Deformation, plastic instability and yield-limited design
Material aspect in engineering applications

Aircraft, such as the one shown here, makes use of aluminium alloys. Why?

Can we permanently deform our raw material and turn into useful applications?
Deformation

To compare specimens of different sizes, the load \( F \) is calculated per unit area \( A \).

\[
\sigma = \frac{F}{A}
\]

Deformation caused by external load

\[
\varepsilon = \frac{\Delta L}{L}
\]
A unidirectional force is applied to a specimen in the tensile test by means of the moveable crosshead. The crosshead movement can be performed using screws or a hydraulic mechanism.
Elastic Deformation

1. Initial
2. Small load
3. Unload

Elastic modulus
(Young’s Modulus)

\[ E = \frac{\sigma}{\varepsilon} \]
Plastic deformation is accomplished by means of a process called slip, which involves the motion of dislocations.
Stress-Strain Behaviour

- Yielding is the stress level at which plastic deformation begins.
- For gradual elastic–plastic transition, point of yielding may be determined as the initial departure from linearity of the stress–strain curve, called proportional limit $P$.
- The proof stress can be used to determine yield strength ($\sigma_y$) using the 0.2% strain offset method (some use 0.1% strain offset).
Yield point phenomenon

- In some materials, the elastic–plastic transition is very well defined.
- At upper yield point, plastic deformation is initiated with an apparent decrease in engineering stress.
- **Yield strength** is taken as the average stress that is associated with the lower yield point.
- Not necessary to employ strain offset method.
Yield Strength

- Magnitude of the yield strength is a measure of its resistance to plastic deformation.
- Pure metals are very soft and have high ductility. By alloying (now they become alloys) the strength can be improved.
- Most ceramics have enormous yield strength. Ceramics almost all fracture before they yield.
- Polymers in general have lower yield strengths than metals.
Strength of materials

- Tensile strength $\sigma_{TS}$
- Yield strength $\sigma_y$

Fracture

- Necking

Stress vs. Strain diagram:

- Point 1: Elastic region
- Point 2: Yield point
- Point 3: Ultimate tensile strength
- Point 4: Ductile-to-brittle transition
- Point 5: Fracture
Ductility

Ductility is a measure of the deformation at fracture.

Lower toughness: ceramics

Higher toughness: metals

Stress

Brittle

Ductile

Strain

A

B

C

B'

C'
## Mechanical Properties

Typical mechanical properties of several metals and alloys

<table>
<thead>
<tr>
<th>Metal Alloy</th>
<th>Yield Strength, MPa (ksi)</th>
<th>Tensile Strength, MPa (ksi)</th>
<th>Ductility, %EL [in 50 mm (2 in.)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>35 (5)</td>
<td>90 (13)</td>
<td>40</td>
</tr>
<tr>
<td>Copper</td>
<td>69 (10)</td>
<td>200 (29)</td>
<td>45</td>
</tr>
<tr>
<td>Brass (70Cu–30Zn)</td>
<td>75 (11)</td>
<td>300 (44)</td>
<td>68</td>
</tr>
<tr>
<td>Iron</td>
<td>130 (19)</td>
<td>262 (38)</td>
<td>45</td>
</tr>
<tr>
<td>Nickel</td>
<td>138 (20)</td>
<td>480 (70)</td>
<td>40</td>
</tr>
<tr>
<td>Steel (1020)</td>
<td>180 (26)</td>
<td>380 (55)</td>
<td>25</td>
</tr>
<tr>
<td>Titanium</td>
<td>450 (65)</td>
<td>520 (75)</td>
<td>25</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>565 (82)</td>
<td>655 (95)</td>
<td>35</td>
</tr>
</tbody>
</table>
Hardness test

- A simple non-destructive test on materials or finished components
- Widely used to estimate the yield strength of hard brittle materials

\[ H = \frac{F}{A} = 3\sigma_y \]

The softer the material, the larger the indents
Loading criteria: Tensile / Compressive / Shear

\[
\left( \frac{F}{A} \right) \leq \sigma_y
\]

\[
\left( \frac{F}{A} \right) \leq \frac{\sigma_y}{2}
\]
Example 1: Considering loading criteria

\[ \sigma = \frac{F}{A} \leq \sigma_y \]

\[ \sigma = \frac{50,000}{(0.02 \times 0.05)} = 50 \, \text{MPa} \]

<table>
<thead>
<tr>
<th>Materials</th>
<th>Stainless Steel</th>
<th>Aluminium</th>
<th>Plastic (Polypropylene)</th>
<th>Carbon Fiber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield strength, MPa</td>
<td>215</td>
<td>110</td>
<td>30</td>
<td>200</td>
</tr>
<tr>
<td>Density, g/cm³</td>
<td>7.9</td>
<td>2.7</td>
<td>1.3</td>
<td>1.6</td>
</tr>
</tbody>
</table>
A tensile stress, $\sigma = F/A$, produces a shear stress, $\tau$, on an inclined plane in the stressed material.

Shear stress is

$$\tau = \frac{F \sin \theta}{A/cos\theta} = \sigma \sin \theta \cos \theta$$
Shear stresses in a material have their maximum value on planes at 45° to the tensile axis.
Continuum aspects of plastic flow

In single crystal, it will slip on the nearest lattice plane to the 45° plane on which the dislocation can glide.

In a polycrystalline material, slip occurs on a zigzag path, but the average slip path is at 45° to the tensile axis.
Shear yield strength

- Treating the material as continuum (we just ignore the details of grains in the polycrystalline)
- Shear stress on 45° plane when yielding occur is therefore \( \tau = \theta / 2 \)

We define this as the shear yield strength

\[
k = \tau_y = \frac{\theta_y}{2}
\]
Strengthening mechanism of materials

- Solid solution strengthening
- Precipitation and dispersion strengthening
- **Work (strain) hardening**

Work hardening is the strengthening of a metal by plastic deformation (the increase in stress needed to produce further strain in plastic region)

\[ \sigma_T = K \varepsilon^n \]

- \( n = \text{strain hardening exponent} \) – measures the ability of a metal to harden
## Work hardening: Typical values

### Values for $n$ and $K$ for metals at room temperature

<table>
<thead>
<tr>
<th>Metal</th>
<th>Condition</th>
<th>$n$</th>
<th>$K$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05% C steel</td>
<td>Annealed</td>
<td>0.26</td>
<td>530</td>
</tr>
<tr>
<td>SAE 4340 steel</td>
<td>Annealed</td>
<td>0.15</td>
<td>640</td>
</tr>
<tr>
<td>0.60% C steel</td>
<td>Quenched and tempered 540°C</td>
<td>0.10</td>
<td>1570</td>
</tr>
<tr>
<td>0.60% C steel</td>
<td>Quenched and tempered 700°C</td>
<td>0.19</td>
<td>1230</td>
</tr>
<tr>
<td>Copper</td>
<td>Annealed</td>
<td>0.54</td>
<td>320</td>
</tr>
<tr>
<td>70/30 brass</td>
<td>Annealed</td>
<td>0.49</td>
<td>900</td>
</tr>
<tr>
<td>2024 aluminum alloy</td>
<td>Heat treated - T3</td>
<td>0.17</td>
<td>780</td>
</tr>
</tbody>
</table>
Localized deformation of a ductile material during a tensile test produces a necked region. The micrograph shows necked region in a fractured sample.
Plastic instability

- If one section deforms a little more than the rest
  - Section area is less
  - Stress in it is larger

- If work-hardening has raised the yield strength enough
  - Reduced section can still carry the force and remain stable

- At higher true stress, rate of work-hardening become insufficient
  - Extra stress in the neck cannot be accommodate and becomes instable.
  - The neck grows faster and faster, until final fracture
Critical condition for Necking

\[ A\sigma = F = \text{constant} \]

\[ Ad\sigma + \sigma dA = 0 \]

\[ \frac{d\sigma}{d\varepsilon} = \sigma \]

In terms of nominal stress-strain

\[ \frac{d\sigma_n}{d\varepsilon_n} = 0 \]

On the point of instability, the nominal stress-strain curve is at its maximum (as known experimentally)
Consequence of plastic instability

- Plastic instability is very important in metal processing such as deep drawing sheet metal to form beverage can.
- We must ensure that material and process design are chosen carefully to avoid instability.
Mild steel can be drawn out a lot before it fails by necking.

Aluminium alloy quickly necks when it is drawn out.
Stable necking

When a stress is applied, a neck develops as chains become aligned locally. The strength of the polymer is increased.

In Polythene, the neck never come unstable. It simply grows in length. This is because material work-hardens considerably at high strain.
**Stable necking**

Mild steel (low-carbon steel) often shows both stable and unstable necks.

Luders band (or slip band or yield point elongation) is localized band of plastic deformation in metals experiencing tensile stresses.

This band is undesirable during forming because it leads to a non-uniform surface.
A leaf spring under bending load, leading to deflection ($\delta$)

$$\delta = \frac{Fl^3}{4Ebt^3}$$

The elastic energy stored in the spring, per unit volume, is

$$U^{el} = \frac{F\delta}{2btl} = \frac{F^2l^2}{8E b^2 t^4}$$

Primary function of spring is that of storing elastic energy when required and releasing it again.
Designing leaf spring

Stresses inside a leaf spring

Stress is maximum at the surface, at the midpoint of the beam, because the bending moment is biggest there.

Maximum surface stress is

$$\sigma = \frac{3Fl}{2bt^2} < \sigma_y$$

Spring must not undergo a permanent set during use (it must spring back).
Designing leaf spring

\[ U_{el} = \frac{F^2 l^2}{8E b^2 t^4} \]

\[ \sigma_y \geq \frac{3Fl}{2bt^2} \]

\[ U_{el} = \frac{1}{18} \left( \frac{\sigma_y^2}{E} \right) \]

<table>
<thead>
<tr>
<th>Materials</th>
<th>Brass</th>
<th>Beryllium Copper</th>
<th>Spring Steel</th>
<th>Stainless Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_y^2/E ) (MJ/m(^3))</td>
<td>3.38</td>
<td>15.9</td>
<td>8.45</td>
<td>5.0</td>
</tr>
</tbody>
</table>
Designing thin-walled pressure vessel

Stresses in pressure vessel \[ \sigma = \frac{pr}{2t} \]

Body of an aircraft and casing of solid rocket booster are examples of pressure vessels. Thus it must be as light as possible

Vessel Mass \[ M = 4\pi r^2 t\rho \]

Designing with a safety factor \((S)\) gives

\[ \frac{\sigma_y}{S} \geq \sigma = \frac{2\pi pr^3 \rho}{M} \]

\[ M = S2\pi pr^3 \left(\frac{\rho}{\sigma_y}\right) \]

<table>
<thead>
<tr>
<th>Materials</th>
<th>Reinforced concrete</th>
<th>Aluminium alloys</th>
<th>Mild steel</th>
<th>CFRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho/\sigma_y \times 10^3)</td>
<td>50</td>
<td>6.8</td>
<td>36</td>
<td>7.5</td>
</tr>
</tbody>
</table>
Designing metal rolling

Rolling, forging and sheet drawing are metal forming processes used to reduce section of metal billet by compressive plastic deformation.

The rolling force \( F \) must cause the metal to yield over the length \( l \) and width \( w \).

The torque on each roll

\[
\Gamma = F \frac{l}{2} = \frac{\sigma_y w r (t_1 - t_2)}{2}
\]

If \( x \) is small then

\[
l^2 + (r - x)^2 = r^2
\]

\[
l = \sqrt{r(t_1 - t_2)}
\]
Designing metal rolling

Torque on each roll

\[ \Gamma = F \frac{l}{2} = \frac{\sigma_y w r (t_1 - t_2)}{2} \]

Torque required to drive the rolls increases with

- **Yield strength** of deforming materials
  - So hot-rolling (when yield strength is low) takes less power than cold-rolling
- **Reduction in thickness** \((t_1 - t_2)\)
- **Size of rollers** – small diameter rolls are typically used, which are often backed by larger rolls