Faculty of Engineering Chulalongkorn University

2102-502 Rand Signal & System, Final Exam, 30 Sep 2008, 9:00-12:00

1) Let $\phi_X(\omega) = \dfrac{(1-\cos 4\omega)}{(1-\cos\omega)}$ be a power spectral density of random signal X(t).
   a) Find the innovation filter transfer function H(Z) of X(t). (4)
   b) Find the impulse response h(t) of this filter. (4)
   c) What is the average power of X(t)? (2)

2) Show that $C_{XX}(\tau) = 4 - |\tau|$, for $\tau = 0, \pm 1, \pm 2$ and zero elsewhere, is not realizable. (5)
   Suggest the minimum change only for $C_{XX}(0)$ so that it becomes realizable. (5)

3) Let X(t) be a stationary zero mean unit white Gaussian noise. Define a new signal Y(t) by

$$Y(t) = \int_{t-2}^{t} X(\lambda)d\lambda$$

   Find $C_{YY}(t,s) = E\{Y(t)Y(s)\}$ and also plot graph of the result. (10)

4) Show that the *Kahunen-Loeve* expansion coefficients α_n's of a random process X(t) are mutually orthogonal. Describe the relationship of its *basis functions* and *variances* of α_n's to the statistical properties of the random process X(t) itself. (10)

5) (*Courant-Hilbert*). Let $C_{XX}(s,t) = \dfrac{1}{2\pi}\cdot\dfrac{1-h^2}{1-2h\cos(s-t)+h^2}$ with $|h| < 1$, for $0 \le t, s \le 2\pi$.
   a) Show that the Fourier series coefficients of X(t), $0 \le t \le 2\pi$, will be uncorrelated. (4)
   b) The basis functions of the K-L expansion of X(t) will be cos(nt) and sin(nt), the same as Fourier series. What are the variances for each K-L expansion coefficients. (6)

6) If you drive a car starting from rest with *constant* but *unknown* acceleration. The car can measure the velocity every 0.1 second with zero mean unit white Gaussian measurement noise. Assume that the car follows the Newton's Law of Motion exactly (noiseless). Formulate the Kalman filter for this case to estimate the position, speed and acceleration at all time. Write only the state equation and output equation with numerical parameters specified, *no need* to solve for the solution. (10)

7) Show that the innovation sequence $\nu(t)$ of a *Kalman* filter is in fact the output from the *Gram-Schmidt* orthogonalization on measurement sequence Y(t). Rewrite $\nu(t)$ in terms of *prediction error* $\underline{\mathbf{\Delta}}(t)$ and then find the variances of each $\nu(t)$ as functions of *prediction error covariance matrix* $\mathbf{M}(t)$. (10)