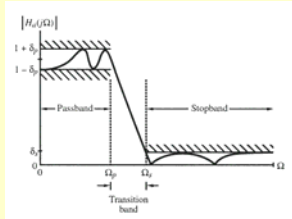


## Analog Lowpass Filter Specifications

- Typical magnitude response  $|H_a(j\Omega)|$  of an analog lowpass filter may be given as indicated below



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## Analog Lowpass Filter Specifications

- In the **passband**, defined by  $0 \leq \Omega \leq \Omega_p$ , we require
 
$$1 - \delta_p \leq |H_a(j\Omega)| \leq 1 + \delta_p, \quad |\Omega| \leq \Omega_p$$
 i.e.,  $|H_a(j\Omega)|$  approximates unity within an error of  $\pm \delta_p$
- In the **stopband**, defined by  $\Omega_s \leq \Omega < \infty$ , we require
 
$$|H_a(j\Omega)| \leq \delta_s, \quad \Omega_s \leq |\Omega| < \infty$$
 i.e.,  $|H_a(j\Omega)|$  approximates zero within an error of  $\delta_s$

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## Analog Lowpass Filter Specifications

- $\Omega_p$  - **passband edge frequency**
- $\Omega_s$  - **stopband edge frequency**
- $\delta_p$  - **peak ripple value** in the passband
- $\delta_s$  - **peak ripple value** in the stopband
- **Peak passband ripple**

$$\alpha_p = -20 \log_{10}(1 - \delta_p) \text{ dB}$$
- **Minimum stopband attenuation**

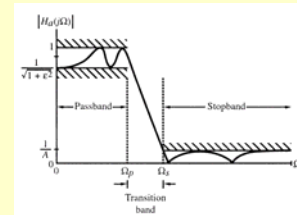
$$\alpha_s = -20 \log_{10}(\delta_s) \text{ dB}$$

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## Analog Lowpass Filter Specifications

- Magnitude specifications may alternately be given in a normalized form as indicated below



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## Analog Lowpass Filter Specifications

- Here, the maximum value of the magnitude in the passband assumed to be unity
- $1/\sqrt{1+\epsilon^2}$  - Maximum passband deviation, given by the minimum value of the magnitude in the passband
- $\frac{1}{A}$  - Maximum stopband magnitude

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## Analog Lowpass Filter Design

- Two additional parameters are defined -

(1) **Transition ratio**  $k = \frac{\Omega_p}{\Omega_s}$

For a lowpass filter  $k < 1$

(2) **Discrimination parameter**  $k_1 = \frac{\epsilon}{\sqrt{A^2 - 1}}$   
Usually  $k_1 \ll 1$

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## Butterworth Approximation

- The magnitude-square response of an  $N$ -th order analog lowpass **Butterworth filter** is given by

$$|H_a(j\Omega)|^2 = \frac{1}{1 + (\Omega/\Omega_c)^{2N}}$$

- First  $2N - 1$  derivatives of  $|H_a(j\Omega)|^2$  at  $\Omega = 0$  are equal to zero
- The Butterworth lowpass filter thus is said to have a **maximally-flat magnitude** at  $\Omega = 0$

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## Butterworth Approximation

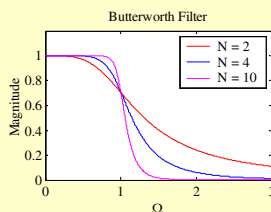
- Gain in dB is  $G(\Omega) = 10 \log_{10} |H_a(j\Omega)|^2$
- As  $G(0) = 0$  and  $G(\Omega_c) = 10 \log_{10}(0.5) = -3.0103 \cong -3$  dB  $\Omega_c$  is called the **3-dB cutoff frequency**

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## Butterworth Approximation

- Typical magnitude responses with  $\Omega_c = 1$



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## Butterworth Approximation

- Two parameters completely characterizing a Butterworth lowpass filter are  $\Omega_c$  and  $N$
- These are determined from the specified bandedges  $\Omega_p$  and  $\Omega_s$ , and minimum passband magnitude  $1/\sqrt{1 + \epsilon^2}$ , and maximum stopband ripple  $1/A$

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## Butterworth Approximation

- $\Omega_c$  and  $N$  are thus determined from

$$|H_a(j\Omega_p)|^2 = \frac{1}{1 + (\Omega_p/\Omega_c)^{2N}} = \frac{1}{1 + \epsilon^2}$$

$$|H_a(j\Omega_s)|^2 = \frac{1}{1 + (\Omega_s/\Omega_c)^{2N}} = \frac{1}{A^2}$$

- Solving the above we get

$$N = \frac{1}{2} \cdot \frac{\log_{10}[(A^2 - 1)/\epsilon^2]}{\log_{10}(\Omega_s/\Omega_p)} = \frac{\log_{10}(1/k_1)}{\log_{10}(1/k)}$$

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## Butterworth Approximation

- Since order  $N$  must be an integer, value obtained is rounded up to the next highest integer
- This value of  $N$  is used next to determine  $\Omega_c$  by satisfying either the stopband edge or the passband edge specification exactly
- If the stopband edge specification is satisfied, then the passband edge specification is exceeded providing a safety margin

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## Butterworth Approximation

- Transfer function of an analog Butterworth lowpass filter is given by

$$H_a(s) = \frac{C}{D_N(s)} = \frac{\Omega_c^N}{s^N + \sum_{\ell=0}^{N-1} d_\ell s^\ell} = \frac{\Omega_c^N}{\prod_{\ell=1}^N (s - p_\ell)}$$

where

$$p_\ell = \Omega_c e^{j\pi(N+2\ell-1)/2N}, \quad 1 \leq \ell \leq N$$

- Denominator  $D_N(s)$  is known as the **Butterworth polynomial** of order  $N$

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## Butterworth Approximation

- Example** - Determine the lowest order of a Butterworth lowpass filter with a 1-dB cutoff frequency at 1 kHz and a minimum attenuation of 40 dB at 5 kHz

Now  $10 \log_{10} \left( \frac{1}{1 + \varepsilon^2} \right) = -1$

which yields  $\varepsilon^2 = 0.25895$

and  $10 \log_{10} \left( \frac{1}{A^2} \right) = -40$

which yields  $A^2 = 10,000$

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## Butterworth Approximation

- Therefore  $\frac{1}{k_1} = \frac{\sqrt{A^2 - 1}}{\varepsilon} = 196.51334$

and  $\frac{1}{k} = \frac{\Omega_s}{\Omega_p} = 5$

- Hence

$$N = \frac{\log_{10}(1/k_1)}{\log_{10}(1/k)} = 3.2811$$

- We choose  $N = 4$

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## Chebyshev Approximation

- The magnitude-square response of an  $N$ -th order analog lowpass **Type 1 Chebyshev filter** is given by

$$|H_a(s)|^2 = \frac{1}{1 + \varepsilon^2 T_N^2(\Omega/\Omega_p)}$$

where  $T_N(\Omega)$  is the **Chebyshev polynomial** of order  $N$ :

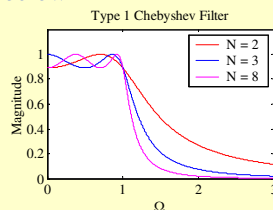
$$T_N(\Omega) = \begin{cases} \cos(N \cos^{-1} \Omega), & |\Omega| \leq 1 \\ \cosh(N \cosh^{-1} \Omega), & |\Omega| > 1 \end{cases}$$

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## Chebyshev Approximation

- Typical magnitude response plots of the analog lowpass **Type 1 Chebyshev filter** are shown below



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## Chebyshev Approximation

- If at  $\Omega = \Omega_s$  the magnitude is equal to  $1/A$ , then

$$|H_a(j\Omega_s)|^2 = \frac{1}{1 + \varepsilon^2 T_N^2(\Omega_s/\Omega_p)} = \frac{1}{A^2}$$

- Solving the above we get

$$N = \frac{\cosh^{-1}(\sqrt{A^2 - 1}/\varepsilon)}{\cosh^{-1}(\Omega_s/\Omega_p)} = \frac{\cosh^{-1}(1/k_1)}{\cosh^{-1}(1/k)}$$

- Order  $N$  is chosen as the nearest integer greater than or equal to the above value

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## Chebyshev Approximation

- The magnitude-square response of an  $N$ -th order analog lowpass **Type 2 Chebyshev** (also called **inverse Chebyshev**) filter is given by

$$|H_a(j\Omega)|^2 = \frac{1}{1 + \varepsilon^2 \left[ \frac{T_N(\Omega_s/\Omega_p)}{T_N(\Omega_s/\Omega)} \right]^2}$$

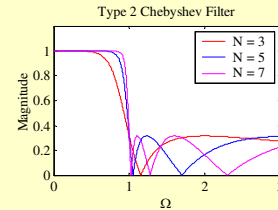
where  $T_N(\Omega)$  is the **Chebyshev polynomial** of order  $N$

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## Chebyshev Approximation

- Typical magnitude response plots of the analog lowpass **Type 2 Chebyshev filter** are shown below



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## Chebyshev Approximation

- The order  $N$  of the **Type 2 Chebyshev filter** is determined from given  $\varepsilon$ ,  $\Omega_s$ , and  $A$  using

$$N = \frac{\cosh^{-1}(\sqrt{A^2 - 1}/\varepsilon)}{\cosh^{-1}(\Omega_s/\Omega_p)} = \frac{\cosh^{-1}(1/k_1)}{\cosh^{-1}(1/k)}$$

- Example** - Determine the lowest order of a Chebyshev lowpass filter with a 1-dB cutoff frequency at 1 kHz and a minimum attenuation of 40 dB at 5 kHz -

$$N = \frac{\cosh^{-1}(1/k_1)}{\cosh^{-1}(1/k)} = 2.6059$$

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## Elliptic Approximation

- The square-magnitude response of an elliptic lowpass filter is given by

$$|H_a(j\Omega)|^2 = \frac{1}{1 + \varepsilon^2 R_N^2(\Omega/\Omega_p)}$$

where  $R_N(\Omega)$  is a rational function of order  $N$  satisfying  $R_N(1/\Omega) = 1/R_N(\Omega)$ , with the roots of its numerator lying in the interval  $0 < \Omega < 1$  and the roots of its denominator lying in the interval  $1 < \Omega < \infty$

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## Elliptic Approximation

- For given  $\Omega_p$ ,  $\Omega_s$ ,  $\varepsilon$ , and  $A$ , the filter order can be estimated using

$$N \cong \frac{2 \log_{10}(4/k_1)}{\log_{10}(1/\rho)}$$

where  $k' = \sqrt{1 - k^2}$

$$\rho_0 = \frac{1 - \sqrt{k'}}{2(1 + \sqrt{k'})}$$

$$\rho = \rho_0 + 2(\rho_0)^5 + 15(\rho_0)^9 + 150(\rho_0)^{13}$$

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## Elliptic Approximation

- Example** - Determine the lowest order of an elliptic lowpass filter with a 1-dB cutoff frequency at 1 kHz and a minimum attenuation of 40 dB at 5 kHz  
**Note:**  $k = 0.2$  and  $1/k_1 = 196.5134$

- Substituting these values we get

$$k' = 0.979796, \quad \rho_0 = 0.00255135, \\ \rho = 0.0025513525$$

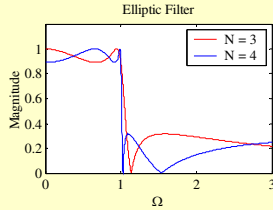
- and hence  $N = 2.23308$
- Choose  $N = 3$

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## Elliptic Approximation

- Typical magnitude response plots with  $\Omega_p = 1$  are shown below



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## Analog Lowpass Filter Design

- **Example** - Design an elliptic lowpass filter of lowest order with a 1-dB cutoff frequency at 1 kHz and a minimum attenuation of 40 dB at 5 kHz

- Code fragments used

```
[N, Wn] = ellipord(Wp, Ws, Rp, Rs, 's');
[b, a] = ellip(N, Rp, Rs, Wn, 's');
```

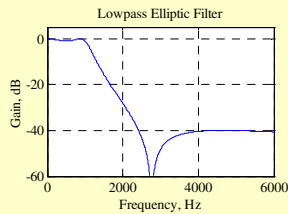
```
with Wp = 2*pi*1000;
     Ws = 2*pi*5000;
     Rp = 1;
     Rs = 40;
```

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## Analog Lowpass Filter Design

- Gain plot



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## Design of Analog Highpass, Bandpass and Bandstop Filters

- Steps involved in the design process:

**Step 1** - Develop of specifications of a prototype analog lowpass filter  $H_{LP}(s)$  from specifications of desired analog filter  $H_D(s)$  using a frequency transformation

**Step 2** - Design the prototype analog lowpass filter

**Step 3** - Determine the transfer function  $H_D(s)$  of desired analog filter by applying the inverse frequency transformation to  $H_{LP}(s)$

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## Design of Analog Highpass, Bandpass and Bandstop Filters

- Let  $s$  denote the Laplace transform variable of prototype analog lowpass filter  $H_{LP}(s)$  and  $\hat{s}$  denote the Laplace transform variable of desired analog filter  $H_D(\hat{s})$
- The mapping from  $s$ -domain to  $\hat{s}$ -domain is given by the invertible transformation

$$s = F(\hat{s})$$

- Then  $H_D(\hat{s}) = H_{LP}(s)|_{s=F(\hat{s})}$   
 $H_{LP}(s) = H_D(\hat{s})|_{\hat{s}=F^{-1}(s)}$

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## Analog Highpass Filter Design

- Spectral Transformation:

$$s = \frac{\Omega_p \hat{\Omega}_p}{\hat{\Omega}}$$

where  $\Omega_p$  is the passband edge frequency of  $H_{LP}(s)$  and  $\hat{\Omega}_p$  is the passband edge frequency of  $H_{HP}(\hat{s})$

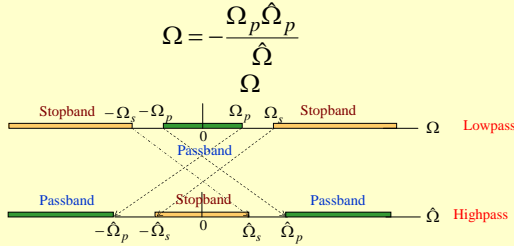
- On the imaginary axis the transformation is

$$\Omega = -\frac{\Omega_p \hat{\Omega}_p}{\hat{\Omega}}$$

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## Analog Highpass Filter Design



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## Analog Highpass Filter Design

- **Example** - Design an analog Butterworth highpass filter with the specifications:

$$\hat{F}_p = 4 \text{ kHz}, \hat{F}_s = 1 \text{ kHz}, \alpha_p = 0.1 \text{ dB}, \alpha_s = 40 \text{ dB}$$

- Choose  $\Omega_p = 1$

$$\Omega_s = \frac{2\pi\hat{F}_p}{2\pi\hat{F}_s} = \frac{\hat{F}_p}{\hat{F}_s} = \frac{4000}{1000} = 4$$

- Analog lowpass filter specifications:  $\Omega_p = 1$ ,  $\Omega_s = 4$ ,  $\alpha_p = 0.1 \text{ dB}$ ,  $\alpha_s = 40 \text{ dB}$

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## Analog Highpass Filter Design

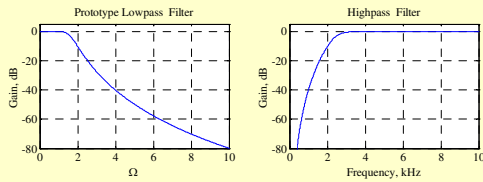
- Code fragments used

```
[N, Wn] = buttord(1, 4, 0.1, 40, 's');
```

```
[B, A] = butter(N, Wn, 's');
```

```
[num, den] = lp2hp(B, A, 2*pi*4000);
```

- Gain plots



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## Analog Bandpass Filter Design

- Spectral Transformation

$$s = \Omega_p \frac{\hat{s}^2 + \hat{\Omega}_o^2}{\hat{s}(\hat{\Omega}_{p2} - \hat{\Omega}_{p1})}$$

where  $\Omega_p$  is the passband edge frequency of  $H_{LP}(s)$ , and  $\hat{\Omega}_{p1}$  and  $\hat{\Omega}_{p2}$  are the lower and upper passband edge frequencies of desired bandpass filter  $H_{BP}(\hat{s})$

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## Analog Bandpass Filter Design

- On the imaginary axis the transformation is

$$\Omega = -\Omega_p \frac{\hat{\Omega}_o^2 - \hat{\Omega}^2}{\hat{\Omega} B_w}$$

where  $B_w = \hat{\Omega}_{p2} - \hat{\Omega}_{p1}$  is the width of passband and  $\hat{\Omega}_o$  is the **passband center frequency** of the bandpass filter

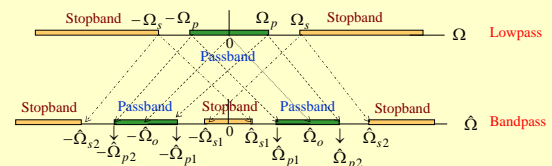
- Passband edge frequency  $\pm\Omega_p$  is mapped into  $\mp\hat{\Omega}_{p1}$  and  $\pm\hat{\Omega}_{p2}$ , lower and upper passband edge frequencies

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## Analog Bandpass Filter Design

$$\Omega = -\Omega_p \frac{\hat{\Omega}_o^2 - \hat{\Omega}^2}{\hat{\Omega} B_w}$$



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## Analog Bandpass Filter Design

- Stopband edge frequency  $\pm\Omega_s$  is mapped into  $\mp\hat{\Omega}_{s1}$  and  $\pm\hat{\Omega}_{s2}$ , lower and upper stopband edge frequencies

- Also,

$$\hat{\Omega}_o^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2} = \hat{\Omega}_{s1}\hat{\Omega}_{s2}$$

- If bandedge frequencies do not satisfy the above condition, then one of the frequencies needs to be changed to a new value so that the condition is satisfied

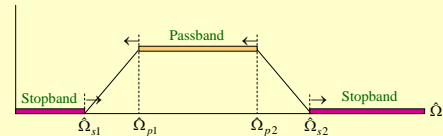
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## Analog Bandpass Filter Design

- **Case 1:**  $\hat{\Omega}_{p1}\hat{\Omega}_{p2} > \hat{\Omega}_{s1}\hat{\Omega}_{s2}$

To make  $\hat{\Omega}_{p1}\hat{\Omega}_{p2} = \hat{\Omega}_{s1}\hat{\Omega}_{s2}$  we can either increase any one of the stopband edges or decrease any one of the passband edges as shown below



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## Analog Bandpass Filter Design

- (1) Decrease  $\hat{\Omega}_{p1}$  to  $\hat{\Omega}_{s1}\hat{\Omega}_{s2} / \hat{\Omega}_{p2}$

⇒ larger passband and shorter leftmost transition band

- (2) Increase  $\hat{\Omega}_{s1}$  to  $\hat{\Omega}_{p1}\hat{\Omega}_{p2} / \hat{\Omega}_{s2}$

⇒ No change in passband and shorter leftmost transition band

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## Analog Bandpass Filter Design

- **Note:** The condition  $\hat{\Omega}_o^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2} = \hat{\Omega}_{s1}\hat{\Omega}_{s2}$  can also be satisfied by decreasing  $\hat{\Omega}_{p2}$  which is not acceptable as the passband is reduced from the desired value
- Alternately, the condition can be satisfied by increasing  $\hat{\Omega}_{s2}$  which is not acceptable as the upper stop band is reduced from the desired value

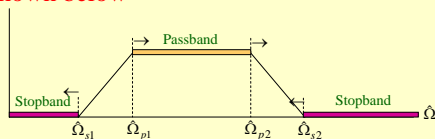
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## Analog Bandpass Filter Design

- **Case 2:**  $\hat{\Omega}_{p1}\hat{\Omega}_{p2} < \hat{\Omega}_{s1}\hat{\Omega}_{s2}$

To make  $\hat{\Omega}_{p1}\hat{\Omega}_{p2} = \hat{\Omega}_{s1}\hat{\Omega}_{s2}$  we can either decrease any one of the stopband edges or increase any one of the passband edges as shown below



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## Analog Bandpass Filter Design

- (1) Increase  $\hat{\Omega}_{p2}$  to  $\hat{\Omega}_{s1}\hat{\Omega}_{s2} / \hat{\Omega}_{p1}$

⇒ larger passband and shorter rightmost transition band

- (2) Decrease  $\hat{\Omega}_{s2}$  to  $\hat{\Omega}_{p1}\hat{\Omega}_{p2} / \hat{\Omega}_{s1}$

⇒ No change in passband and shorter rightmost transition band

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## Analog Bandpass Filter Design

- **Note:** The condition  $\hat{\Omega}_o^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2} = \hat{\Omega}_{s1}\hat{\Omega}_{s2}$  can also be satisfied by increasing  $\hat{\Omega}_{p1}$  which is not acceptable as the passband is reduced from the desired value
- Alternately, the condition can be satisfied by decreasing  $\hat{\Omega}_{s1}$  which is not acceptable as the lower stopband is reduced from the desired value

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## Analog Bandpass Filter Design

- **Example - Design an analog elliptic bandpass filter with the specifications:**

$$\hat{F}_{p1} = 4 \text{ kHz}, \hat{F}_{p2} = 7 \text{ kHz}, \hat{F}_{s1} = 3 \text{ kHz}$$

$$\hat{F}_{s2} = 8 \text{ kHz}, \alpha_p = 1 \text{ dB}, \alpha_s = 22 \text{ dB}$$

- Now  $\hat{F}_{p1}\hat{F}_{p2} = 28 \times 10^6$  and  $\hat{F}_{s1}\hat{F}_{s2} = 24 \times 10^6$

- Since  $\hat{F}_{p1}\hat{F}_{p2} > \hat{F}_{s1}\hat{F}_{s2}$  we choose

$$\hat{F}_{p1} = \hat{F}_{s1}\hat{F}_{s2} / \hat{F}_{p2} = 3.571428 \text{ kHz}$$

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## Analog Bandpass Filter Design

- We choose  $\Omega_p = 1$

- Hence

$$\Omega_s = \frac{24 - 9}{(25/7) \times 3} = 1.4$$

- Analog lowpass filter specifications:  $\Omega_p = 1$ ,  $\Omega_s = 1.4$ ,  $\alpha_p = 1 \text{ dB}$ ,  $\alpha_s = 22 \text{ dB}$

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## Analog Bandpass Filter Design

- Code fragments used

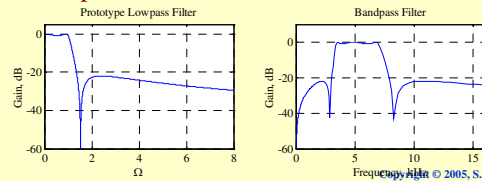
```
[N, Wn] = ellipord(1, 1.4, 1, 22, 's');
```

```
[B, A] = ellip(N, 1, 22, Wn, 's');
```

```
[num, den]
```

```
= lp2bp(B, A, 2*pi*4.8989795, 2*pi*25/7);
```

- Gain plot



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## Analog Bandstop Filter Design

- Spectral Transformation

$$s = \Omega_s \frac{\hat{s}(\hat{\Omega}_{s2} - \hat{\Omega}_{s1})}{\hat{s}^2 + \hat{\Omega}_o^2}$$

where  $\Omega_s$  is the stopband edge frequency of  $H_{LP}(s)$ , and  $\hat{\Omega}_{s1}$  and  $\hat{\Omega}_{s2}$  are the lower and upper stopband edge frequencies of the desired bandstop filter  $H_{BS}(\hat{s})$

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## Analog Bandstop Filter Design

- On the imaginary axis the transformation is

$$\Omega = \Omega_s \frac{\hat{\Omega} B_w}{\hat{\Omega}_o^2 - \hat{\Omega}^2}$$

where  $B_w = \hat{\Omega}_{s2} - \hat{\Omega}_{s1}$  is the width of stopband and  $\hat{\Omega}_o$  is the **stopband center frequency** of the bandstop filter

- Stopband edge frequency  $\pm \Omega_s$  is mapped into  $\mp \hat{\Omega}_{s1}$  and  $\pm \hat{\Omega}_{s2}$ , lower and upper stopband edge frequencies

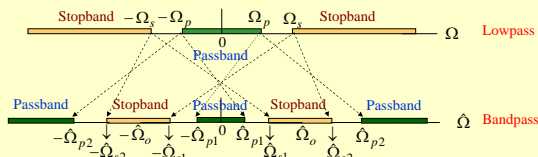
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## Analog Bandstop Filter Design

- Passband edge frequency  $\pm\Omega_p$  is mapped into  $\mp\hat{\Omega}_{p1}$  and  $\pm\hat{\Omega}_{p2}$ , lower and upper passband edge frequencies



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## Analog Bandstop Filter Design

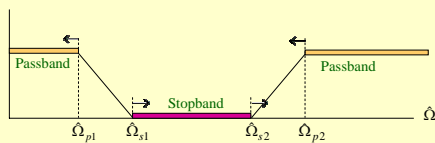
- Also,  $\hat{\Omega}_o^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2} = \hat{\Omega}_{s1}\hat{\Omega}_{s2}$
- If bandedge frequencies do not satisfy the above condition, then one of the frequencies needs to be changed to a new value so that the condition is satisfied

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## Analog Bandstop Filter Design

- Case 1:**  $\hat{\Omega}_{p1}\hat{\Omega}_{p2} > \hat{\Omega}_{s1}\hat{\Omega}_{s2}$
- To make  $\hat{\Omega}_{p1}\hat{\Omega}_{p2} = \hat{\Omega}_{s1}\hat{\Omega}_{s2}$  we can either increase any one of the stopband edges or decrease any one of the passband edges as shown below



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## Analog Bandstop Filter Design

- Decrease  $\hat{\Omega}_{p2}$  to  $\hat{\Omega}_{s1}\hat{\Omega}_{s2} / \hat{\Omega}_{p2}$   
 ⇒ larger high-frequency passband and shorter rightmost transition band
- Increase  $\hat{\Omega}_{s2}$  to  $\hat{\Omega}_{p1}\hat{\Omega}_{p2} / \hat{\Omega}_{s2}$   
 ⇒ No change in passbands and shorter rightmost transition band

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## Analog Bandstop Filter Design

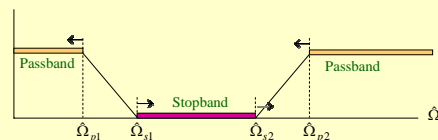
- Note:** The condition  $\hat{\Omega}_o^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2} = \hat{\Omega}_{s1}\hat{\Omega}_{s2}$  can also be satisfied by decreasing  $\hat{\Omega}_{p1}$  which is not acceptable as the low-frequency passband is reduced from the desired value
- Alternately, the condition can be satisfied by increasing  $\hat{\Omega}_{s1}$  which is not acceptable as the stopband is reduced from the desired value

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- Case 1:**  $\hat{\Omega}_{p1}\hat{\Omega}_{p2} < \hat{\Omega}_{s1}\hat{\Omega}_{s2}$
- To make  $\hat{\Omega}_{p1}\hat{\Omega}_{p2} = \hat{\Omega}_{s1}\hat{\Omega}_{s2}$  we can either decrease any one of the stopband edges or increase any one of the passband edges as shown below



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- (1) Increase  $\hat{\Omega}_{p1}$  to  $\hat{\Omega}_{s1}\hat{\Omega}_{s2}/\hat{\Omega}_{p1}$   
⇒ larger passband and shorter leftmost transition band
- (2) Decrease  $\hat{\Omega}_{s1}$  to  $\hat{\Omega}_{p1}\hat{\Omega}_{p2}/\hat{\Omega}_{s1}$   
⇒ No change in passbands and shorter leftmost transition band

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- Note: The condition  $\hat{\Omega}_o^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2} = \hat{\Omega}_{s1}\hat{\Omega}_{s2}$  can also be satisfied by increasing  $\hat{\Omega}_{p2}$  which is not acceptable as the high-frequency passband is decreased from the desired value
- Alternately, the condition can be satisfied by decreasing  $\hat{\Omega}_{s2}$  which is not acceptable as the stopband is decreased

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