Chapter 13

Fast Fracture and Toughness

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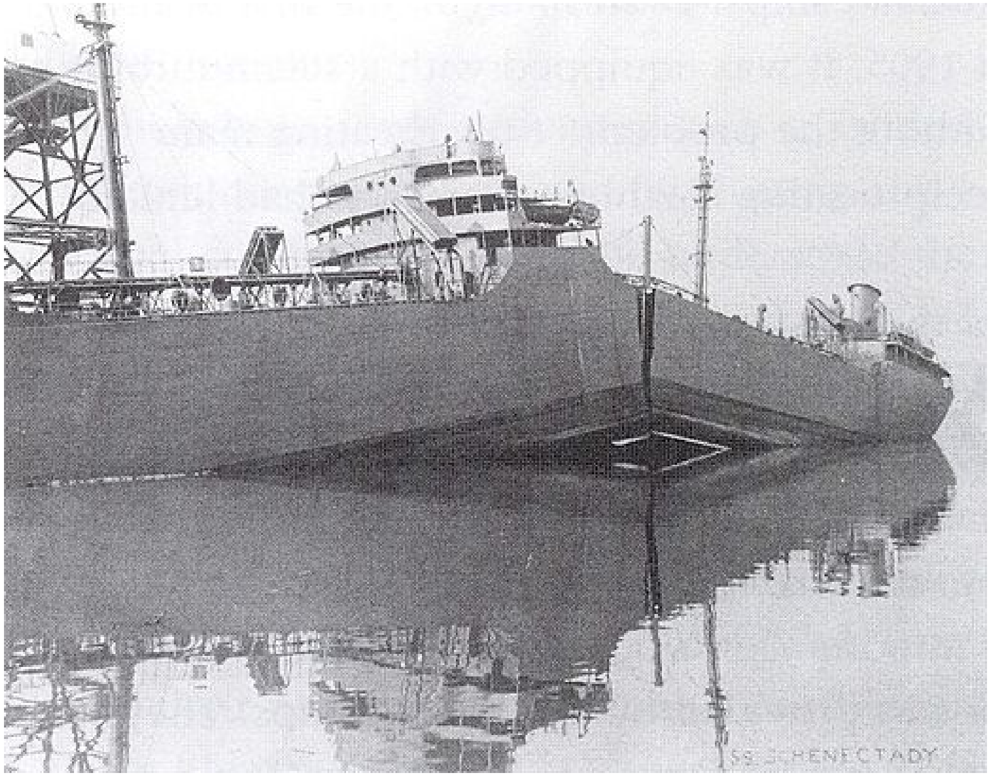
Why is fracture mechanic is important?

Brittle fracture can cause sudden catastrophic failures.



The S.S. *Schenectady* split apart by brittle fracture while in harbor, 1943.

Fracture mechanic

Brittle fracture can cause sudden catastrophic failures.

Moonie to Brisbane pipeline incident (2007)

Why is fracture mechanic is important?

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Brittle fracture can cause sudden catastrophic failures.

Fuselage of Aloha Airlines Flight 243 after the explosive decompression(1988)

Fracture mechanic

Fracture mechanic is the field of **mechanics** concerned with the study of the propagation of cracks in materials. It uses methods of analytical solid **mechanics** to calculate the driving force on a crack and those of experimental solid **mechanics** to characterize the material's resistance to **fracture**.

https://en.wikipedia.org/wiki/**Fracture_mechanics**

Why Fracture mechanic

- Conventional design produces based only on some maximum stress criterion are not adequate under all circumstances.

- Fracture mechanics determines failure based on the interaction between the ***applied stress*****,** the ***crack*** (or flaw) and the materials parameters (e.g. fracture toughness)

Why Fracture mechanic

- Instead of the magnitude of the stress or strain, fracture mechanics is concerned primarily with distribution of stresses and displacement in the vicinity of a **crack tip.**

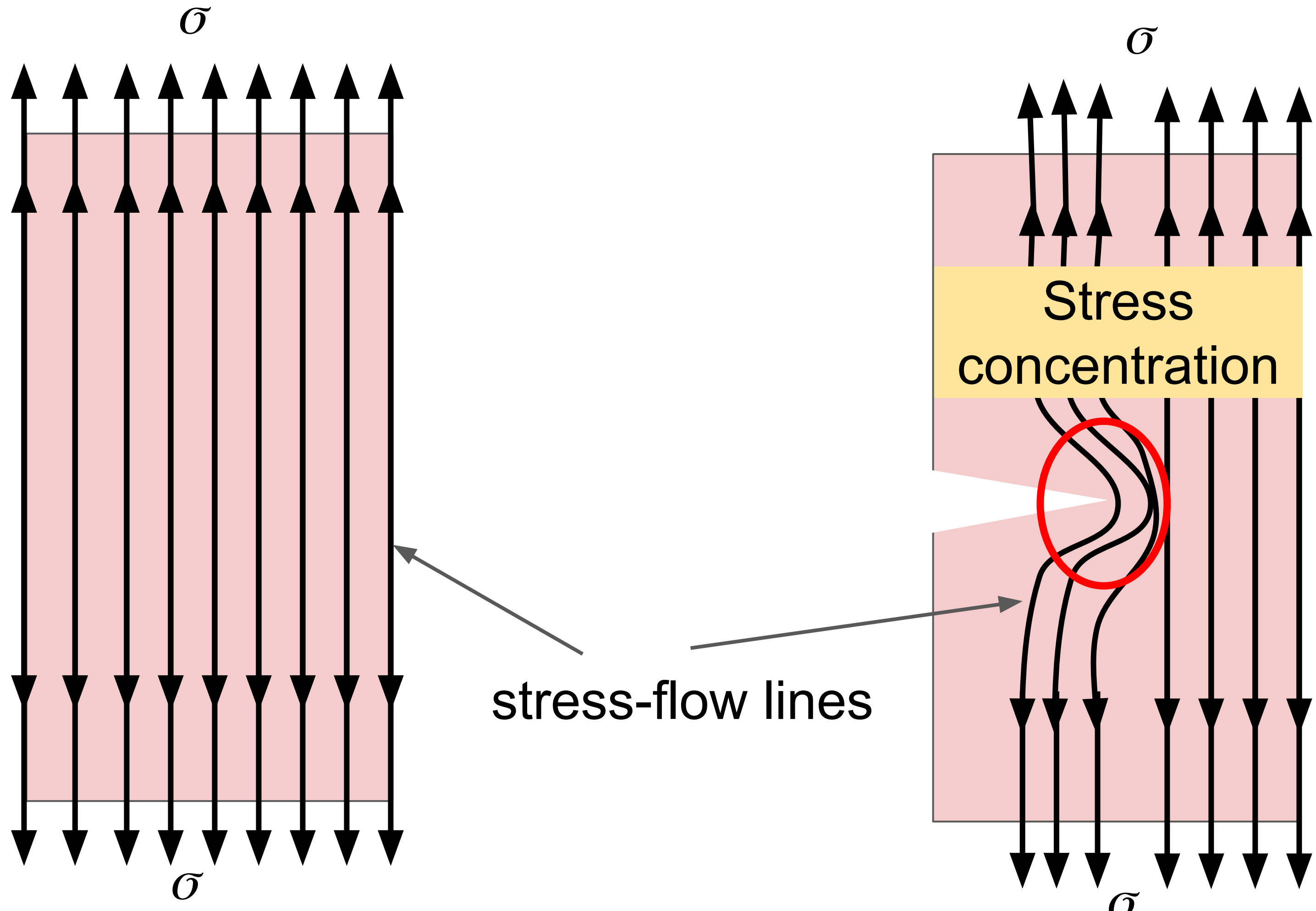
- Fracture mechanics is particularly applicable to the failure of brittle material but under certain circumstances to other material as well.

Why Fracture mechanic

- The difference between the theoretical material fracture (cohesive) strength calculated from the interatomic bonding energy and the value actually enormous, perhaps 2-3 orders of magnitude.
- Griffin, in 1992, concluded that any real material has flaws, microcracks or other defects that would have the effect of concentrating the stress sufficient to reach the theoretical fracture stress in highly localized regions,cracks would grow under an applied stress until failure occurred

Why Fracture mechanic

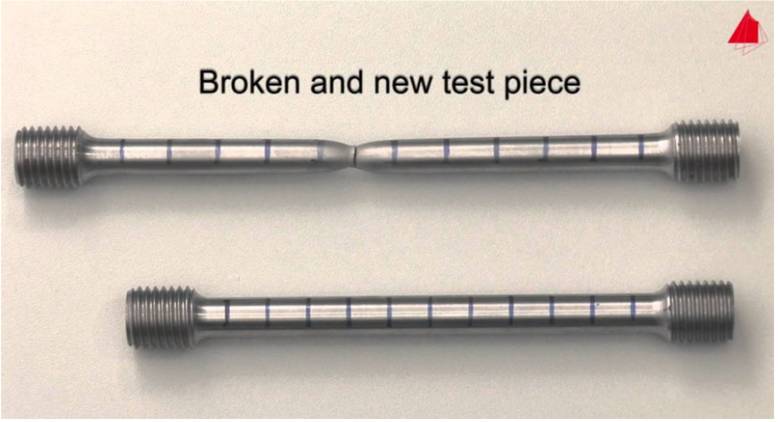
Stress concentration : can be visualized as the concentration of stress-flow lines due to a geometrical discontinuity in the continuum.



Definition of fast fast fracture

- Fast fracture may be defined as an *unstable propagation of crack (or flaw)* which leads to fracture of the part. The crack propagates at a rapid speed (which is usually in the same order of magnitude as the speed of sound in that material).

- This is different from "static fracture" such as one observed in uni-axial tension test in which fracture is induced by extensive plastic deformation and breaking by plastic instability (necking).



Energy Criterion for Fast Fracture: Blowing Up a Balloon

- Energy may be stored in (1) compressed gas in the balloon and (2) elastic energy (U^{el}) of the rubber membrane.

- Bursting can occur by either (1) a crack or flaw is larger than a critical size or (2) the pressure inside reaches a critical value for a particular size of crack.

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Criterion for Unstable Crack Propagation

For a differential (very small) advance of crack, if more energy is released than is absorbed, then the crack can propagate in an unstable manner.

- If the crack is to advance, the work done by loads must be equal to or more than the sum of the elastic energy and the energy absorbed at crack tip.

$$\delta W \geq \delta U^{\mathrm{el}} + G_c \, t\delta a$$

- G_c is the energy absorbed per unit area of crack. It is *a material property and always has positive value*.

Criterion for Unstable Crack Propagation

- G_c is the energy absorbed in making a unit area of crack, and it is called "toughness" or "critical strain energy release rate." Its unit is J m^{-2}.

Determination of Gc for Sellotape Adhesive

- This same quantity G_c measures the strength of adhesives.
- You can measure it for the adhesive used on sticky tape (e.g., Sellotape) by hanging a weight on a partly peeled length while supporting the roll so that it can freely rotate (hang it on a pencil) as shown in Figure 13.1.

Determination of Gc for Sellotape Adhesive

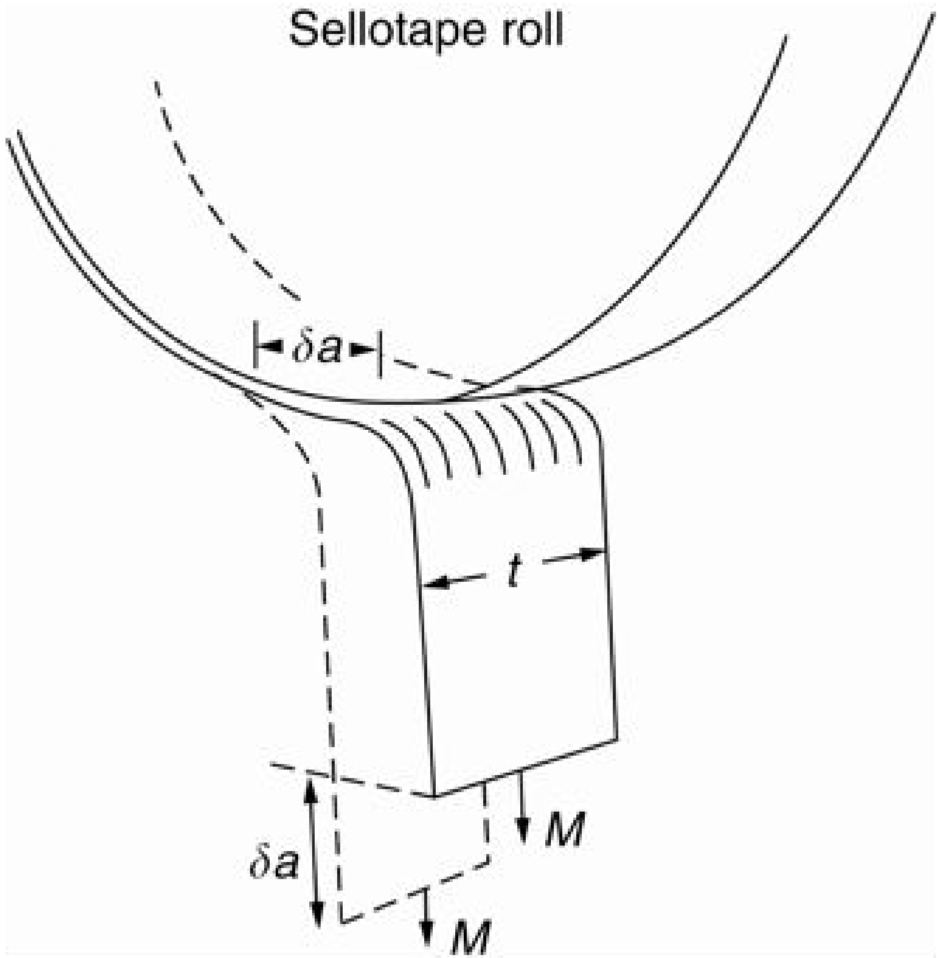


Figure 13.1

How to determine G_c for Sellotape adhesive.

The “Strength” of Sellotape Adhesive.

- Increase the pulling mass to the value M that just causes rapid peeling (= fast fracture). For this geometry, the quantity δU^{el} is small compared to the work done by M (the tape has comparatively little “give”) and it can be neglected [$\delta U^{el} \ll \delta W$]. Then, from our energy formula,

$$\delta W = G_c \, t\delta a$$

for fast fracture. In our case,

The “Strength” of Sellotape Adhesive.

$$\begin{aligned} Mg\delta a &= G_c t \delta a \\ Mg &= G_c t \end{aligned}$$

and therefore,

$$G_c = \frac{Mg}{t}$$

Typically, $t = 0.02$ m, $M = 0.15$ kg, and $g \approx 10$ m s^{-2}, giving

$$G_c \approx 75\,\mathrm{J\,m^{-2}}$$

Fast Fracture at Fixed Displacements

- Naturally, in most cases, we cannot neglect δU^{el}, and must derive more general relationships. Let us first consider a cracked plate of material loaded so that the displacements at the boundary of the plate are fixed.

- This is a common mode of loading a material—it occurs frequently in welds between large pieces of steel, for example—and is one that allows us to calculate δU^{el} quite easily.

Fast fracture at Fixed displacement.

Fast Fracture at Fixed Displacements

- The plate shown in Figure 13.2 is clamped under tension so that its upper and lower ends are **fixed**. Since the ends cannot move, the forces acting on them can do no work. and $\delta W = 0$.

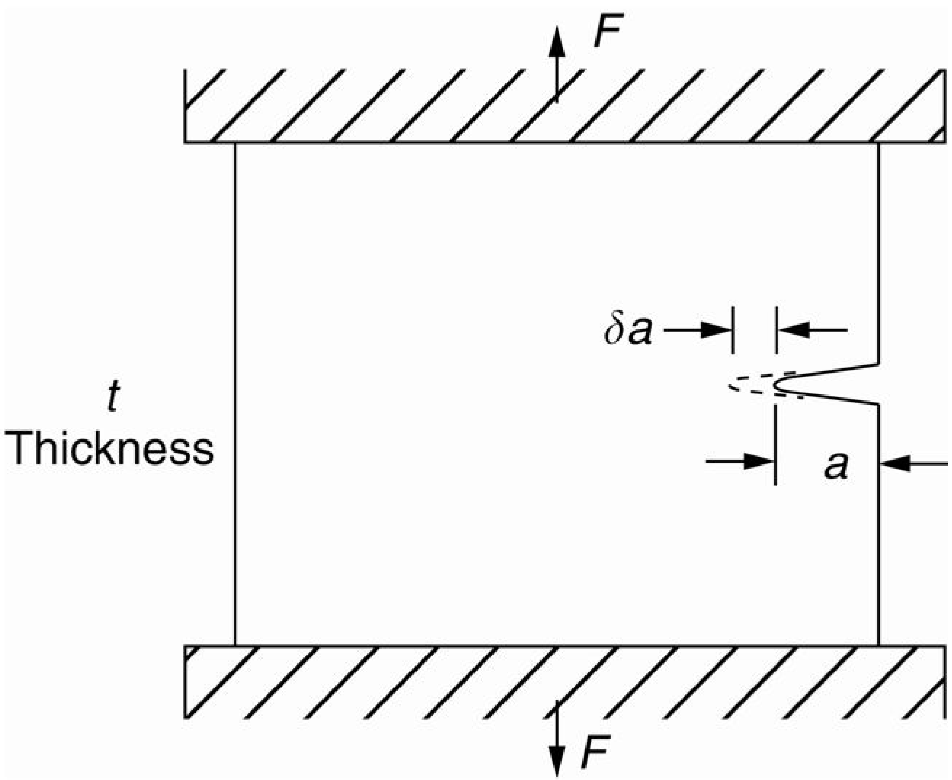


Figure 13.2

Fast fracture in a fixed plate.

Fast Fracture at Fixed Displacements

- Because the upper and lower ends are fixed, there is no

displacement, there is no work done by F (or $\delta W = 0$)

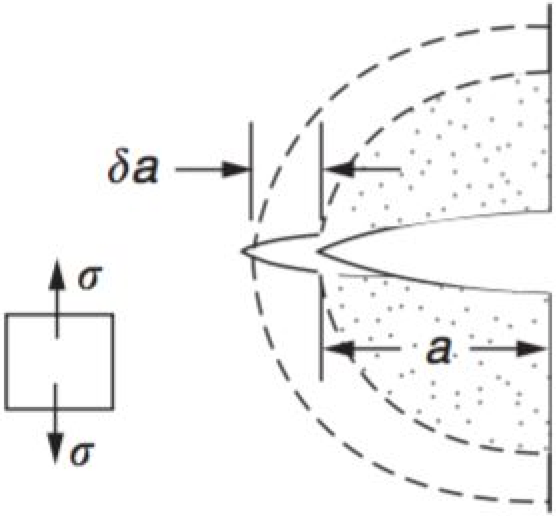
$$\delta W \geq \delta U^{el} + G_c t \delta a$$

- For the onset of fast fracture, the criterion for this particular case is

$$-\delta U^{el} = G_c t \delta a \qquad \text{--- (13.2)}$$

Elastic Energy or Elastic Strain Energy U^{el}

- Let us examine a small cube of material of unit volume inside our plate. Due to the load F this cube is subjected to a stress σ, producing a strain ε. Each unit cube therefore has strain energy U^{el} of $1/2\sigma\varepsilon$, or $\sigma^2/2E$. If we now introduce a crack of length **a**, we can consider that the material in the dotted region relaxes (to zero stress) so as to lose all its strain energy. The energy change is shown in the following equation.



$$U^{\text{el}} = \frac{\sigma^2}{2E}\frac{\pi a^2 t}{2}$$

-

Elastic Energy or Elastic Strain Energy U^{el}

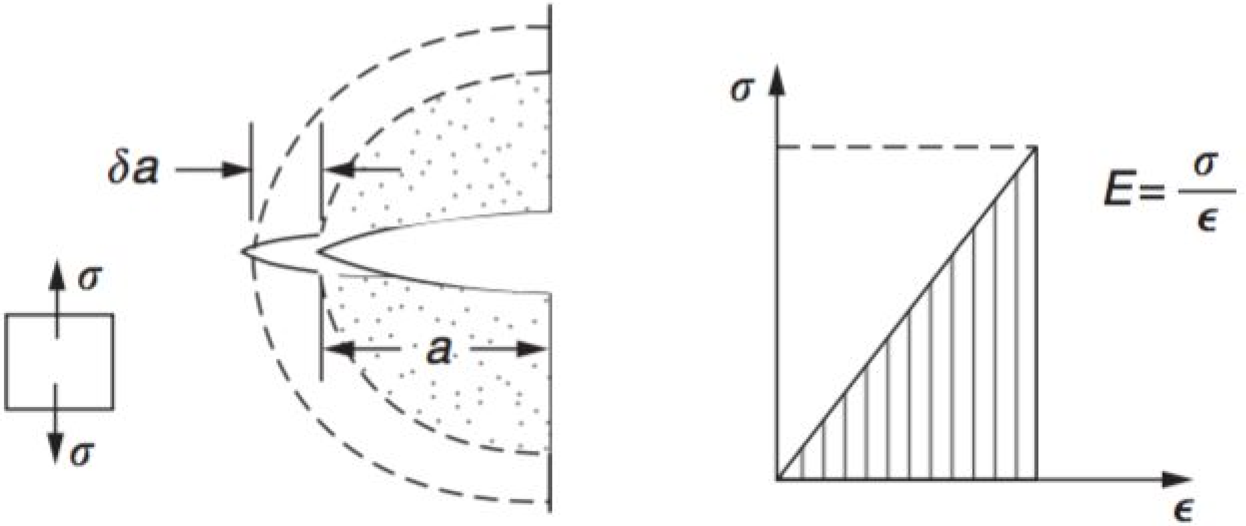


Figure 13.3

The release of stored strain energy as a crack grows.

Fast Fracture at Fixed Displacements

- As the crack advances by a (differential) distance δa, we can find the differential change in elastic energy (δU^{el}) to be:

$$\delta U^{\text{el}} = \frac{\sigma^2}{2E}\frac{2\pi a t}{2}\delta a$$

- The criterion equation ($-\delta U^{\text{el}} = G_c t \delta a$) then becomes

$$\frac{\sigma^2 \pi a}{2E} = G_c$$

Fast Fracture at Fixed Displacements

The critical condition (Equation (13.2)) then gives

$$\frac{\sigma^2 \pi a}{2E} = G_c$$

at onset of fast fracture.

Actually, our assumption about the way in which the plate material relaxes is obviously rather crude, and a rigorous mathematical solution of the elastic stresses and strains indicates that our estimate of δU^{el} is too low by exactly a factor of 2. Thus, correctly, we have

$$\frac{\sigma^2 \pi a}{E} = G_c$$

which reduces to

$$\sigma\sqrt{\pi a} = \sqrt{EG_c} \tag{13.3}$$

at fast fracture.

Fast fracture at Fixed load.

Fast Fracture at Fixed Loads

- Here the situation is a little more complicated than it was in the case of fixed displacements. As the crack grows, the plate becomes less stiff, and relaxes so that the applied forces move and do work.

- δW is therefore finite and positive. However, δU^{el} is now positive also (it turns out that some of δW goes into increasing the strain energy of the plate.)

- The final result for fast fracture is in fact found to be unchanged.

Fast Fracture at Fixed Loads

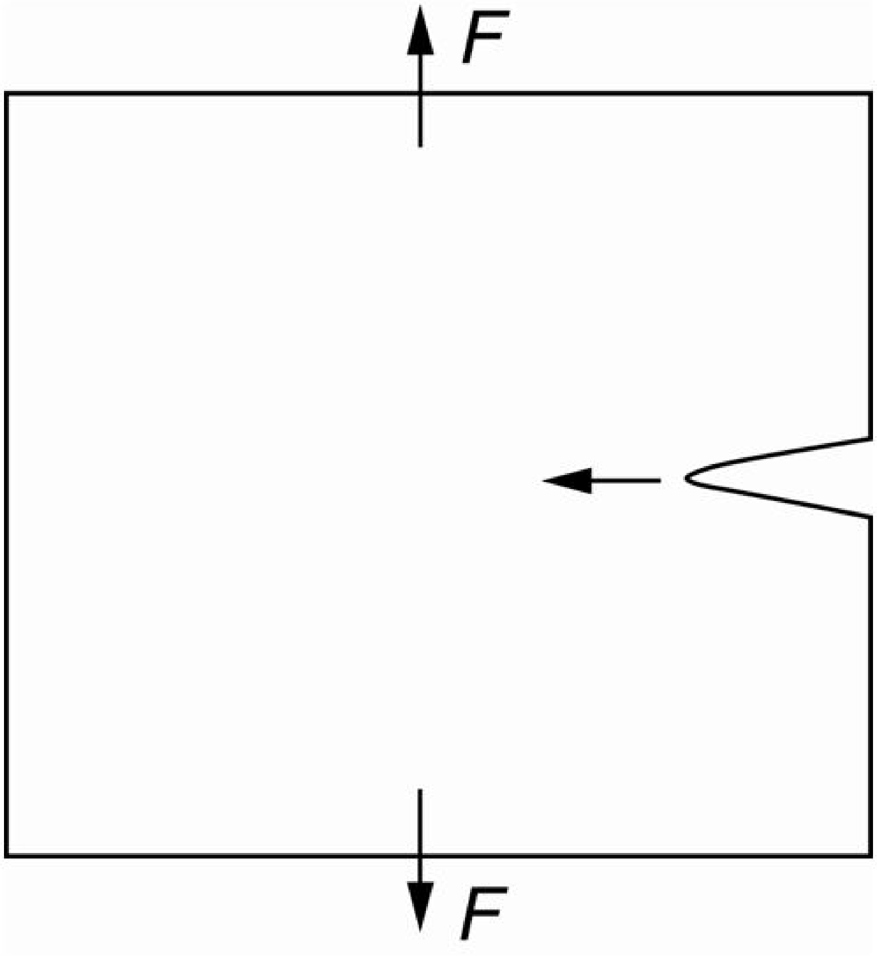


Figure 13.4

Fast fracture of a dead-loaded plate.

The Fast-Fracture Condition

- The condition for the onset of fast fracture, which is general for all engineering structures

$$\sigma\sqrt{\pi a} = \sqrt{EG_c}$$

- The left side of the equation says that fast fracture will occur when, in a material subjected to a stress σ, a crack reaches some critical size a: or, alternatively, when material containing cracks of size a is subjected to some critical stress σ.

The Fast-Fracture Condition

- The right side of the equation depends on material properties only; E is obviously a material constant, and G_c, the energy required to generate a unit area of crack, again must depend only on the basic properties of our material.

- Thus, the important point about the equation is that **the critical combination of stress and crack length at which fast fracture commences is a material constant.**

The Fast-Fracture Condition

- Fast fracture therefore occurs when $K = K_c$

- The terms $\sigma\sqrt{\pi a}$ crops up so frequently in discussing fast fracture that it is usually abbreviated to a single symbol, K, having units MN $m^{-3/2}$; it is called the stress intensity factor.

- Fast fracture therefore occurs when $K = K_c$

- where K_c (= $\sqrt{EG_c}$) is the critical stress intensity factor,

more usually called the fracture toughness.

The Fast-Fracture Condition

To summarize:

G_c = *toughness* (sometimes, critical strain energy release rate); usual units: kJ m^{-2}

$K_c = \sqrt{EG_c}$ = *fracture toughness* (sometimes: critical stress intensity factor); usual units: MN $m^{-3/2}$

$K = \sigma\sqrt{\pi a}$ = stress intensity factor; usual units: MN $m^{-3/2}$

Fast fracture occurs when $K = K_c$.

Data for K_c and G_c

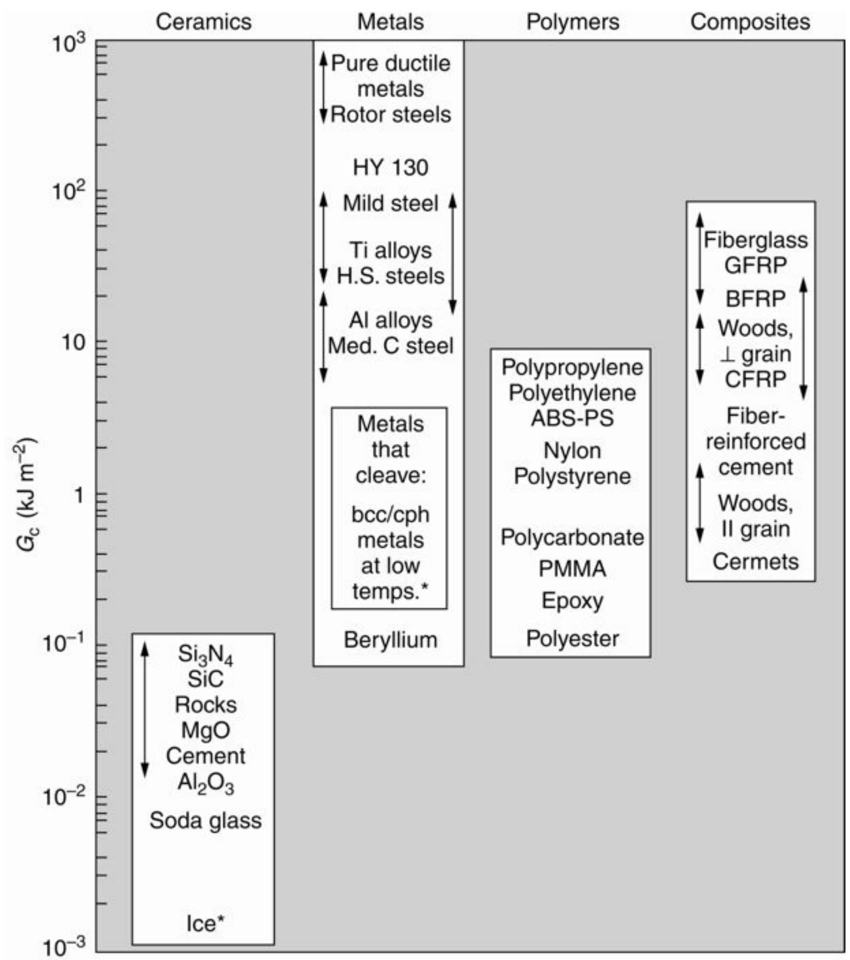


Figure 13.5

Toughness, G_c (values at room temperature unless starred).

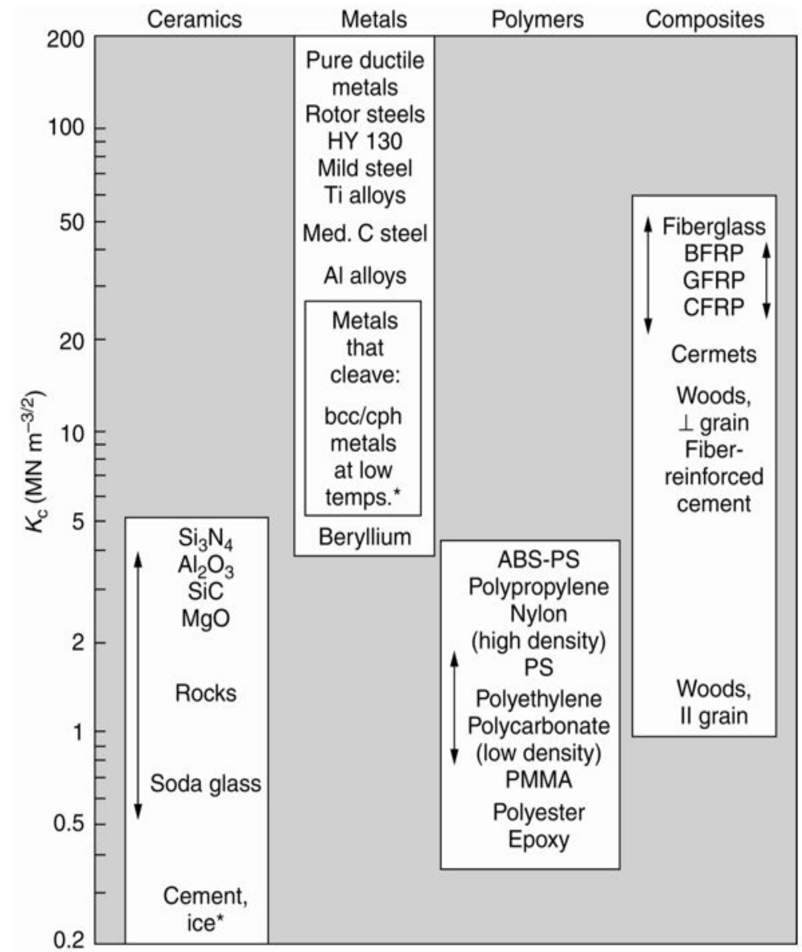


Figure 13.6

Fracture toughness, K_c (values at room temperature unless starred).

A Note on the Stress Intensity Factor,K

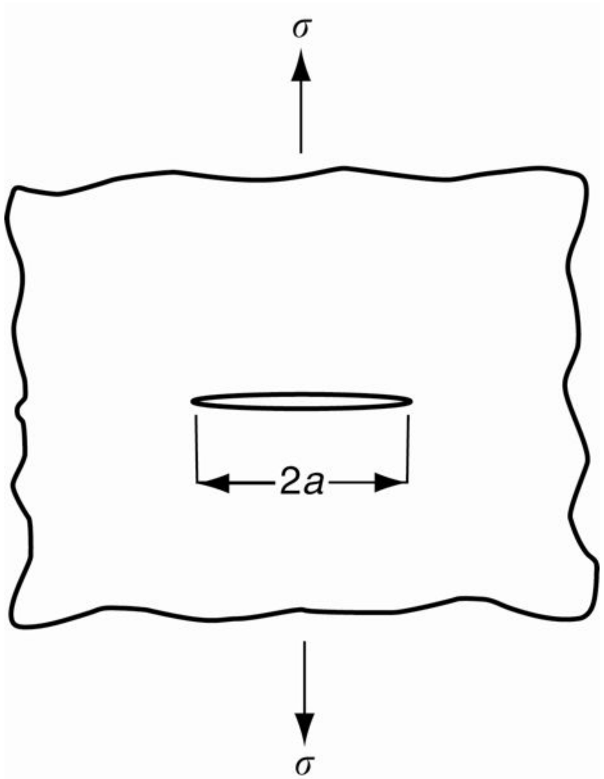
- In general we write:

$$K = Y\sigma\sqrt{\pi a}$$

- where Y is the numerical correction factor.

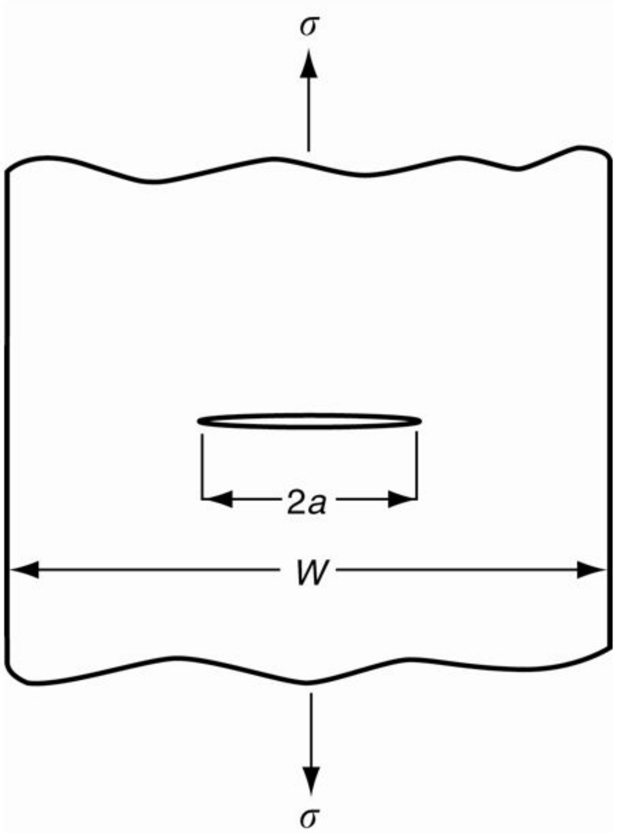
- Values of Y are given at the end of this chapter. However, provided the crack length a is small compared to the width of the plate W, it is usually safe to assume that $Y \approx 1$.

Y values



Page 198 *Case 1*

Y=1



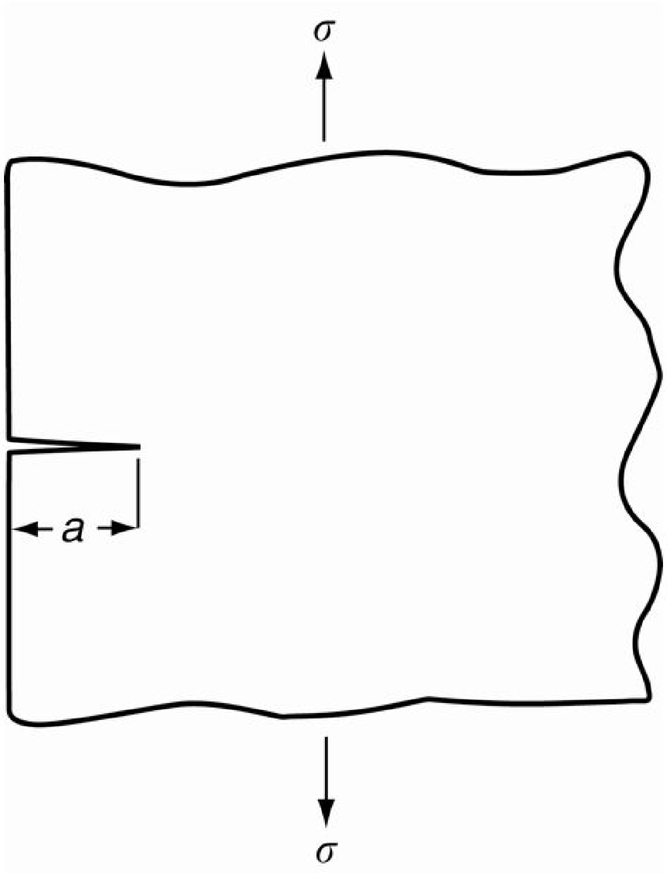
Page 198 *Case 2*

$$Y = \left\{ \cos\left(\frac{\pi a}{W}\right) \right\}^{-1/2}$$

Note: When $W \gg a$, $Y = 1$ (Case 1).

Examples: When $W = 4a$, $Y = 1.20$; when $W = 3a$, $Y = 1.43$.

Y values

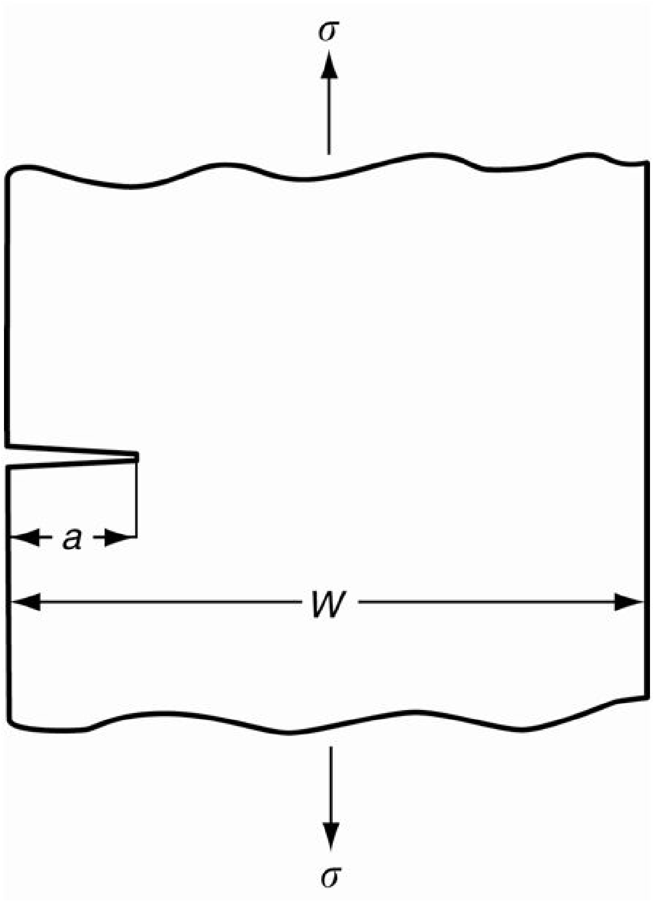


Page 199 *Case 3*

Y=1.12 This situation is like one half of Case 1.

The factor Y=1.12 is added to compensate for introducing a free surface.

Y values



Y	*a/W*
1.12	0 (Case 3)
1.37	0.2
2.11	0.4
2.83	0.5

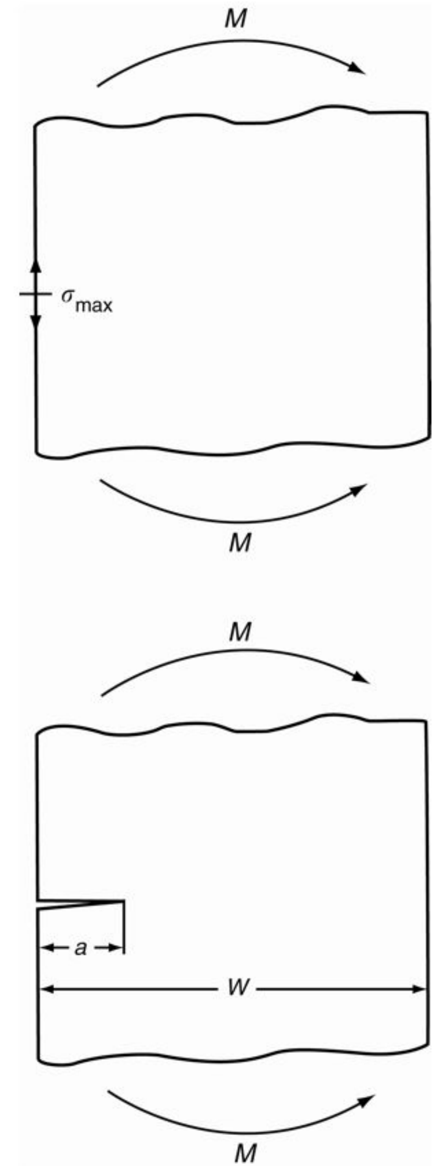
Page 199 *Case 4*

Y values

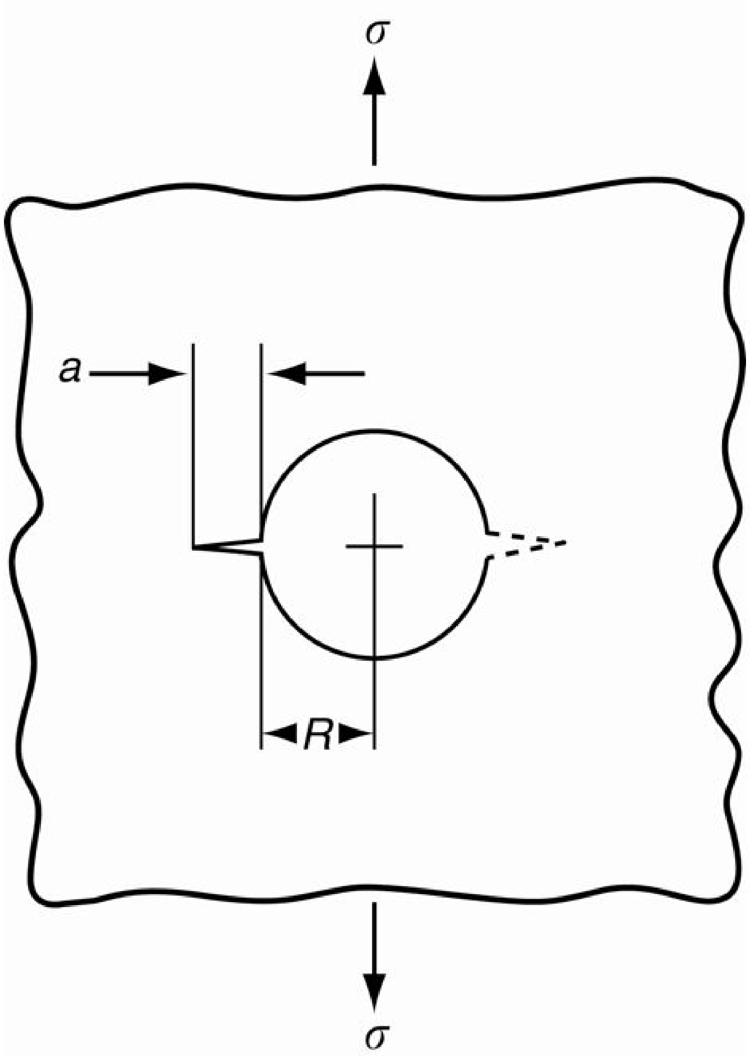
$$K = Y\sigma_{\max}\sqrt{\pi a}$$

Y	*a/W*
1.00	0
1.06	0.2
1.32	0.4
1.62	0.5
2.10	0.6

Page 200 *Case 5*



Y values

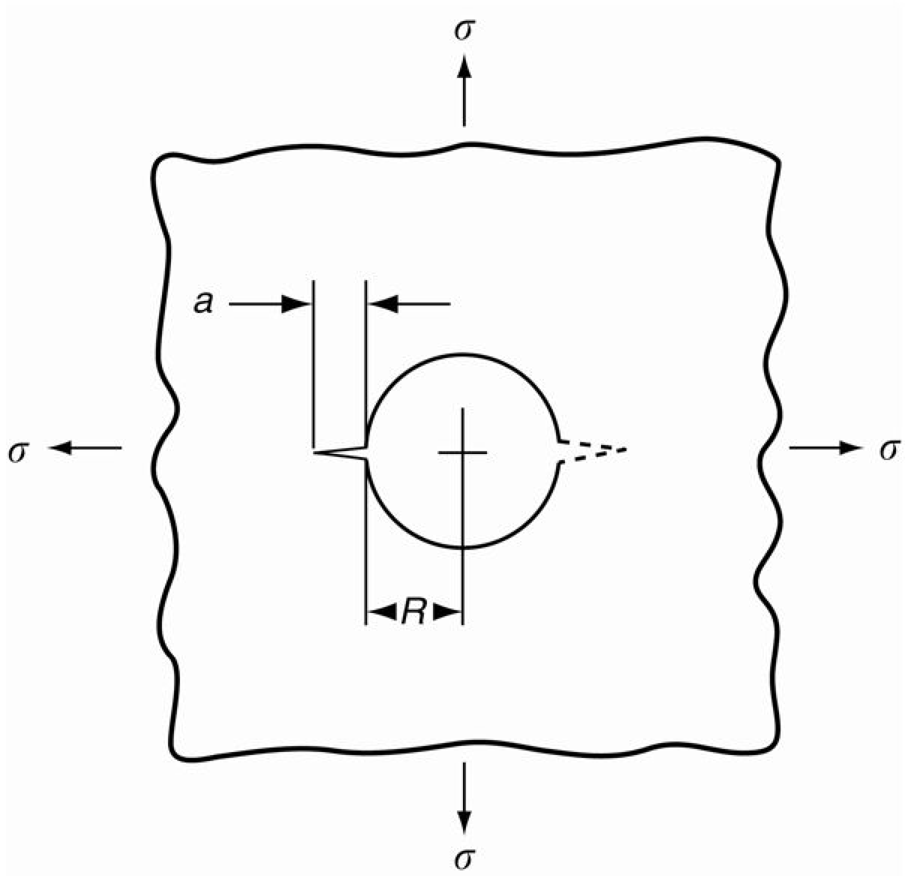


Page 201 *Case 6*

Y (one crack)	*Y* (two cracks)	*a*/*R*
3.36	3.36	0
2.73	2.73	0.1
2.30	2.41	0.2
1.86	1.96	0.4
1.64	1.71	0.6
1.47	1.58	0.8
1.37	1.45	1.0
1.18	1.29	1.5
0.71	1.00 (Case 1)	∞

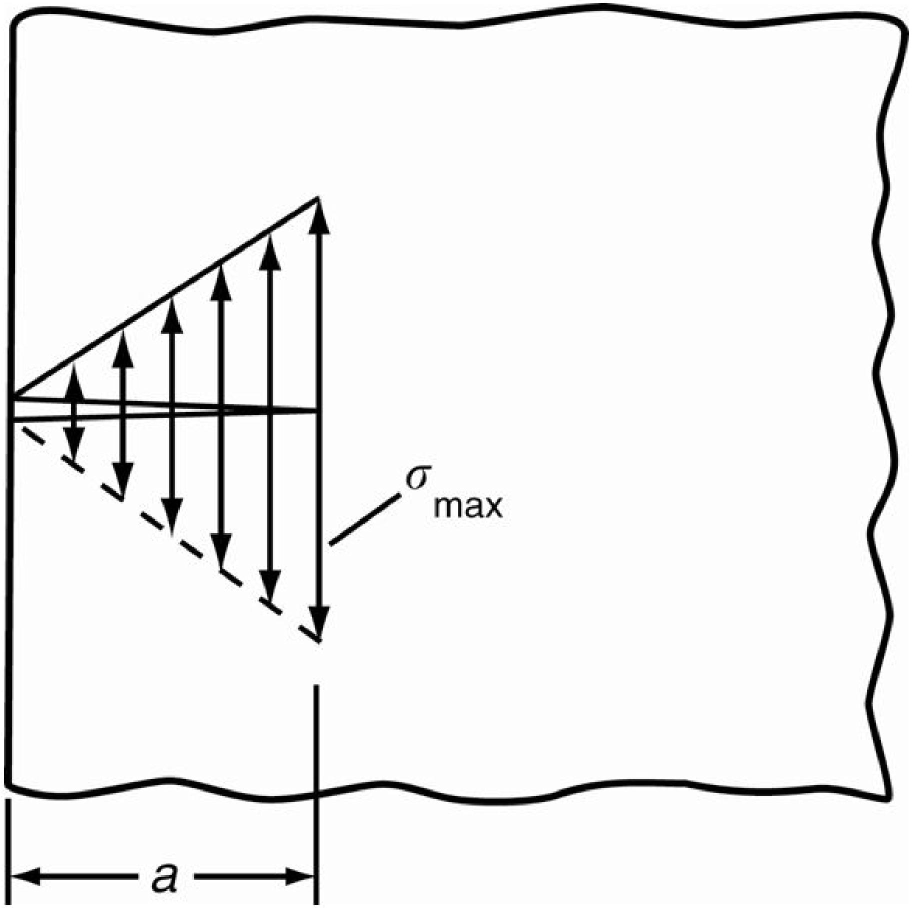
ote that, for a round hole in uniaxial tension, the stress concentration factor 3. For $a/R = 0$ we have a Case 3 crack embedded in a local stress field of 3σ. ıus $Y = 3 \times 1.12 = 3.36$ as shown in the table.

Y values



Page 202 *Case 7*

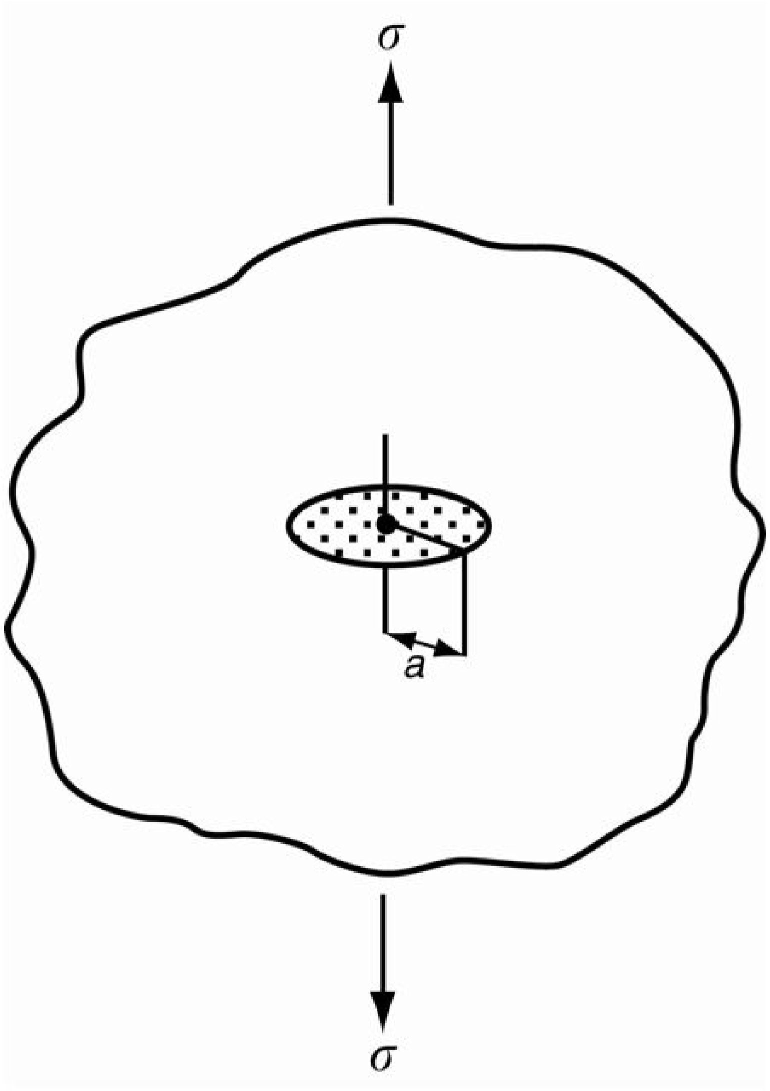
Y values



Page 202 *Case 8*

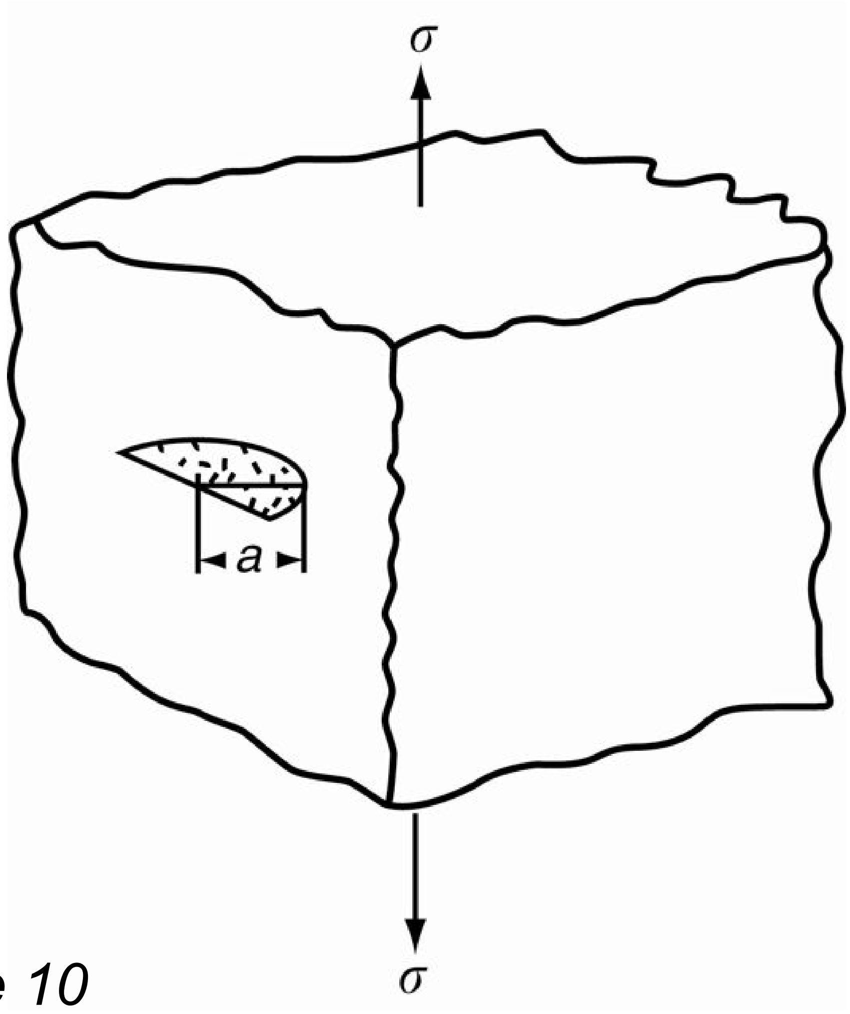
Y values

Y=0.64



Page 203 *Case 9*

Y values



Y= 112x0.64

Page 203 *Case 10*