ (2)^2 + (1)^2 \]

\[ y = \frac{1}{2} \frac{w}{T_0} x^2, \quad T^2 = T_0^2 + w^2 x^2 \]
Parabolic Cables Formula

\[ y = \frac{1}{2} \frac{w}{T_0} x^2 = \frac{1}{2} ax^2, \quad a = \frac{w}{T_0} \]

\[ T_0 = \frac{1}{2} \frac{w}{h} l^2 \quad \text{at} \quad x = l, \quad y = h \]

\[ T = \sqrt{T_0^2 + w^2 x^2} = T_0 \sqrt{1 + a^2 x^2} \quad \rightarrow \quad T_{\text{max}} = T_0 \sqrt{1 + a^2 l^2} \quad \text{at} \quad x = l \]

\[ ds^2 = dx^2 + dy^2 \quad \rightarrow \quad s = \frac{1}{2} \left( x \sqrt{1 + a^2 x^2} \right) + \frac{1}{2} \left( \frac{1}{a} \ln(ax + \sqrt{1 + a^2 x^2}) \right) \]
Example Parabolic Cables 1 #1

Find tension in the cable at A and the angle $\theta$ made by the cable with the horizontal at B.
Example Parabolic Cables 1 #2

At point A

\[ T_0 = \frac{wx^2}{2y} = \frac{wl^2}{2h} \]

\[ T_0 = \frac{(294.21 \text{ N/m})(70 \text{ m})^2}{2(40 \text{ m})} \]

\[ T_0 = 18020.4 \text{ N} = 18.02 \text{ kN} \quad \text{Ans} \]

\[ y = \frac{wx^2}{2T_0} \quad \rightarrow \quad \frac{dy}{dx} = \frac{wx}{T_0} = \tan \theta \]

At point B

\[ \tan \theta = \frac{wx}{T_0} = \frac{(294.21 \text{ N/m})(70 \text{ m})}{18020.4 \text{ N}} \]

\[ \tan \theta = 1.14286 \]

\[ \theta = 48.8^\circ \quad \text{Ans} \]
Flexible Cables  Catenary Cables

Vertical load is the cable’s own weight.
Catenary Cables Analysis #1

\[ \sum F_x = 0 \]
\[ T \cos \theta - T_0 = 0 \quad \rightarrow \quad T \cos \theta = T_0 \quad (1) \]

\[ \sum F_y = 0 \]
\[ T \sin \theta - \mu s = 0 \quad \rightarrow \quad T \sin \theta = \mu s \quad (2) \]

\[ \tan \theta = \frac{\mu}{T_0} s = \frac{dy}{dx} \quad \rightarrow \quad \frac{d^2 y}{dx^2} = \frac{\mu}{T_0} \frac{ds}{dx} = \frac{\mu}{T_0} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \]

\( \mu = \text{uniform vertical load per unit of cable length} \)
Catenary Cables Analysis #2

Let \( \sigma = \frac{dy}{dx} = \tan \theta \), \( a = \frac{\mu}{T_0} \)

At \( x = 0 \): \( y = 0 \), \( \sigma = \frac{dy}{dx} = 0 \)

\[
\frac{d^2y}{dx^2} = \frac{\mu}{T_0} \sqrt{1 + \left( \frac{dy}{dx} \right)^2}
\]

\[
\frac{d\sigma}{dx} = a \sqrt{1 + \sigma^2} \quad \rightarrow \quad \frac{d\sigma}{\sqrt{1 + \sigma^2}} = a \, dx
\]

\[
\int_0^\sigma \frac{d\sigma}{\sqrt{1 + \sigma^2}} = \int_0^x a \, dx
\]
Catenary Cables Formula

Let \( a = \mu / T_0 \)

\[
y = \frac{1}{a} (\cosh(ax) - 1)
\]

\[
\frac{dy}{dx} = \tan \theta = as
\]

\[
s = \frac{1}{a} \sinh(ax)
\]

As \( T \cos \theta = T_0 \), \( dx = ds \cos \theta \)

\[
T = T_0 \cosh(ax)
\]

\[
T = \frac{T_0}{\cos \theta} = T_0 \frac{ds}{dx}
\]

\[
T_{\text{max}} = T_0 \cosh(al) \text{ at } x = l
\]

http://mathworld.wolfram.com/topics/HyperbolicFunctions.html
Catenary Cables  Hyperbolic Functions

\[
\sinh(x) = \frac{e^x - e^{-x}}{2}
\]
\[
\cosh(x) = \frac{e^x + e^{-x}}{2}
\]
\[
\tanh(x) = \frac{\sinh(x)}{\cosh(x)}
\]
\[
\frac{d}{dx} \sinh(x) = \cosh(x)
\]
\[
\frac{d}{dx} \cosh(x) = \sinh(x)
\]

http://en.wikipedia.org/wiki/Hyperbolic_function
Example Catenary Cables 1 #1

Find the length of the cable.

\[ h = 100 \text{ m}, \quad l = 200 \text{ m} \]
Example Catenary Cables 1 #2

From \( y = \frac{1}{a} (\cosh ax - 1) \)

At \( A \): \( x = 200 \, \text{m} \)

\[ 100 = \frac{1}{a} (\cosh(200a) - 1) \]

\[ 100a = \cosh(200a) - 1 \quad (1) \]

Solve (1), \( a = 4.6541 \times 10^{-3} \, \text{m}^{-1} \)

From \( s = \frac{1}{a} \sinh ax \)

\[ s = \frac{\sinh(4.6541 \times 10^{-3} \times 200)}{4.6541 \times 10^{-3}} \]

\[ s = 230.158 \, \text{m} \]

Total length \( S = 2s = 460.3 \, \text{m} \quad \text{Ans} \)
Example Catenary Cables 1 #3

100a = \cosh(200a) - 1

f(x) = 100x

g(x) = \cosh(200x) - 1

f = g at x = 0.004654 m\(^{-1}\)  \hspace{1cm} \text{Ans}
**Example: Catenary Cables 1 #4**

For an equation \( f(x) = 0 \): \( x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \)

\[
f(x) = 100x - \cosh(200x) + 1, \quad f'(x) = 100 - 200 \sinh(200x)
\]

Guess \( x_0 = 0.01, \quad f(x_0) = -1.7622, \quad f'(x_0) = -625.37 \)

\[
x_1 = x_0 - f(x_0)/f'(x_0) = 0.0071822
\]

\[
x_2 = x_1 - f(x_1)/f'(x_1) = 0.0054857
\]

\[
x_3 = x_2 - f(x_2)/f'(x_2) = 0.0047868
\]

\[
x_4 = x_3 - f(x_3)/f'(x_3) = 0.0046541
\]

\[
x_5 = x_4 - f(x_4)/f'(x_4) = 0.0046541
\]

\[
x_6 = x_5 - f(x_5)/f'(x_5) = 0.0046541
\]

\( \text{Ans} \)

See file `chap4_catenary_1.xls`
Example Catenary Cables 1 #5

\[ 100a = \cosh(200a) - 1 \]

Newton-Raphson
Example Catenary Cables 2 #1

Find the length of the cable.

$h_1 = 6 \text{ m}$
$h_2 = 14 \text{ m}$
$l_1 + l_2 = 40 \text{ m}$
Example: Catenary Cables 2 #2

\[ h_1 = 6 \text{ m} \]
\[ h_2 = 14 \text{ m} \]
\[ l_1 + l_2 = 40 \text{ m} \]

From \[ y = \frac{1}{a}(\cosh ax - 1) \]

At B: \[ 6 = \frac{1}{a}(\cosh a l_1 - 1) \] \hspace{1cm} (1)

A: \[ 14 = \frac{1}{a}(\cosh a l_2 - 1) = \frac{1}{a}\left[\cosh(a(40-l_1)) - 1\right] \] \hspace{1cm} (2)

Solve (1) and (2): \[ a = 0.04453 \text{ m}^{-1}, \ l_1 = 16.07 \text{ m} \]
Example Catenary Cables 2 #3

\[ s = \frac{1}{a} \sinh ax \]

\[ s_1 = \frac{1}{0.04453} \sinh(0.04453 \times 16.07) = 17.477 \text{ m} \]

\[ s_2 = \frac{1}{0.04453} \sinh (0.04453(40 - 16.07)) = 28.723 \text{ m} \]

Total cable length \( S = s_1 + s_2 = 46.2 \text{ m} \) \textbf{Ans}
Example Catenary Cables 2 #4

For an equation \( f(x) = 0 \): \( x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \)

For equations \( f(x, y) = 0 \) and \( g(x, y) = 0 \),
\[
\begin{pmatrix}
\frac{\partial f(x_k, y_k)}{\partial x} & \frac{\partial f(x_k, y_k)}{\partial y} \\
\frac{\partial g(x_k, y_k)}{\partial x} & \frac{\partial g(x_k, y_k)}{\partial y}
\end{pmatrix}
\begin{pmatrix}
\Delta x_{k+1} \\
\Delta y_{k+1}
\end{pmatrix}
= -
\begin{pmatrix}
f(x_k, y_k) \\
g(x_k, y_k)
\end{pmatrix}
\]

\( x_{k+1} = x_k + \Delta x_{k+1} \) and \( y_{k+1} = y_k + \Delta y_{k+1} \)
Example Catenary Cables 2 #5

let \( x = a, \ y = l_1 \):

\[ f(x, y) = \cosh xy - 6x - 1 = 0 \]
\[ g(x, y) = \cosh (x(40 - y)) - 14x - 1 = 0 \]
\[ \partial f(x, y)/\partial x = y \sinh xy - 6 \]
\[ \partial f(x, y)/\partial y = x \sinh xy \]
\[ \partial g(x, y)/\partial x = (40 - y) \sinh (x(40 - y)) - 14 \]
\[ \partial g(x, y)/\partial y = -x \sinh (x(40 - y)) \]
Example Catenary Cables 2 #7

Guess \( x_0 = 1.0, \ y_0 = 20 \):

\[
\begin{align*}
 f(x_0, y_0) &= 2.4258 \times 10^8, \\
 g(x_0, y_0) &= 2.4258 \times 10^8, \\
 \partial f / \partial x &= 4.8517 \times 10^9, \\
 \partial f / \partial y &= 2.4258 \times 10^8, \\
 \partial g / \partial x &= 4.8517 \times 10^9, \\
 \partial g / \partial y &= -2.4258 \times 10^8.
\end{align*}
\]

\[
\begin{pmatrix}
 \partial f(x_k, y_k) / \partial x \\
 \partial g(x_k, y_k) / \partial x
\end{pmatrix}
\begin{pmatrix}
 \partial f(x_k, y_k) / \partial y \\
 \partial g(x_k, y_k) / \partial y
\end{pmatrix}
\begin{pmatrix}
 \Delta x_{k+1} \\
 \Delta y_{k+1}
\end{pmatrix}
= -
\begin{pmatrix}
 f(x_k, y_k) \\
 g(x_k, y_k)
\end{pmatrix}
\]

\[
\begin{pmatrix}
 4.8517 \times 10^9 & 2.4258 \times 10^8 \\
 4.8517 \times 10^9 & 2.4258 \times 10^8
\end{pmatrix}
\begin{pmatrix}
 \Delta x_2 \\
 \Delta y_2
\end{pmatrix}
= 
\begin{pmatrix}
 -2.4258 \times 10^8 \\
 -2.4258 \times 10^8
\end{pmatrix}
\]

\[
\Delta x_1 = -5.0000 \times 10^{-2}, \ \Delta y_1 = -1.5665 \times 10^{-8}
\]

\[
\begin{align*}
 x_1 &= x_0 + \Delta x_1 = 1.0 - 0.5, \\
 y_1 &= y_0 + \Delta y_1 = 20.0 - 1.566 \times 10^{-8} = 20.0
\end{align*}
\]
### Example Catenary Cables 2 #8

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<th>$y$</th>
<th>$Dx$</th>
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Concepts #1

- **Centroid, center of mass** and **center of gravity** respectively represent the centers of **geometry**, **mass** and **weight**.

- **Pappus theorems** can be used to find the **surface area and volume of revolutions**.

- Apart from moments of forces, there are several kinds of moments. The **first moment of area** is a property of cross section that is used to predict its resistance to shear stress. The **moment of inertia** quantifies the rotational inertia of a body. The **polar moment of inertia** is a property of cross section that is used to predict its resistance to torsion.
Concepts #2

- The *equivalent resultant* of the distributed loads can be used in the analyses instead of the distributed loads. The *area and volume analogies* help visualize the line and surface distributed loads on bodies.

- The *fluid statics* deal with the effects of fluid at rest.
  - The pressure, exerted by the fluid in the perpendicular direction with respect to the surface in contact with the fluid, *varies linearly with depth*.
  - The block of fluid is an addition method to determine resultants.

- Flexible cables can support tension only.
  - *Parabolic cables* are loaded with uniform force *per unit span length*.
  - *Catenary cables* are loaded with uniform force *per unit of cable length*.