Faculty of Engineering Chulalongkorn University

2102-502 Rand Signal & System, Final Exam, 3 Oct 2006, 9:00-12:00

1) A discrete-time linear system has the impulse response $h(t) = (\frac{1}{2})^t + (\frac{1}{3})^t, t \ge 0$. If the input is zero mean unit WGN.
   a) Find the autocovariance of the output from h(t). (4)
   b) Find the power spectral density of the output from H(z). (4)
   c) What is the average power of the output? (2)

2) Let X(t) be a stationary zero mean continuous-time Gaussian process with power spectral density $\phi_{XX}(\omega) = \dfrac{\omega^2}{1+\omega^4}$. Find the probability density function of random variable X(5). What is the transfer function of the whitenning filter of X(t)? (10)

3) Let X(t), a zero mean random signal defined over the interval $0 \le t, s \le T$, have auto-covariance function $c_{XX}(t,s)$ with orthonormal eigenfunctions and eigenvalues $\phi_n(t)$ and λ_n.
   a) If $Y(t) = X(t) + \mu$, we can use $\phi_n(t)$ as the Kahunen-Loeve expansion basis for Y(t) to get $\alpha_n = \int_0^T \phi_n(t)Y(t)dt$. Evaluate $E\{\alpha_n\}$ (*will be non-zero*) and also $Cov(\alpha_n, \alpha_m)$ to show that all α_n are still uncorrelated. (8)
   b) If $c_{XX}(t,s) = \sigma^2\cos(\omega_0(t-s))$ and we require an uncorrelated Fourier series expansion of X(t). What is the suitable time interval for this Fourier series expansion? (2)

4) Let X(t) be a zero mean random signal defined over the interval $-1 \le t, s \le 1$ with auto-covariance function $c_{XX}(t,s) = 1 + 6ts + 5(3t^2-1)(3s^2-1)$. For Kahunen-Loeve expansion,
   a) use Mercer's theorem to find all *orthonormal* eigenfunctions $\phi_n(t)$ and corresponding eigenvalues λ_n. What is the average signal energy of X(t)? (7)
   b) If one realization is $X(t) = t$, for $-1 \le t \le 1$, find all its K-L coefficients α_n. (3)

5) Let $x_1(t)$ and $x_2(t)$ are the position and velocity of an object then $x_1(t) = \int_0^t x_2(\tau)d\tau + x_1(0)$.
   With sampling time of 1 second, this can be approximated with difference equation as $x_1(t+1) = x_2(t) + x_1(t)$. If the randomness in velocity is $x_2(t+1) = x_2(t) + v(t)$ where v(t) is zero mean unit WGN. We can observe $Y(t) = x_1(t) + N(t)$, only the position, where N(t) is zero mean unit WGN measurement noise. Let state vector $\mathbf{X}(t) = [\ x_1(t)\ x_2(t)\ ]^T$ with known initial state $\mathbf{X}(0) = [\ 0 \quad 0\ ]^T$. Formulate this problem as Kalman filtering and find the Kalman gain $\mathbf{k}(t)$, the estimation error covariance matrix $\mathbf{P}(t)$ and the estimation $\hat{\mathbf{X}}(t)$ if the measurement Y(t) = 0.1, 0.2, 0.1 for t = 1, 2, 3 second respectively. (10)